EDWARD J. COGAN and
ROBERT Z. NORMAN

Handbook of CALCULUS, DIFFERENCE and DIFFERENTIAL EQUATIONS

GEOMETRIC FORMULAS

Triangle: altitude h, base b:

$$\text{Area} = hb/2.$$

Rectangle: length a, width b:

$$\text{Area} = ab.$$

Parallelogram: altitude h, base b:

$$\text{Area} = hb.$$

Trapezoid: altitude h, two parallel sides a and b:

$$\text{Area} = (a + b)h/2.$$

Circle: radius r:

$$\text{Area} = \pi r^2,$$

$$\text{Circumference} = 2\pi r.$$

Arc of a circle: radius r, central angle θ (in radians):

$$\text{Length} = r\theta.$$

Sector of a circle: radius r, central angle θ (in radians):

$$\text{Area} = r^2\theta/2.$$

Ellipse: semiaxes a and b:

$$\text{Area} = \pi ab,$$

$$\text{Circumference} = 2\pi\sqrt{(a^2 + b^2)/2}.$$

Rectangular parallepiped: length a, width b, height c:

$$\text{Volume} = abc.$$

Pyramid: altitude h, area A:

$$\text{Volume} = \tfrac{1}{3}\, hA.$$

Sphere: radius r:

$$\text{Area of surface} = 4\pi r^2,$$

$$\text{Volume} = 4\pi r^3/3.$$

Right circular cylinder: height h, base radius r:

$$\text{Area of lateral surface} = 2\pi rh,$$

$$\text{Volume} = \pi r^2 h.$$

Right circular cone: height h, base radius r:

$$\text{Area of lateral surface} = \pi r\sqrt{r^2 + h^2},$$

$$\text{Volume} = \pi r^2 h/3.$$

IMPORTANT CONSTANTS

$\pi = 3.14159\ 26536$	$e = 2.71828\ 18284$
$\pi^2 = 9.86960$	$e^2 = 7.38906$
$\sqrt{\pi} = 1.77245$	$1/e = 0.36788$
$\sqrt{2\pi} = 2.50663$	$\ln 10 = 2.30258$
$\sqrt[3]{\pi} = 1.46459$	$\log_{10} e = 0.43429$
$1/\pi = 0.31831$	$\ln 2 = 0.69315$
$1/\sqrt{\pi} = 0.56419$	$\log_2 e = 1.44269$
$1/\sqrt{2\pi} = 0.39896$	$\gamma = 0.57722$

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and
DIFFERENTIAL EQUATIONS

PRENTICE-HALL MATHEMATICS SERIES

Albert A. Bennett, Editor

Handbook of
CALCULUS, DIFFERENCE
and
DIFFERENTIAL EQUATIONS

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PREFACE

THIS HANDBOOK offers to the student of calculus and differential equations a review of properties of often-used functions as well as tables of values of the more common transcendental functions and a table of integrals. Since the intention of the authors is that this book be a self-contained supplement to a textbook in such a course, the expository material is necessarily concise.

The key new feature of this Handbook is its tables for the solution of differential equations. To solve an elementary differential equation, the first step is to classify it so as to determine what technique to apply; the second step is to apply the indicated technique. The Handbook does just this. For each large class of differential equations, there is a classification table which refers to an associated table of techniques for solution. These tables are preceded by a general discussion of differential equations and their solutions. Thus these tables do for elementary differential equations what an integral table does for indefinite integrals. Because of the numerous analogies that exist between the difference and differential calculus, we have included tables for differences, antidifferences, difference equations, and sums.

There is an increasing awareness that the undergraduate mathematics curriculum should include a deeper study of basic concepts and more topics outside the calculus. We feel that the use of a handbook wherever possible to replace extensive memorization of techniques releases sufficient time for the introduction of these topics into the program of the first two years.

The original idea for this Handbook was conceived by the Committee on the Undergraduate Program of the Mathematical Association of America. It was intended as a companion to *Modern*

Mathematical Methods and Models, an experimental text for sophomore students of the behavioral sciences developed for the Committee by the Dartmouth College Writing Group. On writing the first version of the Handbook, it became clear that using the Handbook is a good way to teach all students of differential equations. The present Handbook is extensively rewritten from the earlier version and is now designed as well to supplement a basic calculus and differential equations course for engineering and science students.

We are most grateful to the Committee on the Undergraduate Program for their encouragement to prepare this Handbook, and for the release to us of the original version. We received a number of valuable suggestions from members of the Dartmouth College Writing Group. Special thanks go to our students Sheldon Lippe and Richard Schneider, who painstakingly helped check the Handbook's entries and who thrust upon us the viewpoint of the student, notably indispensable in a work such as this.

E. J. Cogan
R. Z. Norman

CONTENTS

Chapter 1:

FUNCTIONS 1

1.1 Definition: Function, domain, range 1
1.2 Definition: Argument, value 1
1.3 Symbolism 2
1.4 Defining functions explicitly 2
1.5 Definition: Constant function 2
1.6 Definition: Identity function 2
1.7 Composition of functions 3
1.8 Example 3
1.9 Example 3

Chapter 2:

ALGEBRAIC FUNCTIONS 4

2.1 Definition: Polynomial, degree, coefficient 4
2.2 Definition: Linear, quadratic, cubic 4
2.3 Prime polynomials 4
2.4 Properties of polynomials 4
2.5 Definition: Multiplicity 5
2.6 Definition: Rational function 5
2.7 Partial fractions decomposition 5
2.8 Example 6
2.9 Definition: Algebraic operations 6
2.10 Definition: Algebraic expressions 7
2.11 Definition: Algebraic function 7
2.12 Examples 7
2.13 Definition: Inverse function 8
2.14 Example 8

Chapter 3:

FUNCTION—FUNCTION OPERATIONS 9

3.1 Definition: Derivative 9
3.2 Definition: Translate 9

3.3 Definition: Difference 10
3.4 Operator polynomials 10
3.5 Relations between E and Δ 11
3.6 Examples 11

Chapter 4:

FUNCTIONAL EQUATIONS 12

4.1 Definition: Functional equation 12
4.2 Definition: Particular solution 12
4.3 Definition: General solution, general solution form . . 12
4.4 Definition: Differential equation, order 12
4.5 Definition: Difference equation, order 13
4.6 Examples 13
4.7 Definition: Antiderivative, indefinite integral 13
4.8 Definition: Antidifference, indefinite finite integral . . 14
4.9 Arbitrary constants 14
4.10 Methods of defining functions 16
4.11 Examples 17

Chapter 5:

SPECIAL FUNCTIONS DEFINED FOR INTEGERS 18

5.1 Definition: The factorial function 18
5.2 Table: The factorial function 19
5.3 Definition: Binomial coefficients 19
5.4 Table: Binomial coefficients 20
5.5 Definition: Factorial powers 20
5.6 Stirling numbers 21
5.7 Table: Stirling numbers, first kind 22
5.8 Table: Stirling numbers, second kind 23
5.9 Example 23
5.10 Example 23

Chapter 6:

THE TRANSCENDENTAL FUNCTIONS 24

6.1 Definition: Transcendental functions 24
6.2 Trigonometric functions 24
6.3 Definition: Trigonometric functions 25
6.4 Properties of trigonometric functions 25
6.5 Table: Values of trigonometric functions 28
6.6 Inverses of the trigonometric functions 69
6.7 Definition: Inverses of the trigonometric functions . . 69
6.8 The logarithmic and exponential functions 70

6.9 Definition: Natural logarithm function 70
6.10 Definition: The number e 70
6.11 Definition: Logarithm function to the base a 70
6.12 Properties of logarithmic function 70
6.13 Table: Natural logarithms 72
6.14 Table: Common logarithms 74
6.15 Table: Common logarithms of trigonometric functions . 96
6.16 Definition: Exponential function e^x 140
6.17 Definition: Exponential function a^x 140
6.18 Properties of exponential functions 140
6.19 Definition: Hyperbolic functions and their inverses . . 140
6.20 Properties of hyperbolic functions 141
6.21 Table: Values of exponential and hyperbolic functions . 142

Chapter 7:

TWO- AND THREE-DIMENSIONAL GRAPHS 149

7.1 The straight line 149
7.2 The circle 151
7.3 The parabola 152
7.4 The ellipse 153
7.5 The hyperbola 154
7.6 The cubic 156
7.7 The trigonometric functions 158
7.8 The inverse trigonometric functions 160
7.9 The logarithmic functions 161
7.10 The exponential functions 161
7.11 The hyperbolic functions 163
7.12 The inverse hyperbolic functions 164
7.13 Three-dimensional graphs 166
7.14 The conic sections 171

Chapter 8:

TYPES OF FUNCTIONAL EQUATIONS 173

8.1 Derivatives, antiderivatives, and indefinite integrals . . 173
8.2 Definite integrals 173
8.3 Other differential equations 174
8.4 Differences, antidifferences and indefinite finite integrals 174
8.5 Finite sums 175
8.6 Other difference equations 175

Chapter 9:

DERIVATIVES 176

9.1 Table: Properties of derivatives 176
9.2 Table: Derivatives of basic functions 176

15.6 Homogeneous equations, order n 248
15.7 Table: General solution forms, linear homogeneous equations, nth order 249
15.8 Example 249
15.9 Particular solutions: Linear equations with constant coefficients 250
15.10 Table: Particular solutions: Linear equations with constant coefficients 250
15.11 Remark on the extension of Table 15.10 251
15.12 Examples 252

Chapter 16:

OTHER DIFFERENCE EQUATIONS 254

INDEX 257

Handbook of
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and
DIFFERENTIAL EQUATIONS

1

FUNCTIONS

A FUNDAMENTAL notion in mathematics is that of a **function.** In order to specify a function, two sets called the **domain** and the **range** of the function must be given, as well as a **law of correspondence,** or **mapping,** which assigns to each member of its domain exactly one element in its range.

1.1 DEFINITION

A **function** is a law of correspondence which assigns to each element of a set, called the **domain** of the function, exactly one element of another set, called the **range** of the function. The range is restricted by the property that at least one domain element is mapped into each range element.

The domain and range of functions treated in the calculus are generally sets of real numbers. In finite differences, the domain is usually a set of nonnegative integers.

1.2 DEFINITION

Any element of the domain of a function is called an **argument** of the function; any element of its range is called a **value** of the function.

1.3 SYMBOLISM

In this handbook, the letters **f, g,** and **h,** with or without subscripts, are used to stand for unspecified functions. An unspecified value of **f** is denoted by y. The value of the function **f** which corresponds to an unspecified argument x is denoted by $\mathbf{f}(x)$. Similarly, the value of a function **f** for a specified argument, such as the number 2, is denoted by $\mathbf{f}(2)$.

1.4 DEFINING FUNCTIONS EXPLICITLY

A function may be defined explicitly by specifying its domain, its range, and a rule for computing its value for any given argument. However, when only the rule is given, the domain and range are taken to be the greatest sets of real numbers for which the rule applies. Thus the formula

$$y = x^2 - x$$

defines a function whose value for each argument is found by subtracting the argument from its square. The domain of this function is the set of all real numbers; the range can be shown to be the set of all real numbers greater than or equal to $-\frac{1}{4}$.

When a function is defined by an equation in which y occurs alone in the left-hand member, we say that the function is **defined explicitly,** and we use the right-hand member of the equation as a name for the function. Thus we may call the function defined by the above equation $x^2 - x$, even though this combination of symbols actually stands for the value of the function for an unspecified argument.

1.5 DEFINITION

Given a real number a, the function **f** for which $\mathbf{f}(x) = a$ for every x is called a **constant function** and is denoted by **a.**

1.6 DEFINITION

The function **f** for which $\mathbf{f}(x) = x$ for every x is called the **identity function** and is denoted by **x.**

1.7 COMPOSITION OF FUNCTIONS

Given two functions **f** and **g**, we call a function **h** the **composition of f on g** if, for an unspecified argument x, the value $\mathbf{h}(x)$ is found by first evaluating $\mathbf{g}(x)$ and then finding the value of **f** for the argument $\mathbf{g}(x)$. Thus, $\mathbf{h}(x) = \mathbf{f}(\mathbf{g}(x))$ for every argument x. The composition of **f** on **g** will be denoted by $\mathbf{f}(\mathbf{g})$. The domain of $\mathbf{f}(\mathbf{g})$ is the set of all real numbers in the domain of **g** for which $\mathbf{g}(x)$ is in the domain of **f**; the range of $\mathbf{f}(\mathbf{g})$ is the corresponding part of the range of **f**.

1.8 EXAMPLE

Let $\mathbf{f}(x) = x^2 + x$ and $\mathbf{g}(x) = x - 1$. Then

$$\mathbf{f}(\mathbf{g}(x)) = (x - 1)^2 + (x - 1) = x^2 - x$$

and

$$\mathbf{g}(\mathbf{f}(x)) = (x^2 + x) - 1.$$

1.9 EXAMPLE

Let $\mathbf{h}(x) = (x + 2)^3 - (x + 2) + 1/(x + 2)$. We can write $\mathbf{h}(x)$ as a composite of two functions **f** and **g**. By choosing $\mathbf{f}(x) = x^3 - x + (1/x)$ and $\mathbf{g}(x) = x + 2$, then $\mathbf{h} = \mathbf{f}(\mathbf{g})$.

2

ALGEBRAIC FUNCTIONS

2.1 DEFINITION

If n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers, then the function **f** for which $\mathbf{f}(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$ is called a **polynomial.** If $a_0 \neq 0$, then n is called the **degree** of the polynomial. Each number a_i (where $0 \leqq i \leqq n$) is called the **coefficient** of the corresponding x^{n-i}.

2.2 DEFINITION

(1) A polynomial of degree 1 is called a **linear function.**
(2) A polynomial of degree 2 is called a **quadratic function.**
(3) A polynomial of degree 3 is called a **cubic function.**

2.3 PRIME POLYNOMIALS

A polynomial $\mathbf{p}(x)$ of degree n, with real coefficients which cannot be written as the product of two polynomials of degree less than n with real coefficients, is called a **prime polynomial.**

2.4 PROPERTIES OF POLYNOMIALS

(1) The sum, difference, and product of two polynomials are polynomials.

(2) Every polynomial can be expressed as a product of prime factors which are polynomials of degree at most 2. Thus if $\mathbf{p}(x)$ is a polynomial,

$$\mathbf{p}(x) = a(x - b_1)(x - b_2) \cdots (x - b_k)(x^2 + c_1x + d_1) \cdots (x^2 + c_lx + d_l),$$

where the numbers a, b_i, c_i, and d_i are all real.

2.5 DEFINITION

The number of times that a given polynomial occurs as a factor in the product form of (2) of Sec. 2.4 is called the **multiplicity** of the factor.

2.6 DEFINITION

If $\mathbf{f}$ and $\mathbf{g}$ are polynomials, then the function $\mathbf{h}$ for which $\mathbf{h}(x) = \mathbf{f}(x)/\mathbf{g}(x)$ is called a **rational function.**

2.7 PARTIAL FRACTIONS DECOMPOSITION

Let $\mathbf{p}(x)/\mathbf{q}(x)$ be a rational function in which the degree of the polynomial $\mathbf{p}(x)$ is m and that of $\mathbf{q}(x)$ is n. If $m \geqq n$, then $\mathbf{p}(x)/\mathbf{q}(x)$ can be written in the form $\mathbf{s}(x) + \mathbf{t}(x)/\mathbf{q}(x)$, where $\mathbf{s}(x)$ is a polynomial of degree $m - n$ and $\mathbf{t}(x)$ is a polynomial of degree less than n.

Assume that $\mathbf{p}(x)/\mathbf{q}(x)$ is a rational function for which the degree of $\mathbf{p}(x)$ is less than that of $\mathbf{q}(x)$. The denominator can be written as a product

$$[\mathbf{q}_1(x)]^{r_1} \cdot [\mathbf{q}_2(x)]^{r_2} \cdots [\mathbf{q}_k(x)]^{r_k},$$

where the functions $\mathbf{q}_i(x)$ are prime polynomials and the numbers r_i are their respective multiplicities. In this case, $\mathbf{p}(x)/\mathbf{q}(x)$ is expressible as a sum of fractions of the form $\mathbf{P}(x)/\mathbf{Q}(x)$, where $\mathbf{Q}(x)$ is a power less than or equal to r_i of the factor $\mathbf{q}_i(x)$ and $\mathbf{P}(x)$ is a polynomial of degree less than that of $\mathbf{q}_i(x)$.

2.8 EXAMPLE

We shall obtain the partial fractions decomposition of the rational function

$$\frac{x^4 - 2x^3 + 3x^2 - x + 1}{x^3 + 2x^2 + x}.$$

On dividing the numerator by the denominator, a quotient of $x - 4$ is obtained, with a remainder of $10x^2 + 3x + 1$. Hence, the fraction may be written in the form

$$x - 4 + \frac{10x^2 + 3x + 1}{x^3 + 2x^2 + x}.$$

Consider the fractional part of this expression. Its numerator has lower degree than its denominator, and its denominator may be factored as the product $x(x + 1)^2$. According to the procedure in Sec. 2.7, there are numbers A, B, and C such that

$$\frac{10x^2 + 3x + 1}{x^3 + 2x^2 + x} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}.$$

By clearing fractions, we find that

$$\begin{aligned} 10x^2 + 3x + 1 &= A(x + 1)^2 + Bx(x + 1) + Cx \\ &= (A + B)x^2 + (2A + B + C)x + A. \end{aligned}$$

Setting coefficients of like powers of x equal to each other, we obtain the system

$$\begin{aligned} A + B &= 10, \\ 2A + B + C &= 3, \\ A &= 1; \end{aligned}$$

thus $A = 1$, $B = 9$, and $C = -8$. Combining all our results, we have

$$\frac{x^4 - 2x^3 + 3x^2 - x + 1}{x^3 + 2x^2 + x} = x - 4 + \frac{1}{x} + \frac{9}{x + 1} - \frac{8}{(x + 1)^2}.$$

2.9 DEFINITION

The operations of addition, subtraction, multiplication, division, raising to an integral power, and extraction of a root are called **algebraic operations.**

2.10 DEFINITION

If x and y designate unspecified numbers, then an expression which results from the combination of x, y, and a finite number of real numbers by means of a finite number of algebraic operations is called an **algebraic expression** in x and y. $\mathbf{A}(x, y)$ designates an unspecified algebraic expression in x and y.

2.11 DEFINITION

If $\mathbf{A}(x, y)$ is an algebraic expression in x and y, then the equation $\mathbf{A}(x, y) = 0$ may define implicitly one or more functions $\mathbf{f}$ such that $y = \mathbf{f}(x)$. A function $\mathbf{f}$ defined by such an equation is called an **algebraic function.**

2.12 EXAMPLES

The expressions $x^2 + y^2 - 1$, $\dfrac{y\sqrt{x} - x^2\sqrt[3]{y - 2}}{\sqrt{x^2 + y^2}}$, and $xy - 1$ are examples of algebraic expressions. The equations

$$x^2 + y^2 - 1 = 0,$$

$$\frac{y\sqrt{x} - x^2\sqrt[3]{y - 2}}{\sqrt{x^2 + y^2}} = 0,$$

$$xy - 1 = 0$$

each define implicitly at least one function $\mathbf{f}$ for which $y = \mathbf{f}(x)$. The function defined by the third equation may also be defined explicitly by the equation

$$y = \frac{1}{x}.$$

The equation $x^2 + y^2 - 1 = 0$ defines two functions implicitly, namely, those which are defined explicitly by the equations

$$y = \sqrt{1 - x^2}$$

and

$$y = -\sqrt{1 - x^2}.$$

It is not practical to express explicitly the functions determined by the second equation.

2.13 DEFINITION

Let $\mathbf{f}$ be a function, and let y be a number in the range of $\mathbf{f}$. Then the rule which assigns to each y those numbers x in the domain of $\mathbf{f}$ for which $\mathbf{f}(x) = y$ is called the **inverse** of $\mathbf{f}$. If for every y in the range of $\mathbf{f}$ there is exactly one number x in the domain of $\mathbf{f}$ such that $\mathbf{f}(x) = y$, then the inverse of $\mathbf{f}$ is a function called the **inverse function** of $\mathbf{f}$ and denoted $\mathbf{f}^{-1}$. Thus the composition of $\mathbf{f}$ and $\mathbf{f}^{-1}$ is always the identity function: $\mathbf{f}^{-1}(\mathbf{f}(x)) = \mathbf{f}(\mathbf{f}^{-1}(x)) = x$ for every x common to the domains of both $\mathbf{f}$ and $\mathbf{f}^{-1}$.

2.14 EXAMPLE

If $\mathbf{f}(x) = x + 1$ and $\mathbf{g}(x) = x - 1$, then

$$\mathbf{f}(\mathbf{g}(x)) = (x - 1) + 1 = x$$

and

$$\mathbf{g}(\mathbf{f}(x)) = (x + 1) - 1 = x,$$

so that $\mathbf{f}$ and $\mathbf{g}$ are inverses of each other.

3

FUNCTION-FUNCTION OPERATIONS

A NUMBER OF LAWS of correspondence, or mappings, whose domains and ranges are sets of functions rather than sets of numbers arise in the calculus. Those used in this handbook are defined below.

3.1 DEFINITION

Given a function **f**, if there is a function **g** such that

$$\mathbf{g}(x) = \lim_{h \to 0} \frac{\mathbf{f}(x + h) - \mathbf{f}(x)}{h},$$

then **g** is called the **first derivative** of **f** and is denoted by **Df.** The domain of **Df** is that part of the domain of **f** for which values can be assigned to **Df.**

The **second derivative** of **f**, denoted by $\mathbf{D}^2\mathbf{f}$, is the first derivative of **Df**; thus $\mathbf{D}^2\mathbf{f} = \mathbf{D}(\mathbf{Df})$.

The **third derivative** of **f**, denoted by $\mathbf{D}^3\mathbf{f}$, is the first derivative of $\mathbf{D}^2\mathbf{f}$; thus $\mathbf{D}^3\mathbf{f} = \mathbf{D}(\mathbf{D}^2\mathbf{f})$.

In general, for any positive integer n, the **n-th derivative** of **f**, denoted by $\mathbf{D}^n\mathbf{f}$, is the first derivative of $\mathbf{D}^{n-1}\mathbf{f}$; thus $\mathbf{D}^n\mathbf{f} = \mathbf{D}(\mathbf{D}^{n-1}\mathbf{f})$.

The inverse of the derivative operator **D**, called the **indefinite integral**, will be considered in Sec. 4.7.

3.2 DEFINITION

Given a function **f**, the function **g** for which $\mathbf{g}(x) = \mathbf{f}(x + 1)$ is called the **translate** (or **first translate**) of **f** and is denoted **Ef.**

For every x for which $x + 1$ is in the domain of **f**, we find that $\mathbf{Ef}(x) = \mathbf{f}(x + 1)$.

The **second translate** of **f**, denoted by $\mathbf{E}^2\mathbf{f}$, is the translate of **Ef**; thus, $\mathbf{E}^2\mathbf{f} = \mathbf{E}(\mathbf{Ef})$. Note that $\mathbf{E}^2\mathbf{f}(x) = \mathbf{f}(x + 2)$.

The **third translate** of **f**, denoted by $\mathbf{E}^3\mathbf{f}$, is the translate of $\mathbf{E}^2\mathbf{f}$; thus, $\mathbf{E}^3\mathbf{f} = \mathbf{E}(\mathbf{E}^2\mathbf{f})$. Note that $\mathbf{E}^3\mathbf{f}(x) = \mathbf{f}(x + 3)$.

In general, for any positive integer n, the **n-th translate** of **f**, denoted by $\mathbf{E}^n\mathbf{f}$, is the translate of $\mathbf{E}^{n-1}\mathbf{f}$; thus, $\mathbf{E}^n\mathbf{f} = \mathbf{E}(\mathbf{E}^{n-1}\mathbf{f})$. Note that $\mathbf{E}^n\mathbf{f}(x) = \mathbf{f}(x + n)$.

3.3 DEFINITION

Given a function **f**, the function **g** for which $\mathbf{g}(x) = \mathbf{f}(x + 1) - \mathbf{f}(x)$ is called the **difference** (or **first difference**) of **f** and is denoted by $\Delta\mathbf{f}$. For every x for which $x + 1$ is in the domain of $\Delta\mathbf{f}$, we have $\Delta\mathbf{f}(x) = \mathbf{f}(x + 1) - \mathbf{f}(x)$.

The **second difference** of **f**, denoted by $\Delta^2\mathbf{f}$, is the difference of $\Delta\mathbf{f}$; thus, $\Delta^2\mathbf{f} = \Delta(\Delta\mathbf{f})$.

The **third difference** of **f**, denoted by $\Delta^3\mathbf{f}$, is the difference of $\Delta^2\mathbf{f}$; thus, $\Delta^3\mathbf{f} = \Delta(\Delta^2\mathbf{f})$.

In general, for any positive integer n, the **n-th difference** of **f**, denoted by $\Delta^n\mathbf{f}$, is the difference of $\Delta^{n-1}\mathbf{f}$; thus, $\Delta^n\mathbf{f} = \Delta(\Delta^{n-1}\mathbf{f})$.

The inverse of the difference operator Δ, called the **indefinite finite integral**, will be considered in Sec. 4.8.

3.4 OPERATOR POLYNOMIALS

In terms of the operators defined in Secs. 3.1 to 3.3, it is possible to define a large class of operators by using the operations of ordinary arithmetic. For any real number k, we define an operator **k** such that the values of **kf** are k times the corresponding values of **f**. We can then form polynomials in **D**, **E**, or Δ. For example, $\mathbf{E}^2 + 2\mathbf{E} - \mathbf{1}$ may be considered as a single operator which for any function **f** has the property that

$$(\mathbf{E}^2 + 2\mathbf{E} - \mathbf{1})\mathbf{f}(x) = \mathbf{f}(x + 2) + 2\mathbf{f}(x + 1) - \mathbf{f}(x).$$

If we assume the properties

$$(\mathbf{M} + \mathbf{N})\mathbf{f} = \mathbf{M}\mathbf{f} + \mathbf{N}\mathbf{f},$$
$$\mathbf{M}(\mathbf{k}\mathbf{f}) = \mathbf{k}(\mathbf{M}\mathbf{f}),$$

whenever $\mathbf{M}$ and $\mathbf{N}$ are operators, $\mathbf{k}$ is a constant operator, and $\mathbf{f}$ is a function, then the meaning of the operator polynomial is clear.

3.5 RELATIONS BETWEEN E AND Δ

The definitions of $\mathbf{E}$ and Δ establish the following relations between them:

(1) $\Delta = \mathbf{E} - \mathbf{1},$

(2) $\mathbf{E} = \Delta + \mathbf{1},$

(3) $\Delta^2 = (\mathbf{E} - \mathbf{1})^2 = \mathbf{E}^2 - 2\mathbf{E} + \mathbf{1},$

(4) $\mathbf{E}^2 = (\Delta + \mathbf{1})^2 = \Delta^2 + 2\Delta + \mathbf{1}.$

If n is a positive integer, then

(5) $$\begin{aligned}\Delta^n &= (\mathbf{E} - \mathbf{1})^n \\ &= \mathbf{E}^n - \binom{n}{1}\mathbf{E}^{n-1} + \binom{n}{2}\mathbf{E}^{n-2} - \ldots + (-\mathbf{1})^n,\end{aligned}$$

(6) $$\begin{aligned}\mathbf{E}^n &= (\Delta + \mathbf{1})^n \\ &= \Delta^n + \binom{n}{1}\Delta^{n-1} + \binom{n}{2}\Delta^{n-2} + \ldots + \mathbf{1},\end{aligned}$$

where the numbers $\binom{n}{i}$, $1 \leqq i \leqq n$, are the binomial coefficients given in Table 5.4.

3.6 EXAMPLES

Let $\mathbf{f}(x) = x^2 - 2x + 3$; then

$$\begin{aligned}\mathbf{E}\mathbf{f}(x) &= (x+1)^2 - 2(x+1) + 3 = x^2 + 2. \\ \mathbf{E}^2\mathbf{f}(x) &= (x+2)^2 - 2(x+2) + 3 = x^2 + 2x + 3. \\ \Delta\mathbf{f}(x) &= (\mathbf{E} - \mathbf{1})\mathbf{f}(x) \\ &= [(x+1)^2 - 2(x+1) + 3] - [x^2 - 2x + 3] \\ &= 2x - 1. \\ \Delta^2\mathbf{f}(x) &= \Delta(\Delta\mathbf{f})(x) = \Delta(2x - 1) \\ &= [2(x+1) - 1] - [2x - 1] = 2. \\ \Delta^3\mathbf{f}(x) &= \Delta(\Delta^2\mathbf{f})(x) = \Delta(\mathbf{2}) = 2 - 2 = 0.\end{aligned}$$

4

FUNCTIONAL EQUATIONS

4.1 DEFINITION

An equation which sets forth a condition to be satisfied by a function is called a **functional equation.**

4.2 DEFINITION

Any function **f** which satisfies the conditions set forth in a functional equation is called a **particular solution** of the equation.

4.3 DEFINITION

The set of all functions which satisfy a given functional equation is called the **general solution** of the equation. A symbolic representation of the general solution of a functional equation is called a **general solution form** of the equation.

4.4 DEFINITION

A functional equation which involves a finite number of powers of the operator **D** is called a **differential equation.** The highest power of the operator is called the **order** of the differential equation.

4.5 DEFINITION

A functional equation which involves a finite number of powers of the operator $\mathbf{E}$ or the operator Δ is called a **difference equation.** The highest power of the operator involved is called the **order** of the difference equation.

4.6 EXAMPLES

(1) The equation $\mathbf{f}(x_1 + x_2) = \mathbf{f}(x_1) + \mathbf{f}(x_2)$, where x_1 and x_2 are unspecified arguments of $\mathbf{f}$, is an example of a functional equation. The function $\mathbf{f}$ for which $\mathbf{f}(x) = x$ is a particular solution of this equation, and so is the function $\mathbf{f}$ for which $\mathbf{f}(x) = 3x$. In fact, if C is any constant, then Cx is a solution of the equation, and every continuous solution has this form. However, Cx is not the general solution form, since there are discontinuous solutions not of this form.

(2) The equation $\mathbf{D}\mathbf{f}(x) = 1$ is a simple example of a differential equation. A particular solution of this equation is $\mathbf{f}(x) = x$, and another solution is $\mathbf{f}(x) = x + 3$. In fact, if C is an unspecified constant, then $\mathbf{f}(x) = x + C$ is a general solution form for the equation, since every solution can be written in this form for some value of C.

(3) The equation $(\mathbf{E} - 2)\mathbf{f}(x) = 0$ is an example of a difference equation. A particular solution of this equation is $\mathbf{f}(x) = 2^x$, since $\mathbf{E}(2^x) = 2^{x+1} = 2 \cdot 2^x$, and $(\mathbf{E} - 2)2^x = 0$. Another particular solution is $\mathbf{f}(x) = 3 \cdot 2^x$. In fact, if C is an unspecified constant, then $C \cdot 2^x$ is a general solution form of the equation, since every particular solution can be written in this form for some value of C.

4.7 DEFINITION

Given a function $\mathbf{g}$, any particular solution of the equation $\mathbf{D}\mathbf{f} = \mathbf{g}$ is called an **antiderivative** of $\mathbf{g}$. The general solution of the equation $\mathbf{D}\mathbf{f} = \mathbf{g}$ is called the **indefinite integral** of $\mathbf{g}$ and is denoted by $\int \mathbf{g}(x)\, dx$.

4.8 DEFINITION

Given a function $\mathbf{g}$, any particular solution of the equation $\Delta\mathbf{f} = \mathbf{g}$ is called an **antidifference** of $\mathbf{g}$. The general solution of the equation $\Delta\mathbf{f} = \mathbf{g}$ is called the **indefinite finite integral** of $\mathbf{g}$ and is denoted by $\Delta^{-1}\mathbf{g}$.

4.9 ARBITRARY CONSTANTS

In most cases, the representation of a general solution form is made convenient by the fact that the particular solutions of the equation share common features. In particular, the general solution form of a difference or differential equation of order n can be written so that it involves n arbitrary (unspecified) constants, usually designated by $C_1, C_2, \ldots, C_n$. A particular solution of such an equation is obtained by replacing each occurrence of C_1 by the same fixed number, each occurrence of C_2 by the same fixed number, and so on. The assignment of a number to any one of the arbitrary constants is independent of the assignments made to the others.

The general solution of a given equation may be written in many ways. Some ways are simpler than others; some may involve more arbitrary constants than necessary.

Example 1. The general solution of the equation of first order $\mathbf{D}\mathbf{f}(x) = 1$ can obviously be written $\mathbf{f}(x) = x + C$. (We write C instead of C_1 when only one arbitrary constant appears.) However, by integrating both sides of this equation as follows,

$$\int \mathbf{D}\mathbf{f}(x)\,dx = \int dx,$$

we may also obtain the less desirable representations

$$\mathbf{f}(x) + C_1 = x + C_2 \quad \text{or} \quad \mathbf{f}(x) = x + C_2 - C_1.$$

We say that one representation of the general solution of an equation is simpler than another under one or more of the following circumstances:

(1) Fewer different arbitrary constants occur in the first than in the second.

(2) The total number of occurrences of arbitrary constants in the first is smaller than in the second.

(3) Computations with the first representation can be made more conveniently than with the second.

The first step toward satisfying requirement (1) is to reduce the number of different arbitrary constants in a representation of the general solution of an equation to a minimum. This minimum is equal to the order of the equation.

Example 2. If one representation of the general solution of a given equation is $\mathbf{f} = 3C_1x^2 \ln (1 + C_2)$, then a simpler one is $\mathbf{f} = Cx^2$ (see Example 4).

Suppose that an algebraic combination of terms, or a factor of a term, consists entirely of constants at least one of which is arbitrary. The first two conditions for simplicity can be met by giving such a collection of terms or factor of a term a single new name, C_k, provided that C_k does not occur elsewhere in the representation. It will frequently be convenient to make use of some algebraic manipulation before applying this rule. In simplifying an expression which already contains the minimum number of constants, it is often more convenient to express these constants as factors where possible.

Example 3. $\log_a C_1x = \log_a x + \log_a C_1$ may be replaced in any representation by $\log_a x + C$.

Example 4. $3C_1 \ln (1 + C_2)$ may be replaced by C (see Example 2).

Example 5. $C_2 - C_1$ may be replaced by C (see Example 1).

Example 6. Since $3C_1e^{x^2+C_2} = 3C_1e^{C_2}e^{x^2}$, the expression $3C_1e^{x^2+C_2} + 6C_1e^{C_2}$ may be replaced by $Ce^{x^2} + 2C$.

In order to meet the third condition for simplicity, we make use of known identity relations between functions to introduce into a representation of the general solution functions which combine more readily with other parts of the representation than those which already occur. Thus, an expression which is part of a representation, and which involves a nonconstant function, may be replaced by a second expression identically equal to it. Such a replacement often achieves a different distribution of arbitrary constants since it may make possible some simplifications of the type discussed in the preceding paragraph. Of course, simplicity here will depend not so much on the

intrinsic form of the representation as on the particular way in which it is to be used.

Example 7. Since $\mathbf{sin}^2 x = 1 - \mathbf{cos}^2 x$, we have

$$\mathbf{sin}^2 x + C = -\mathbf{cos}^2 x + 1 + C.$$

We may replace $1 + C$ by C_1, and since the letter C is no longer involved, we may use C instead of C_1. Thus $\mathbf{sin}^2 x + C$ may be replaced by $-\mathbf{cos}^2 x + C$.

Example 8. $C_1 \mathbf{sin}\, x + C_2 \mathbf{cos}\, x$ may be replaced by $C_3 \mathbf{sin}\, (x + C_4)$, since

$$C_3 \mathbf{sin}\, (x + C_4) = C_3(\mathbf{sin}\, x \mathbf{cos}\, C_4 + \mathbf{cos}\, x \mathbf{sin}\, C_4)$$

and we may take

$$C_3 = \sqrt{C_1^2 + C_2^2} \quad \text{and} \quad C_4 = \mathbf{arctan}\, \frac{C_2}{C_1}.$$

Again, after replacement, C_1 and C_2 are no longer involved, so that we may relabel the new expression to arrive at $C_1 \mathbf{sin}\, (x + C_2)$.

Example 9. Solve for y in $C_1 x = C_2 \mathbf{e}^{2y+C_3}$.

$$\begin{aligned} C_1 x &= C_2 \mathbf{e}^{C_3} \mathbf{e}^{2y} = C_2 C_4 \mathbf{e}^{2y}, \\ \mathbf{e}^{2y} &= C_5 x, \\ 2y &= \mathbf{ln}\, C_5 x = \mathbf{ln}\, x + C_6, \\ y &= \tfrac{1}{2} \mathbf{ln}\, x + C. \end{aligned}$$

4.10 METHODS OF DEFINING FUNCTIONS

Functions can be defined in essentially two different ways: **explicitly** or **implicitly.**

An explicit definition of a function furnishes a procedure by which a value of the function for a given argument may be obtained. Some algebraic functions may be defined explicitly, namely, those which may be represented by an equation of the form $y = \mathbf{f}(x)$ which indicates the sequences of algebraic operations that must be applied to an argument of the function to yield a value. The method of defining the trigonometric functions is an explicit one, since it furnishes a direct procedure by which a value of such a function is obtained for a given argument.

An implicit definition of a function furnishes no such procedure.

Instead it sets forth conditions on the function. One way of specifying such conditions is by a functional equation. In general, a differential or a difference equation is satisfied by several functions. In order to specify a single function by means of such an equation, additional conditions must be furnished in order that the particular function to be defined may be distinguished from the other functions in the general solution. These additional conditions are called **boundary conditions.**

4.11 EXAMPLES

(1) The general solution of the equation $\mathbf{D}\mathbf{f}(x) = 1/x$ is a class of functions, one of which is the function $\mathbf{ln}\ x$ (the natural logarithm; see Sec. 6.9). This function is completely defined by the given equation and the additional condition that $\mathbf{f}(1) = 0$.

(2) The equation $(\mathbf{E} - \mathbf{1})\mathbf{f}(x) = 0$ has $\mathbf{f}(x) = C\mathbf{2}^x$ as its general solution. This equation along with the condition that $\mathbf{f}(0) = 1$ suffices to define the function $\mathbf{2}^x$ for integral values of x.

5

SPECIAL FUNCTIONS DEFINED FOR INTEGERS

THE DOMAINS of the functions defined in this section are sets of integers. They occur frequently in situations from which difference equations arise.

5.1 DEFINITION

$$x! = x(x-1)(x-2)\cdots 2\cdot 1, \quad \text{if } x > 0,$$
$$0! = 1.$$

Note that $x!$ is defined only when $x \geqq 0$.

Table 5.2 gives values of $x!$ for $0 \leqq x \leqq 35$. For larger values of x, $x!$ can be approximated by Stirling's formula, which states that

$$x! \approx \sqrt{2x\pi}(x/e)^x$$

More specifically,

$$\sqrt{2x\pi}(x/e)^x < x!$$
$$< \sqrt{2x\pi}(x/e)^x[1 + (1/12x - 1)].$$

The factorial function $x!$ is defined by

5.2 TABLE

VALUES OF $x!$, $0 \leqq x \leqq 35$

x	$x!$
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5 040
8	40 320
9	362 880
10	3 628 800
11	39 916 800
12	479 001 600
13	6 227 020 800
14	87 178 291 200
15	1 307 674 368 000
16	20 922 789 888 000
17	355 687 428 096 000
18	6 402 373 705 728 000
19	121 645 100 408 832 000
20	2 432 902 008 176 640 000
21	51 090 942 171 709 440 000
22	1 124 000 727 777 937 680 000
23	25 852 016 738 892 566 840 000
24	620 448 401 733 421 599 360 000
25	15 511 210 043 335 539 984 000 000
26	403 291 461 126 724 039 584 000 000
27	10 888 869 450 421 549 068 768 000 000
28	304 888 344 611 803 373 925 504 000 000
29	8 841 761 993 742 297 843 839 616 000 000
30	265 252 859 812 268 935 315 188 480 000 000
31	8 222 838 654 180 336 994 770 842 880 000 000
32	263 130 836 933 770 783 832 666 972 160 000 000
33	8 683 317 618 814 435 866 478 010 081 280 000 000
34	295 232 799 039 690 819 460 252 342 763 520 000 000
35	10 333 147 966 389 178 681 108 831 996 723 200 000 000

The following formula makes possible the continuation of Table 5.2:

$$(x + 1)! = (x + 1)x!.$$

5.3 DEFINITION

The **binomial coefficients** $\binom{x}{n}$ are defined for $0 \leqq n \leqq x$ by the equation

$$\binom{x}{n} = \frac{x!}{n!(x - n)!}.$$

5.4 TABLE

VALUES OF THE BINOMIAL COEFFICIENTS, $0 \leqq x \leqq 35$

x \ n	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1 287	1 716	1 716	1 287	715	286
14	1	14	91	364	1 001	2 002	3 003	3 432	3 003	2 002	1 001
15	1	15	105	455	1 365	3 003	5 005	6 435	6 435	5 005	3 003
16	1	16	120	560	1 820	4 368	8 008	11 440	12 870	11 440	8 008
17	1	17	136	680	2 380	6 188	12 376	19 448	24 310	24 310	19 448
18	1	18	153	816	3 060	8 568	18 564	31 824	43 758	48 620	43 758
19	1	19	171	969	3 876	11 628	27 132	50 388	75 582	92 378	92 378
20	1	20	190	1 140	4 845	15 504	38 760	77 520	125 970	167 960	184 756
21	1	21	210	1 330	5 985	20 349	54 264	116 280	203 490	293 930	352 716
22	1	22	231	1 540	7 315	26 334	74 613	170 544	319 770	497 420	646 646
23	1	23	253	1 771	8 855	33 649	100 947	245 157	490 314	817 190	1 144 066
24	1	24	276	2 024	10 626	42 504	134 596	346 104	735 471	1 307 504	1 961 256
25	1	25	300	2 300	12 650	53 130	177 100	480 700	1 081 575	2 042 975	3 268 760
26	1	26	325	2 600	14 950	65 780	230 230	657 800	1 562 275	3 124 550	5 311 735
27	1	27	351	2 925	17 550	80 730	296 010	888 030	2 220 075	4 686 825	8 436 285
28	1	28	378	3 276	20 475	98 280	376 740	1 184 040	3 108 105	6 906 900	13 123 110
29	1	29	406	3 654	23 751	118 755	475 020	1 560 780	4 292 145	10 015 005	20 030 010
30	1	30	435	4 060	27 405	142 506	593 775	2 035 800	5 852 925	14 307 150	30 045 015
31	1	31	465	4 495	31 465	169 911	736 281	2 629 575	7 888 725	20 160 075	44 352 165
32	1	32	496	4 960	35 960	201 376	906 192	3 365 856	10 518 300	28 048 800	64 512 240
33	1	33	528	5 456	40 920	237 336	1 107 568	4 272 048	13 884 156	38 567 100	92 561 040
34	1	34	561	5 984	46 376	278 256	1 344 904	5 379 616	18 156 204	32 451 256	131 128 140
35	1	35	595	6 545	52 360	324 632	1 623 160	6 724 520	23 535 820	70 607 460	183 579 396

The following formulas make possible the continuation of Table 5.4:

$$\binom{x}{n} = \binom{x}{x-n},$$

$$\binom{x+1}{n} = \binom{x}{n} + \binom{x}{n-1}.$$

5.5 THE FACTORIAL POWERS. DEFINITION

The **n-th factorial power** of x is given by the formulas

$$x^{(n)} = x(x-1)(x-2)\cdots(x-n+1), \quad n > 0;$$
$$x^{(0)} = 1.$$

11	12	13	14	15	16	17	n / x
							0
							1
							2
							3
							4
							5
							6
							7
							8
							9
							10
1							11
12	1						12
78	13	1					13
364	91	14	1				14
1 365	455	105	15	1			15
4 368	1 820	560	120	16	1		16
12 736	6 188	2 380	680	136	17	1	17
31 824	18 564	8 568	3 060	816	153	18	18
75 582	50 388	27 132	11 628	3 876	969	171	19
167 960	125 970	77 520	38 760	15 504	4 845	1 140	20
352 716	293 930	203 490	116 280	54 264	20 349	5 985	21
705 432	646 646	497 420	319 770	170 544	74 613	26 334	22
1 352 078	1 352 078	1 144 066	817 190	490 314	245 157	100 947	23
2 496 144	2 704 156	2 496 144	1 961 256	1 307 504	735 471	346 104	24
4 457 400	5 200 300	5 200 300	4 457 400	3 268 760	2 042 975	1 081 575	25
7 726 160	9 657 700	10 400 600	9 657 700	7 726 160	5 311 735	3 124 550	26
13 037 895	17 383 860	20 058 300	20 058 300	17 383 860	13 037 895	8 436 285	27
21 474 180	30 421 755	37 442 160	40 116 600	37 442 160	30 421 755	21 474 180	28
34 597 290	51 895 935	67 863 915	77 558 760	77 558 760	67 863 915	51 895 935	29
54 627 300	86 493 225	119 759 850	145 422 675	155 117 520	145 422 675	119 759 850	30
84 672 315	141 120 525	206 253 075	265 182 525	300 540 195	300 540 195	265 182 525	31
129 024 480	225 792 840	347 373 600	471 435 600	565 722 720	601 080 390	565 722 720	32
193 536 720	354 817 320	573 166 440	818 809 200	1 037 158 320	1 166 803 110	1 166 803 110	33
286 097 760	548 354 040	927 983 760	1 391 975 640	1 855 967 520	2 203 961 430	2 333 606 220	34
417 225 900	834 451 800	1 476 337 800	2 319 959 400	3 247 943 160	4 059 928 950	4 537 567 650	35

5.6 STIRLING NUMBERS

The factorial powers play a role in finite differences analogous to that played by powers in the differential calculus. That is,

$$\Delta x^{(n)} = n x^{(n-1)}. \tag{1}$$

(See Table 14.2.)

Thus, in order to difference a polynomial, it is convenient to write the polynomial in terms of factorial powers instead of ordinary powers. The relationships between ordinary powers and factorial powers are:

(2) For every nonnegative integer n the function $x^{(n)}$ is a polynomial of degree n in x; that is, there are numbers $s_{n1}, s_{n2}, \ldots, s_{nn}$ such that

$$x^{(n)} = s_{n1}x^1 + s_{n2}x^2 + \ldots + s_{nn}x^n.$$

The numbers $s_{n1}, s_{n2}, \ldots, s_{nn}$ are called **Stirling numbers of the first kind** and are listed in Table 5.7.

(3) For every nonnegative integer n the function x^n is a linear combination of factorial powers of x not higher than the n-th; that is, there are numbers $t_{n1}, t_{n2}, \ldots, t_{nn}$ such that

$$x^n = t_{n1}x^{(1)} + t_{n2}x^{(2)} + \ldots + t_{nn}x^{(n)}.$$

The numbers $t_{n1}, t_{n2}, \ldots, t_{nn}$ are called **Stirling numbers of the second kind** and are listed in Table 5.8.

These three considerations enable us to find differences of polynomials in the following way: First, apply (3) to express each power of x in terms of factorial powers; second, find the required differences of these powers by means of (1); third, by means of (2), write each resulting factorial power as a polynomial.

5.7 TABLE

STIRLING NUMBERS OF THE FIRST KIND

n	s_{n1}	s_{n2}	s_{n3}	s_{n4}	s_{n5}	s_{n6}	s_{n7}	s_{n8}
1	1	0	0	0	0	0	0	0
2	−1	1	0	0	0	0	0	0
3	2	−3	1	0	0	0	0	0
4	−6	11	−6	1	0	0	0	0
5	24	−50	35	−10	1	0	0	0
6	−120	274	−225	85	−15	1	0	0
7	720	−1 764	1 624	−735	175	−21	1	0
8	−5 040	13 068	−13 132	6 769	−1 960	322	−28	1

To continue this table to find s_{ni} for $n > 8$, define $s_{n0} = 0$ for all n, and use the formula

$$s_{ni} = s_{n-1,i-1} - (n-1)s_{n-1,i}, \quad i = 1, 2, \ldots, n.$$

5.8 TABLE

STIRLING NUMBERS OF THE SECOND KIND

n	t_{n1}	t_{n2}	t_{n3}	t_{n4}	t_{n5}	t_{n6}	t_{n7}	t_{n8}
1	1	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0
3	1	3	1	0	0	0	0	0
4	1	7	6	1	0	0	0	0
5	1	15	25	10	1	0	0	0
6	1	31	90	65	15	1	0	0
7	1	63	301	350	140	21	1	0
8	1	127	966	1 701	1 050	266	28	1

To continue this table to find t_{ni} for $n > 8$, define $t_{n0} = 0$ for all n, and use the formula

$$t_{ni} = it_{n-1,i} + t_{n-1,i-1}, \quad i = 1, 2, \ldots, n.$$

5.9 EXAMPLE

Express $x^3 + 2x - 1$, using factorial powers. We find from Table 5.8 that

$$x^3 = x^{(1)} + 3x^{(2)} + x^{(3)},$$
$$2x = 2x^{(1)},$$
$$-1 = -1.$$

Then, $x^3 + 2x - 1 = x^{(3)} + 3x^{(2)} + 3x^{(1)} - 1.$

5.10 EXAMPLE

Express $x^{(3)} - 2x^{(2)} + 4x^{(1)}$ as a polynomial.

We find from Table 5.7 that

$$x^{(3)} = 2x - 3x^2 + x^3,$$
$$-2x^{(2)} = 2x - 2x^2,$$
$$4x^{(1)} = 4x.$$

Thus, $$x^{(3)} - 2x^{(2)} + 4x^{(1)} = x^3 - 5x^2 + 8x.$$

6

TRANSCENDENTAL FUNCTIONS

6.1 DEFINITION

Any function which is not an algebraic function is called a **transcendental** function.

6.2 TRIGONOMETRIC FUNCTIONS

Consider a circle whose center is at the origin of a cartesian coordinate system and whose radius is one unit. Label with the letter O the point $(1, 0)$ at which the circle cuts the positive side of the horizontal axis. Let x be an unspecified real number. We assign to the number x a point X on the circle as follows:

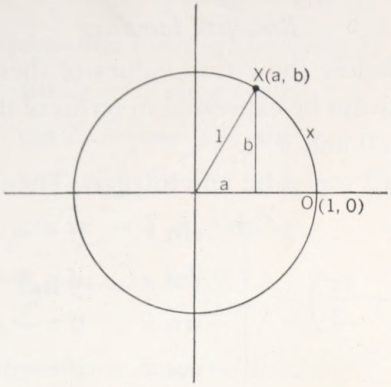
(1) If $x > 0$, the point X is x units from O, the distance being measured along the circumference of the circle in a counterclockwise sense.

(2) If $x < 0$, the point X is x units from O, the distance being measured along the circumference of the circle in a clockwise sense.

(3) If $x = 0$, then X is O.

This assigns a point on the circle to each real number x, but several real numbers are assigned to the same point X on the circle.

For each number x, let the coordinates of the point X be (a, b). Since the numbers a and b are determined as soon as x is specified, the above procedure determines functions $\mathbf{f}$ and $\mathbf{g}$ such that $\mathbf{f}(x) = a$ and $\mathbf{g}(x) = b$.



6.3 DEFINITIONS

(See the last paragraph of Sec. 6.2.)

(1) $$\sin x = b.$$

(2) $$\cos x = a.$$

(3) $$\tan x = \frac{\sin x}{\cos x}.$$

(4) $$\cot x = \frac{1}{\tan x}.$$

(5) $$\sec x = \frac{1}{\cos x}.$$

(6) $$\csc x = \frac{1}{\sin x}.$$

6.4 PROPERTIES OF TRIGONOMETRIC FUNCTIONS

Pythagorean Identities

(1) $$\sin^2 x + \cos^2 x = 1.$$

(2) $$\tan^2 x + 1 = \sec^2 x.$$

(3) $$\cot^2 x + 1 = \csc^2 x.$$

Negative Argument Identities

$$\sin(-x) = -\sin x. \tag{4}$$

$$\cos(-x) = \cos x. \tag{5}$$

$$\tan(-x) = -\tan x. \tag{6}$$

Reduction Identities

The identities below show that values of these functions for any real number can always be expressed in terms of their values for some arc length between 0 and $\pi/2$.

Let $0 \leqq x \leqq \pi/2$ and k be any integer. Then

$$\sin\left(x + \frac{n\pi}{2}\right) = \begin{cases} \sin x & \text{if } n = 4k, \\ \cos x & \text{if } n = 4k + 1, \\ -\sin x & \text{if } n = 4k + 2, \\ -\cos x & \text{if } n = 4k + 3. \end{cases} \tag{7}$$

$$\cos\left(x + \frac{n\pi}{2}\right) = \begin{cases} \cos x & \text{if } n = 4k, \\ -\sin x & \text{if } n = 4k + 1, \\ -\cos x & \text{if } n = 4k + 2, \\ \sin x & \text{if } n = 4k + 3. \end{cases} \tag{8}$$

$$\tan\left(x + \frac{n\pi}{2}\right) = \begin{cases} \tan x & \text{if } n = 4k, \\ -\cot x & \text{if } n = 4k + 1, \\ \tan x & \text{if } n = 4k + 2, \\ -\cot x & \text{if } n = 4k + 3. \end{cases} \tag{9}$$

Let x_1 and x_2 be any real numbers.

Argument Sum Identities

$$\sin(x_1 + x_2) = \sin x_1 \cos x_2 + \cos x_1 \sin x_2. \tag{10}$$

$$\cos(x_1 + x_2) = \cos x_1 \cos x_2 - \sin x_1 \sin x_2. \tag{11}$$

$$\tan(x_1 + x_2) = \frac{\tan x_1 + \tan x_2}{1 - \tan x_1 \tan x_2}. \tag{12}$$

Argument Difference Identities

$$\sin(x_1 - x_2) = \sin x_1 \cos x_2 - \cos x_1 \sin x_2. \tag{13}$$

$$\cos(x_1 - x_2) = \cos x_1 \cos x_2 + \sin x_1 \sin x_2. \tag{14}$$

$$\tan(x_1 - x_2) = \frac{\tan x_1 - \tan x_2}{1 + \tan x_1 \tan x_2}. \tag{15}$$

Product to Sum Identities

(16) $$\sin x_1 \cos x_2 = \tfrac{1}{2}[\sin (x_1 + x_2) + \sin (x_1 - x_2)].$$

(17) $$\cos x_1 \cos x_2 = \tfrac{1}{2}[\cos (x_1 + x_2) + \cos (x_1 - x_2)].$$

(18) $$\sin x_1 \sin x_2 = \tfrac{1}{2}[\cos (x_1 - x_2) - \cos (x_1 + x_2)].$$

Trigonometric Functions Involving $2x$

(19) $$\sin 2x = 2 \sin x \cos x.$$

(20) $$\cos 2x = \cos^2 x - \sin^2 x,$$

(20a) $$\cos 2x = 2 \cos^2 x - 1,$$

(20b) $$\cos 2x = 1 - 2 \sin^2 x.$$

(21) $$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

Trigonometric Functions Involving $\frac{x}{2}$

(22) $$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}.$$

(23) $$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}.$$

6.5 TABLES OF VALUES OF THE TRIGONOMETRIC FUNCTIONS

Table 6.5(a)

NATURAL TRIGONOMETRIC FUNCTIONS

x	Sin	Tan	Ctn	Cos
.00	.0000	.0000	∞	1.0000
.01	.0100	.0100	99.997	1.0000
.02	.0200	.0200	49.993	.9998
.03	.0300	.0300	33.323	.9996
.04	.0400	.0400	24.987	.9992
.05	.0500	.0500	19.983	.9988
.06	.0600	.0601	16.647	.9982
.07	.0699	.0701	14.262	.9976
.08	.0799	.0802	12.473	.9968
.09	.0899	.0902	11.081	.9960
.10	.0998	.1003	9.967	.9950
.11	.1098	.1104	9.054	.9940
.12	.1197	.1206	8.293	.9928
.13	.1296	.1307	7.649	.9916
.14	.1395	.1409	7.096	.9902
.15	.1494	.1511	6.617	.9888
.16	.1593	.1614	6.197	.9872
.17	.1692	.1717	5.826	.9856
.18	.1790	.1820	5.495	.9838
.19	.1889	.1923	5.200	.9820
.20	.1987	.2027	4.933	.9801
.21	.2085	.2131	4.692	.9780
.22	.2182	.2236	4.472	.9759
.23	.2280	.2341	4.271	.9737
.24	.2377	.2447	4.086	.9713
.25	.2474	.2553	3.916	.9689
.26	.2571	.2660	3.759	.9664
.27	.2667	.2768	3.613	.9638
.28	.2764	.2876	3.478	.9611
.29	.2860	.2984	3.351	.9582
.30	.2955	.3093	3.233	.9553
.31	.3051	.3203	3.122	.9523
.32	.3146	.3314	3.018	.9492
.33	.3240	.3425	2.920	.9460
.34	.3335	.3537	2.827	.9428
.35	.3429	.3650	2.740	.9394
.36	.3523	.3764	2.657	.9359
.37	.3616	.3879	2.578	.9323
.38	.3709	.3994	2.504	.9287
.39	.3802	.4111	2.433	.9249
.40	.3894	.4228	2.365	.9211
.41	.3986	.4346	2.301	.9171
.42	.4078	.4466	2.239	.9131
.43	.4169	.4586	2.180	.9090
.44	.4259	.4708	2.124	.9048
.45	.4350	.4831	2.070	.9004
.46	.4439	.4954	2.018	.8961
.47	.4529	.5080	1.969	.8916
.48	.4618	.5206	1.921	.8870
.49	.4706	.5334	1.875	.8823
.50	.4794	.5463	1.830	.8776

x	Sin	Tan	Ctn	Cos
.50	.4794	.5463	1.830	.8776
.51	.4882	.5594	1.788	.8727
.52	.4969	.5726	1.747	.8678
.53	.5055	.5859	1.707	.8628
.54	.5141	.5994	1.668	.8577
.55	.5227	.6131	1.631	.8525
.56	.5312	.6269	1.595	.8473
.57	.5396	.6410	1.560	.8419
.58	.5480	.6552	1.526	.8365
.59	.5564	.6696	1.494	.8309
.60	.5646	.6841	1.462	.8253
.61	.5729	.6989	1.431	.8196
.62	.5810	.7139	1.401	.8139
.63	.5891	.7291	1.372	.8080
.64	.5972	.7445	1.343	.8021
.65	.6052	.7602	1.315	.7961
.66	.6131	.7761	1.288	.7900
.67	.6210	.7923	1.262	.7838
.68	.6288	.8087	1.237	.7776
.69	.6365	.8253	1.212	.7712
.70	.6442	.8423	1.187	.7648
.71	.6518	.8595	1.163	.7584
.72	.6594	.8771	1.140	.7518
.73	.6669	.8949	1.117	.7452
.74	.6743	.9131	1.095	.7385
.75	.6816	.9316	1.073	.7317
.76	.6889	.9505	1.052	.7248
.77	.6961	.9697	1.031	.7179
.78	.7033	.9893	1.011	.7109
.79	.7104	1.009	.9908	.7038
.80	.7174	1.030	.9712	.6967
.81	.7243	1.050	.9520	.6895
.82	.7311	1.072	.9331	.6822
.83	.7379	1.093	.9146	.6749
.84	.7446	1.116	.8964	.6675
.85	.7513	1.138	.8785	.6600
.86	.7578	1.162	.8609	.6524
.87	.7643	1.185	.8437	.6448
.88	.7707	1.210	.8267	.6372
.89	.7771	1.235	.8100	.6294
.90	.7833	1.260	.7936	.6216
.91	.7895	1.286	.7774	.6137
.92	.7956	1.313	.7615	.6058
.93	.8016	1.341	.7458	.5978
.94	.8076	1.369	.7303	.5898
.95	.8134	1.398	.7151	.5817
.96	.8192	1.428	.7001	.5735
.97	.8249	1.459	.6853	.5653
.98	.8305	1.491	.6707	.5570
.99	.8360	1.524	.6563	.5487
1.00	.8415	1.557	.6421	.5403

Table 6.5(a) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS

x	Sin	Tan	Ctn	Cos
1.00	.8415	1.557	.6421	.5403
1.01	.8468	1.592	.6281	.5319
1.02	.8521	1.628	.6142	.5234
1.03	.8573	1.665	.6005	.5148
1.04	.8624	1.704	.5870	.5062
1.05	.8674	1.743	.5736	.4976
1.06	.8724	1.784	.5604	.4889
1.07	.8772	1.827	.5473	.4801
1.08	.8820	1.871	.5344	.4713
1.09	.8866	1.917	.5216	.4625
1.10	.8912	1.965	.5090	.4536
1.11	.8957	2.014	.4964	.4447
1.12	.9001	2.066	.4840	.4357
1.13	.9044	2.120	.4718	.4267
1.14	.9086	2.176	.4596	.4176
1.15	.9128	2.234	.4475	.4085
1.16	.9168	2.296	.4356	.3993
1.17	.9208	2.360	.4237	.3902
1.18	.9246	2.427	.4120	.3809
1.19	.9284	2.498	.4003	.3717
1.20	.9320	2.572	.3888	.3624
1.21	.9356	2.650	.3773	.3530
1.22	.9391	2.733	.3659	.3436
1.23	.9425	2.820	.3546	.3342
1.24	.9458	2.912	.3434	.3248
1.25	.9490	3.010	.3323	.3153
1.26	.9521	3.113	.3212	.3058
1.27	.9551	3.224	.3102	.2963
1.28	.9580	3.341	.2993	.2867
1.29	.9608	3.467	.2884	.2771
1.30	.9636	3.602	.2776	.2675

x	Sin	Tan	Ctn	Cos
1.30	.9636	3.602	.2776	.2675
1.31	.9662	3.747	.2669	.2579
1.32	.9687	3.903	.2562	.2482
1.33	.9711	4.072	.2456	.2385
1.34	.9735	4.256	.2350	.2288
1.35	.9757	4.455	.2245	.2190
1.36	.9779	4.673	.2140	.2092
1.37	.9799	4.913	.2035	.1994
1.38	.9819	5.177	.1931	.1896
1.39	.9837	5.471	.1828	.1798
1.40	.9854	5.798	.1725	.1700
1.41	.9871	6.165	.1622	.1601
1.42	.9887	6.581	.1519	.1502
1.43	.9901	7.055	.1417	.1403
1.44	.9915	7.602	.1315	.1304
1.45	.9927	8.238	.1214	.1205
1.46	.9939	8.989	.1113	.1106
1.47	.9949	9.887	.1011	.1006
1.48	.9959	10.983	.0910	.0907
1.49	.9967	12.350	.0810	.0807
1.50	.9975	14.101	.0709	.0707
1.51	.9982	16.428	.0609	.0608
1.52	.9987	19.670	.0508	.0508
1.53	.9992	24.498	.0408	.0408
1.54	.9995	32.461	.0308	.0308
1.55	.9998	48.078	.0208	.0208
1.56	.9999	92.620	.0108	.0108
1.57	1.0000	1255.8	.0008	.0008
1.58	1.0000	−108.65	−.0092	−.0092

In certain applications, it is convenient to allow trigonometric functions to have angular measurements as arguments. The natural measure of an angle, called its radian measure, is the arclength it subtends from the center of a unit circle. Angles also may be measured in degrees. The entire circumference of the unit circle is subtended by a central angle of 2π radians, or 360°. Each degree is subdivided into 60 minutes; each minute is subdivided into 60 seconds.

Table 6.5(b)

RADIANS TO DEGREES, MINUTES AND SECONDS

Rad		Rad		Rad		Rad		Rad	
1	57°17′44″.8	.1	5°43′46″.5	.01	0°34′22″.6	.001	0° 3′26″.3	.0001	0°0′20″.6
2	114°35′29″.6	.2	11°27′33″.0	.02	1° 8′45″.3	.002	0° 6′52″.5	.0002	0°0′41″.3
3	171°53′14″.4	.3	17°11′19″.4	.03	1°43′07″.9	.003	0°10′18″.8	.0003	0°1′01″.9
4	229°10′59″.2	.4	22°55′05″.9	.04	2°17′30″.6	.004	0°13′45″.1	.0004	0°1′22″.5
5	286°28′44″.0	.5	28°38′52″.4	.05	2°51′53″.2	.005	0°17′11″.3	.0005	0°1′43″.1
6	343°46′28″.8	.6	34°22′38″.9	.06	3°26′15″.9	.006	0°20′37″.6	.0006	0°2′03″.8
7	401° 4′13″.6	.7	40° 6′25″.4	.07	4° 0′38″.5	.007	0°24′03″.9	.0007	0°2′24″.4
8	458°21′58″.4	.8	45°50′11″.8	.08	4°35′01″.2	.008	0°27′30″.1	.0008	0°2′45″.0
9	515°39′43″.3	.9	51°33′58″.3	.09	5° 9′23″.8	.009	0°30′56″.4	.0009	0°3′05″.6

DEGREES, MINUTES AND SECONDS TO RADIAN

Deg.	Rad	Min	Rad	Sec	Rad
1	0.01745 33	1	0.00029 09	1	0.00000 48
2	0.03490 66	2	0.00058 18	2	0.00000 97
3	0.05235 99	3	0.00087 27	3	0.00001 45
4	0.06981 32	4	0.00116 36	4	0.00001 94
5	0.08726 65	5	0.00145 44	5	0.00002 42
6	0.10471 98	6	0.00174 53	6	0.00002 91
7	0.12217 30	7	0.00203 62	7	0.00003 39
8	0.13962 63	8	0.00232 71	8	0.00003 88
9	0.15707 96	9	0.00261 80	9	0.00004 36
10	0.17453 29	10	0.00290 89	10	0.00004 85
20	0.34906 59	20	0.00581 78	20	0.00009 70
30	0.52359 88	30	0.00872 66	30	0.00014 54
40	0.69813 17	40	0.01163 55	40	0.00019 39
50	0.87266 46	50	0.01454 44	50	0.00024 24
60	1.04719 76	60	0.01745 33	60	0.00029 09
70	1.22173 05				
80	1.39626 34				
90	1.57079 63				

Table 6.5(c)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

0° (180°) (359°) **179°**

′	Sin	Tan	Ctn	Cos	′
0	.00000	.00000	∞	1.0000	60
1	.00029	.00029	3437.7	1.0000	59
2	.00058	.00058	1718.9	1.0000	58
3	.00087	.00087	1145.9	1.0000	57
4	.00116	.00116	859.44	1.0000	56
5	.00145	.00145	687.55	1.0000	55
6	.00175	.00175	572.96	1.0000	54
7	.00204	.00204	491.11	1.0000	53
8	.00233	.00233	429.72	1.0000	52
9	.00262	.00262	381.97	1.0000	51
10	.00291	.00291	343.77	1.0000	50
11	.00320	.00320	312.52	.99999	49
12	.00349	.00349	286.48	.99999	48
13	.00378	.00378	264.44	.99999	47
14	.00407	.00407	245.55	.99999	46
15	.00436	.00436	229.18	.99999	45
16	.00465	.00465	214.86	.99999	44
17	.00495	.00495	202.22	.99999	43
18	.00524	.00524	190.98	.99999	42
19	.00553	.00553	180.93	.99998	41
20	.00582	.00582	171.89	.99998	40
21	.00611	.00611	163.70	.99998	39
22	.00640	.00640	156.26	.99998	38
23	.00669	.00669	149.47	.99998	37
24	.00698	.00698	143.24	.99998	36
25	.00727	.00727	137.51	.99997	35
26	.00756	.00756	132.22	.99997	34
27	.00785	.00785	127.32	.99997	33
28	.00814	.00815	122.77	.99997	32
29	.00844	.00844	118.54	.99996	31
30	.00873	.00873	114.59	.99996	30
31	.00902	.00902	110.89	.99996	29
32	.00931	.00931	107.43	.99996	28
33	.00960	.00960	104.17	.99995	27
34	.00989	.00989	101.11	.99995	26
35	.01018	.01018	98.218	.99995	25
36	.01047	.01047	95.489	.99995	24
37	.01076	.01076	92.908	.99994	23
38	.01105	.01105	90.463	.99994	22
39	.01134	.01135	88.144	.99994	21
40	.01164	.01164	85.940	.99993	20
41	.01193	.01193	83.844	.99993	19
42	.01222	.01222	81.847	.99993	18
43	.01251	.01251	79.943	.99992	17
44	.01280	.01280	78.126	.99992	16
45	.01309	.01309	76.390	.99991	15
46	.01338	.01338	74.729	.99991	14
47	.01367	.01367	73.139	.99991	13
48	.01396	.01396	71.615	.99990	12
49	.01425	.01425	70.153	.99990	11
50	.01454	.01455	68.750	.99989	10
51	.01483	.01484	67.402	.99989	9
52	.01513	.01513	66.105	.99989	8
53	.01542	.01542	64.858	.99988	7
54	.01571	.01571	63.657	.99988	6
55	.01600	.01600	62.499	.99987	5
56	.01629	.01629	61.383	.99987	4
57	.01658	.01658	60.306	.99986	3
58	.01687	.01687	59.266	.99986	2
59	.01716	.01716	58.261	.99985	1
60	.01745	.01746	57.290	.99985	0
′	Cos	Ctn	Tan	Sin	′

90° (270°) (269°) **89°**

1° (181°) (358°) **178°**

′	Sin	Tan	Ctn	Cos	′
0	.01745	.01746	57.290	.99985	60
1	.01774	.01775	56.351	.99984	59
2	.01803	.01804	55.442	.99984	58
3	.01832	.01833	54.561	.99983	57
4	.01862	.01862	53.709	.99983	56
5	.01891	.01891	52.882	.99982	55
6	.01920	.01920	52.081	.99982	54
7	.01949	.01949	51.303	.99981	53
8	.01978	.01978	50.549	.99980	52
9	.02007	.02007	49.816	.99980	51
10	.02036	.02036	49.104	.99979	50
11	.02065	.02066	48.412	.99979	49
12	.02094	.02095	47.740	.99978	48
13	.02123	.02124	47.085	.99977	47
14	.02152	.02153	46.449	.99977	46
15	.02181	.02182	45.829	.99976	45
16	.02211	.02211	45.226	.99976	44
17	.02240	.02240	44.639	.99975	43
18	.02269	.02269	44.066	.99974	42
19	.02298	.02298	43.508	.99974	41
20	.02327	.02328	42.964	.99973	40
21	.02356	.02357	42.433	.99972	39
22	.02385	.02386	41.916	.99972	38
23	.02414	.02415	41.411	.99971	37
24	.02443	.02444	40.917	.99970	36
25	.02472	.02473	40.436	.99969	35
26	.02501	.02502	39.965	.99969	34
27	.02530	.02531	39.506	.99968	33
28	.02560	.02560	39.057	.99967	32
29	.02589	.02589	38.618	.99966	31
30	.02618	.02619	38.188	.99966	30
31	.02647	.02648	37.769	.99965	29
32	.02676	.02677	37.358	.99964	28
33	.02705	.02706	36.956	.99963	27
34	.02734	.02735	36.563	.99963	26
35	.02763	.02764	36.178	.99962	25
36	.02792	.02793	35.801	.99961	24
37	.02821	.02822	35.431	.99960	23
38	.02850	.02851	35.070	.99959	22
39	.02879	.02881	34.715	.99959	21
40	.02908	.02910	34.368	.99958	20
41	.02938	.02939	34.027	.99957	19
42	.02967	.02968	33.694	.99956	18
43	.02996	.02997	33.366	.99955	17
44	.03025	.03026	33.045	.99954	16
45	.03054	.03055	32.730	.99953	15
46	.03083	.03084	32.421	.99952	14
47	.03112	.03114	32.118	.99952	13
48	.03141	.03143	31.821	.99951	12
49	.03170	.03172	31.528	.99950	11
50	.03199	.03201	31.242	.99949	10
51	.03228	.03230	30.960	.99948	9
52	.03257	.03259	30.683	.99947	8
53	.03286	.03288	30.412	.99946	7
54	.03316	.03317	30.145	.99945	6
55	.03345	.03346	29.882	.99944	5
56	.03374	.03376	29.624	.99943	4
57	.03403	.03405	29.371	.99942	3
58	.03432	.03434	29.122	.99941	2
59	.03461	.03463	28.877	.99940	1
60	.03490	.03492	28.636	.99939	0
′	Cos	Ctn	Tan	Sin	′

91° (271°) (268°) **88°**

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

2° (182°) (357°) **177°**

′	Sin	Tan	Ctn	Cos	′
0	.03490	.03492	28.636	.99939	60
1	.03519	.03521	28.399	.99938	59
2	.03548	.03550	28.166	.99937	58
3	.03577	.03579	27.937	.99936	57
4	.03606	.03609	27.712	.99935	56
5	.03635	.03638	27.490	.99934	55
6	.03664	.03667	27.271	.99933	54
7	.03693	.03696	27.057	.99932	53
8	.03723	.03725	26.845	.99931	52
9	.03752	.03754	26.637	.99930	51
10	.03781	.03783	26.432	.99929	50
11	.03810	.03812	26.230	.99927	49
12	.03839	.03842	26.031	.99926	48
13	.03868	.03871	25.835	.99925	47
14	.03897	.03900	25.642	.99924	46
15	.03926	.03929	25.452	.99923	45
16	.03955	.03958	25.264	.99922	44
17	.03984	.03987	25.080	.99921	43
18	.04013	.04016	24.898	.99919	42
19	.04042	.04046	24.719	.99918	41
20	.04071	.04075	24.542	.99917	40
21	.04100	.04104	24.368	.99916	39
22	.04129	.04133	24.196	.99915	38
23	.04159	.04162	24.026	.99913	37
24	.04188	.04191	23.859	.99912	36
25	.04217	.04220	23.695	.99911	35
26	.04246	.04250	23.532	.99910	34
27	.04275	.04279	23.372	.99909	33
28	.04304	.04308	23.214	.99907	32
29	.04333	.04337	23.058	.99906	31
30	.04362	.04366	22.904	.99905	30
31	.04391	.04395	22.752	.99904	29
32	.04420	.04424	22.602	.99902	28
33	.04449	.04454	22.454	.99901	27
34	.04478	.04483	22.308	.99900	26
35	.04507	.04512	22.164	.99898	25
36	.04536	.04541	22.022	.99897	24
37	.04565	.04570	21.881	.99896	23
38	.04594	.04599	21.743	.99894	22
39	.04623	.04628	21.606	.99893	21
40	.04653	.04658	21.470	.99892	20
41	.04682	.04687	21.337	.99890	19
42	.04711	.04716	21.205	.99889	18
43	.04740	.04745	21.075	.99888	17
44	.04769	.04774	20.946	.99886	16
45	.04798	.04803	20.819	.99885	15
46	.04827	.04833	20.693	.99883	14
47	.04856	.04862	20.569	.99882	13
48	.04885	.04891	20.446	.99881	12
49	.04914	.04920	20.325	.99879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	.99876	9
52	.05001	.05007	19.970	.99875	8
53	.05030	.05037	19.855	.99873	7
54	.05059	.05066	19.740	.99872	6
55	.05088	.05095	19.627	.99870	5
56	.05117	.05124	19.516	.99869	4
57	.05146	.05153	19.405	.99867	3
58	.05175	.05182	19.296	.99866	2
59	.05205	.05212	19.188	.99864	1
60	.05234	.05241	19.081	.99863	0
′	Cos	Ctn	Tan	Sin	′

92° (272°) (267°) **87°**

3° (183°) (356°) **176°**

′	Sin	Tan	Ctn	Cos	′
0	.05234	.05241	19.081	.99863	60
1	.05263	.05270	18.976	.99861	59
2	.05292	.05299	18.871	.99860	58
3	.05321	.05328	18.768	.99858	57
4	.05350	.05357	18.666	.99857	56
5	.05379	.05387	18.564	.99855	55
6	.05408	.05416	18.464	.99854	54
7	.05437	.05445	18.366	.99852	53
8	.05466	.05474	18.268	.99851	52
9	.05495	.05503	18.171	.99849	51
10	.05524	.05533	18.075	.99847	50
11	.05553	.05562	17.980	.99846	49
12	.05582	.05591	17.886	.99844	48
13	.05611	.05620	17.793	.99842	47
14	.05640	.05649	17.702	.99841	46
15	.05669	.05678	17.611	.99839	45
16	.05698	.05708	17.521	.99838	44
17	.05727	.05737	17.431	.99836	43
18	.05756	.05766	17.343	.99834	42
19	.05785	.05795	17.256	.99833	41
20	.05814	.05824	17.169	.99831	40
21	.05844	.05854	17.084	.99829	39
22	.05873	.05883	16.999	.99827	38
23	.05902	.05912	16.915	.99826	37
24	.05931	.05941	16.832	.99824	36
25	.05960	.05970	16.750	.99822	35
26	.05989	.05999	16.668	.99821	34
27	.06018	.06029	16.587	.99819	33
28	.06047	.06058	16.507	.99817	32
29	.06076	.06087	16.428	.99815	31
30	.06105	.06116	16.350	.99813	30
31	.06134	.06145	16.272	.99812	29
32	.06163	.06175	16.195	.99810	28
33	.06192	.06204	16.119	.99808	27
34	.06221	.06233	16.043	.99806	26
35	.06250	.06262	15.969	.99804	25
36	.06279	.06291	15.895	.99803	24
37	.06308	.06321	15.821	.99801	23
38	.06337	.06350	15.748	.99799	22
39	.06366	.06379	15.676	.99797	21
40	.06395	.06408	15.605	.99795	20
41	.06424	.06438	15.534	.99793	19
42	.06453	.06467	15.464	.99792	18
43	.06482	.06496	15.394	.99790	17
44	.06511	.06525	15.325	.99788	16
45	.06540	.06554	15.257	.99786	15
46	.06569	.06584	15.189	.99784	14
47	.06598	.06613	15.122	.99782	13
48	.06627	.06642	15.056	.99780	12
49	.06656	.06671	14.990	.99778	11
50	.06685	.06700	14.924	.99776	10
51	.06714	.06730	14.860	.99774	9
52	.06743	.06759	14.795	.99772	8
53	.06773	.06788	14.732	.99770	7
54	.06802	.06817	14.669	.99768	6
55	.06831	.06847	14.606	.99766	5
56	.06860	.06876	14.544	.99764	4
57	.06889	.06905	14.482	.99762	3
58	.06918	.06934	14.421	.99760	2
59	.06947	.06963	14.361	.99758	1
60	.06976	.06993	14.301	.99756	0
′	Cos	Ctn	Tan	Sin	′

93° (273°) (266°) **86°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

4° (184°) (355°) **175°**

′	Sin	Tan	Ctn	Cos	′
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	14.241	.99754	59
2	.07034	.07051	14.182	.99752	58
3	.07063	.07080	14.124	.99750	57
4	.07092	.07110	14.065	.99748	56
5	.07121	.07139	14.008	.99746	55
6	.07150	.07168	13.951	.99744	54
7	.07179	.07197	13.894	.99742	53
8	.07208	.07227	13.838	.99740	52
9	.07237	.07256	13.782	.99738	51
10	.07266	.07285	13.727	.99736	50
11	.07295	.07314	13.672	.99734	49
12	.07324	.07344	13.617	.99731	48
13	.07353	.07373	13.563	.99729	47
14	.07382	.07402	13.510	.99727	46
15	.07411	.07431	13.457	.99725	45
16	.07440	.07461	13.404	.99723	44
17	.07469	.07490	13.352	.99721	43
18	.07498	.07519	13.300	.99719	42
19	.07527	.07548	13.248	.99716	41
20	.07556	.07578	13.197	.99714	40
21	.07585	.07607	13.146	.99712	39
22	.07614	.07636	13.096	.99710	38
23	.07643	.07665	13.046	.99708	37
24	.07672	.07695	12.996	.99705	36
25	.07701	.07724	12.947	.99703	35
26	.07730	.07753	12.898	.99701	34
27	.07759	.07782	12.850	.99699	33
28	.07788	.07812	12.801	.99696	32
29	.07817	.07841	12.754	.99694	31
30	.07846	.07870	12.706	.99692	30
31	.07875	.07899	12.659	.99689	29
32	.07904	.07929	12.612	.99687	28
33	.07933	.07958	12.566	.99685	27
34	.07962	.07987	12.520	.99683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	.08046	12.429	.99678	24
37	.08049	.08075	12.384	.99676	23
38	.08078	.08104	12.339	.99673	22
39	.08107	.08134	12.295	.99671	21
40	.08136	.08163	12.251	.99668	20
41	.08165	.08192	12.207	.99666	19
42	.08194	.08221	12.163	.99664	18
43	.08223	.08251	12.120	.99661	17
44	.08252	.08280	12.077	.99659	16
45	.08281	.08309	12.035	.99657	15
46	.08310	.08339	11.992	.99654	14
47	.08339	.08368	11.950	.99652	13
48	.08368	.08397	11.909	.99649	12
49	.08397	.08427	11.867	.99647	11
50	.08426	.08456	11.826	.99644	10
51	.08455	.08485	11.785	.99642	9
52	.08484	.08514	11.745	.99639	8
53	.08513	.08544	11.705	.99637	7
54	.08542	.08573	11.664	.99635	6
55	.08571	.08602	11.625	.99632	5
56	.08600	.08632	11.585	.99630	4
57	.08629	.08661	11.546	.99627	3
58	.08658	.08690	11.507	.99625	2
59	.08687	.08720	11.468	.99622	1
60	.08716	.08749	11.430	.99619	0
′	Cos	Ctn	Tan	Sin	′

94° (274°) (265°) **85°**

5° (185°) (354°) **174°**

′	Sin	Tan	Ctn	Cos	′
0	.08716	.08749	11.430	.99619	60
1	.08745	.08778	11.392	.99617	59
2	.08774	.08807	11.354	.99614	58
3	.08803	.08837	11.316	.99612	57
4	.08831	.08866	11.279	.99609	56
5	.08860	.08895	11.242	.99607	55
6	.08889	.08925	11.205	.99604	54
7	.08918	.08954	11.168	.99602	53
8	.08947	.08983	11.132	.99599	52
9	.08976	.09013	11.095	.99596	51
10	.09005	.09042	11.059	.99594	50
11	.09034	.00971	11.024	.99591	49
12	.09063	.09101	10.988	.99588	48
13	.09092	.09130	10.953	.99586	47
14	.09121	.09159	10.918	.99583	46
15	.09150	.09189	10.883	.99580	45
16	.09179	.09218	10.848	.99578	44
17	.09208	.09247	10.814	.99575	43
18	.09237	.09277	10.780	.99572	42
19	.09266	.09306	10.746	.99570	41
20	.09295	.09335	10.712	.99567	40
21	.09324	.09365	10.678	.99564	39
22	.09353	.09394	10.645	.99562	38
23	.09382	.09423	10.612	.99559	37
24	.09411	.09453	10.579	.99556	36
25	.09440	.09482	10.546	.99553	35
26	.09469	.09511	10.514	.99551	34
27	.09498	.09541	10.481	.99548	33
28	.09527	.09570	10.449	.99545	32
29	.09556	.09600	10.417	.99542	31
30	.09585	.09629	10.385	.99540	30
31	.09614	.09658	10.354	.99537	29
32	.09642	.09688	10.322	.99534	28
33	.09671	.09717	10.291	.99531	27
34	.09700	.09746	10.260	.99528	26
35	.09729	.09776	10.229	.99526	25
36	.09758	.09805	10.199	.99523	24
37	.09787	.09834	10.168	.99520	23
38	.09816	.09864	10.138	.99517	22
39	.09845	.09893	10.108	.99514	21
40	.09874	.09923	10.078	.99511	20
41	.09903	.09952	10.048	.99508	19
42	.09932	.09981	10.019	.99506	18
43	.09961	.10011	9.9893	.99503	17
44	.09990	.10040	9.9601	.99500	16
45	.10019	.10069	9.9310	.99497	15
46	.10048	.10099	9.9021	.99494	14
47	.10077	.10128	9.8734	.99491	13
48	.10106	.10158	9.8448	.99488	12
49	.10135	.10187	9.8164	.99485	11
50	.10164	.10216	9.7882	.99482	10
51	.10192	.10246	9.7601	.99479	9
52	.10221	.10275	9.7322	.99476	8
53	.10250	.10305	9.7044	.99473	7
54	.10279	.10334	9.6768	.99470	6
55	.10308	.10363	9.6493	.99467	5
56	.10337	.10393	9.6220	.99464	4
57	.10366	.10422	9.5949	.99461	3
58	.10395	.10452	9.5679	.99458	2
59	.10424	.10481	9.5411	.99455	1
60	.10453	.10510	9.5144	.99452	0
′	Cos	Ctn	Tan	Sin	′

95° (275°) (264°) **84°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

6° (186°)				(353°) 173°	
′	Sin	Tan	Ctn	Cos	′
0	.10453	.10510	9.5144	.99452	60
1	.10482	.10540	9.4878	.99449	59
2	.10511	.10569	9.4614	.99446	58
3	.10540	.10599	9.4352	.99443	57
4	.10569	.10628	9.4090	.99440	56
5	.10597	.10657	9.3831	.99437	55
6	.10626	.10687	9.3572	.99434	54
7	.10655	.10716	9.3315	.99431	53
8	.10684	.10746	9.3060	.99428	52
9	.10713	.10775	9.2806	.99424	51
10	.10742	.10805	9.2553	.99421	50
11	.10771	.10834	9.2302	.99418	49
12	.10800	.10863	9.2052	.99415	48
13	.10829	.10893	9.1803	.99412	47
14	.10858	.10922	9.1555	.99409	46
15	.10887	.10952	9.1309	.99406	45
16	.10916	.10981	9.1065	.99402	44
17	.10945	.11011	9.0821	.99399	43
18	.10973	.11040	9.0579	.99396	42
19	.11002	.11070	9.0338	.99393	41
20	.11031	.11099	9.0098	.99390	40
21	.11060	.11128	8.9860	.99386	39
22	.11089	.11158	8.9623	.99383	38
23	.11118	.11187	8.9387	.99380	37
24	.11147	.11217	8.9152	.99377	36
25	.11176	.11246	8.8919	.99374	35
26	.11205	.11276	8.8686	.99370	34
27	.11234	.11305	8.8455	.99367	33
28	.11263	.11335	8.8225	.99364	32
29	.11291	.11364	8.7996	.99360	31
30	.11320	.11394	8.7769	.99357	30
31	.11349	.11423	8.7542	.99354	29
32	.11378	.11452	8.7317	.99351	28
33	.11407	.11482	8.7093	.99347	27
34	.11436	.11511	8.6870	.99344	26
35	.11465	.11541	8.6648	.99341	25
36	.11494	.11570	8.6427	.99337	24
37	.11523	.11600	8.6208	.99334	23
38	.11552	.11629	8.5989	.99331	22
39	.11580	.11659	8.5772	.99327	21
40	.11609	.11688	8.5555	.99324	20
41	.11638	.11718	8.5340	.99320	19
42	.11667	.11747	8.5126	.99317	18
43	.11696	.11777	8.4913	.99314	17
44	.11725	.11806	8.4701	.99310	16
45	.11754	.11836	8.4490	.99307	15
46	.11783	.11865	8.4280	.99303	14
47	.11812	.11895	8.4071	.99300	13
48	.11840	.11924	8.3863	.99297	12
49	.11869	.11954	8.3656	.99293	11
50	.11898	.11983	8.3450	.99290	10
51	.11927	.12013	8.3245	.99286	9
52	.11956	.12042	8.3041	.99283	8
53	.11985	.12072	8.2838	.99279	7
54	.12014	.12101	8.2636	.99276	6
55	.12043	.12131	8.2434	.99272	5
56	.12071	.12160	8.2234	.99269	4
57	.12100	.12190	8.2035	.99265	3
58	.12129	.12219	8.1837	.99262	2
59	.12158	.12249	8.1640	.99258	1
60	.12187	.12278	8.1443	.99255	0
′	Cos	Ctn	Tan	Sin	′
96° (276°)				(263°) 83°	

7° (187°)				(352°) 172°	
′	Sin	Tan	Ctn	Cos	′
0	.12187	.12278	8.1443	.99255	60
1	.12216	.12308	8.1248	.99251	59
2	.12245	.12338	8.1054	.99248	58
3	.12274	.12367	8.0860	.99244	57
4	.12302	.12397	8.0667	.99240	56
5	.12331	.12426	8.0476	.99237	55
6	.12360	.12456	8.0285	.99233	54
7	.12389	.12485	8.0095	.99230	53
8	.12418	.12515	7.9906	.99226	52
9	.12447	.12544	7.9718	.99222	51
10	.12476	.12574	7.9530	.99219	50
11	.12504	.12603	7.9344	.99215	49
12	.12533	.12633	7.9158	.99211	48
13	.12562	.12662	7.8973	.99208	47
14	.12591	.12692	7.8789	.99204	46
15	.12620	.12722	7.8606	.99200	45
16	.12649	.12751	7.8424	.99197	44
17	.12678	.12781	7.8243	.99193	43
18	.12706	.12810	7.8062	.99189	42
19	.12735	.12840	7.7882	.99186	41
20	.12764	.12869	7.7704	.99182	40
21	.12793	.12899	7.7525	.99178	39
22	.12822	.12929	7.7348	.99175	38
23	.12851	.12958	7.7171	.99171	37
24	.12880	.12988	7.6996	.99167	36
25	.12908	.13017	7.6821	.99163	35
26	.12937	.13047	7.6647	.99160	34
27	.12966	.13076	7.6473	.99156	33
28	.12995	.13106	7.6301	.99152	32
29	.13024	.13136	7.6129	.99148	31
30	.13053	.13165	7.5958	.99144	30
31	.13081	.13195	7.5787	.99141	29
32	.13110	.13224	7.5618	.99137	28
33	.13139	.13254	7.5449	.99133	27
34	.13168	.13284	7.5281	.99129	26
35	.13197	.13313	7.5113	.99125	25
36	.13226	.13343	7.4947	.99122	24
37	.13254	.13372	7.4781	.99118	23
38	.13283	.13402	7.4615	.99114	22
39	.13312	.13432	7.4451	.99110	21
40	.13341	.13461	7.4287	.99106	20
41	.13370	.13491	7.4124	.99102	19
42	.13399	.13521	7.3962	.99098	18
43	.13427	.13550	7.3800	.99094	17
44	.13456	.13580	7.3639	.99091	16
45	.13485	.13609	7.3479	.99087	15
46	.13514	.13639	7.3319	.99083	14
47	.13543	.13669	7.3160	.99079	13
48	.13572	.13698	7.3002	.99075	12
49	.13600	.13728	7.2844	.99071	11
50	.13629	.13758	7.2687	.99067	10
51	.13658	.13787	7.2531	.99063	9
52	.13687	.13817	7.2375	.99059	8
53	.13716	.13846	7.2220	.99055	7
54	.13744	.13876	7.2066	.99051	6
55	.13773	.13906	7.1912	.99047	5
56	.13802	.13935	7.1759	.99043	4
57	.13831	.13965	7.1607	.99039	3
58	.13860	.13995	7.1455	.99035	2
59	.13889	.14024	7.1304	.99031	1
60	.13917	.14054	7.1154	.99027	0
′	Cos	Ctn	Tan	Sin	′
97° (277°)				(262°) 82°	

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

8° (188°) (351°) **171°**

′	Sin	Tan	Ctn	Cos	′
0	.13917	.14054	7.1154	.99027	60
1	.13946	.14084	7.1004	.99023	59
2	.13975	.14113	7.0855	.99019	58
3	.14004	.14143	7.0706	.99015	57
4	.14033	.14173	7.0558	.99011	56
5	.14061	.14202	7.0410	.99006	55
6	.14090	.14232	7.0264	.99002	54
7	.14119	.14262	7.0117	.98998	53
8	.14148	.14291	6.9972	.98994	52
9	.14177	.14321	6.9827	.98990	51
10	.14205	.14351	6.9682	.98986	50
11	.14234	.14381	6.9538	.98982	49
12	.14263	.14410	6.9395	.98978	48
13	.14292	.14440	6.9252	.98973	47
14	.14320	.14470	6.9110	.98969	46
15	.14349	.14499	6.8969	.98965	45
16	.14378	.14529	6.8828	.98961	44
17	.14407	.14559	6.8687	.98957	43
18	.14436	.14588	6.8548	.98953	42
19	.14464	.14618	6.8408	.98948	41
20	.14493	.14648	6.8269	.98944	40
21	.14522	.14678	6.8131	.98940	39
22	.14551	.14707	6.7994	.98936	38
23	.14580	.14737	6.7856	.98931	37
24	.14608	.14767	6.7720	.98927	36
25	.14637	.14796	6.7584	.98923	35
26	.14666	.14826	6.7448	.98919	34
27	.14695	.14856	6.7313	.98914	33
28	.14723	.14886	6.7179	.98910	32
29	.14752	.14915	6.7045	.98906	31
30	.14781	.14945	6.6912	.98902	30
31	.14810	.14975	6.6779	.98897	29
32	.14838	.15005	6.6646	.98893	28
33	.14867	.15034	6.6514	.98889	27
34	.14896	.15064	6.6383	.98884	26
35	.14925	.15094	6.6252	.98880	25
36	.14954	.15124	6.6122	.98876	24
37	.14982	.15153	6.5992	.98871	23
38	.15011	.15183	6.5863	.98867	22
39	.15040	.15213	6.5734	.98863	21
40	.15069	.15243	6.5606	.98858	20
41	.15097	.15272	6.5478	.98854	19
42	.15126	.15302	6.5350	.98849	18
43	.15155	.15332	6.5223	.98845	17
44	.15184	.15362	6.5097	.98841	16
45	.15212	.15391	6.4971	.98836	15
46	.15241	.15421	6.4846	.98832	14
47	.15270	.15451	6.4721	.98827	13
48	.15299	.15481	6.4596	.98823	12
49	.15327	.15511	6.4472	.98818	11
50	.15356	.15540	6.4348	.98814	10
51	.15385	.15570	6.4225	.98809	9
52	.15414	.15600	6.4103	.98805	8
53	.15442	.15630	6.3980	.98800	7
54	.15471	.15660	6.3859	.98796	6
55	.15500	.15689	6.3737	.98791	5
56	.15529	.15719	6.3617	.98787	4
57	.15557	.15749	6.3496	.98782	3
58	.15586	.15779	6.3376	.98778	2
59	.15615	.15809	6.3257	.98773	1
60	.15643	.15838	6.3138	.98769	0
′	Cos	Ctn	Tan	Sin	′

98° (278°) (261°) **81°**

9° (189°) (350°) **170°**

′	Sin	Tan	Ctn	Cos	′
0	.15643	.15838	6.3138	.98769	60
1	.15672	.15868	6.3019	.98764	59
2	.15701	.15898	6.2901	.98760	58
3	.15730	.15928	6.2783	.98755	57
4	.15758	.15958	6.2666	.98751	56
5	.15787	.15988	6.2549	.98746	55
6	.15816	.16017	6.2432	.98741	54
7	.15845	.16047	6.2316	.98737	53
8	.15873	.16077	6.2200	.98732	52
9	.15902	.16107	6.2085	.98728	51
10	.15931	.16137	6.1970	.98723	50
11	.15959	.16167	6.1856	.98718	49
12	.15988	.16196	6.1742	.98714	48
13	.16017	.16226	6.1628	.98709	47
14	.16046	.16256	6.1515	.98704	46
15	.16074	.16286	6.1402	.98700	45
16	.16103	.16316	6.1290	.98695	44
17	.16132	.16346	6.1178	.98690	43
18	.16160	.16376	6.1066	.98686	42
19	.16189	.16405	6.0955	.98681	41
20	.16218	.16435	6.0844	.98676	40
21	.16246	.16465	6.0734	.98671	39
22	.16275	.16495	6.0624	.98667	38
23	.16304	.16525	6.0514	.98662	37
24	.16333	.16555	6.0405	.98657	36
25	.16361	.16585	6.0296	.98652	35
26	.16390	.16615	6.0188	.98648	34
27	.16419	.16645	6.0080	.98643	33
28	.16447	.16674	5.9972	.98638	32
29	.16476	.16704	5.9865	.98633	31
30	.16505	.16734	5.9758	.98629	30
31	.16533	.16764	5.9651	.98624	29
32	.16562	.16794	5.9545	.98619	28
33	.16591	.16824	5.9439	.98614	27
34	.16620	.16854	5.9333	.98609	26
35	.16648	.16884	5.9228	.98604	25
36	.16677	.16914	5.9124	.98600	24
37	.16706	.16944	5.9019	.98595	23
38	.16734	.16974	5.8915	.98590	22
39	.16763	.17004	5.8811	.98585	21
40	.16792	.17033	5.8708	.98580	20
41	.16820	.17063	5.8605	.98575	19
42	.16849	.17093	5.8502	.98570	18
43	.16878	.17123	5.8400	.98565	17
44	.16906	.17153	5.8298	.98561	16
45	.16935	.17183	5.8197	.98556	15
46	.16964	.17213	5.8095	.98551	14
47	.16992	.17243	5.7994	.98546	13
48	.17021	.17273	5.7894	.98541	12
49	.17050	.17303	5.7794	.98536	11
50	.17078	.17333	5.7694	.98531	10
51	.17107	.17363	5.7594	.98526	9
52	.17136	.17393	5.7495	.98521	8
53	.17164	.17423	5.7396	.98516	7
54	.17193	.17453	5.7297	.98511	6
55	.17222	.17483	5.7199	.98506	5
56	.17250	.17513	5.7101	.98501	4
57	.17279	.17543	5.7004	.98496	3
58	.17308	.17573	5.6906	.98491	2
59	.17336	.17603	5.6809	.98486	1
60	.17365	.17633	5.6713	.98481	0
′	Cos	Ctn	Tan	Sin	′

99° (279°) (260°) **80°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

10° (190°) (349°) **169°**

′	Sin	Tan	Ctn	Cos	′
0	.17365	.17633	5.6713	.98481	60
1	.17393	.17663	5.6617	.98476	59
2	.17422	.17693	5.6521	.98471	58
3	.17451	.17723	5.6425	.98466	57
4	.17479	.17753	5.6329	.98461	56
5	.17508	.17783	5.6234	.98455	55
6	.17537	.17813	5.6140	.98450	54
7	.17565	.17843	5.6045	.98445	53
8	.17594	.17873	5.5951	.98440	52
9	.17623	.17903	5.5857	.98435	51
10	.17651	.17933	5.5764	.98430	50
11	.17680	.17963	5.5671	.98425	49
12	.17708	.17993	5.5578	.98420	48
13	.17737	.18023	5.5485	.98414	47
14	.17766	.18053	5.5393	.98409	46
15	.17794	.18083	5.5301	.98404	45
16	.17823	.18113	5.5209	.98399	44
17	.17852	.18143	5.5118	.98394	43
18	.17880	.18173	5.5026	.98389	42
19	.17909	.18203	5.4936	.98383	41
20	.17937	.18233	5.4845	.98378	40
21	.17966	.18263	5.4755	.98373	39
22	.17995	.18293	5.4665	.98368	38
23	.18023	.18323	5.4575	.98362	37
24	.18052	.18353	5.4486	.98357	36
25	.18081	.18384	5.4397	.98352	35
26	.18109	.18414	5.4308	.98347	34
27	.18138	.18444	5.4219	.98341	33
28	.18166	.18474	5.4131	.98336	32
29	.18195	.18504	5.4043	.98331	31
30	.18224	.18534	5.3955	.98325	30
31	.18252	.18564	5.3868	.98320	29
32	.18281	.18594	5.3781	.98315	28
33	.18309	.18624	5.3694	.98310	27
34	.18338	.18654	5.3607	.98304	26
35	.18367	.18684	5.3521	.98299	25
36	.18395	.18714	5.3435	.98294	24
37	.18424	.18745	5.3349	.98288	23
38	.18452	.18775	5.3263	.98283	22
39	.18481	.18805	5.3178	.98277	21
40	.18509	.18835	5.3093	.98272	20
41	.18538	.18865	5.3008	.98267	19
42	.18567	.18895	5.2924	.98261	18
43	.18595	.18925	5.2839	.98256	17
44	.18624	.18955	5.2755	.98250	16
45	.18652	.18986	5.2672	.98245	15
46	.18681	.19016	5.2588	.98240	14
47	.18710	.19046	5.2505	.98234	13
48	.18738	.19076	5.2422	.98229	12
49	.18767	.19106	5.2339	.98223	11
50	.18795	.19136	5.2257	.98218	10
51	.18824	.19166	5.2174	.98212	9
52	.18852	.19197	5.2092	.98207	8
53	.18881	.19227	5.2011	.98201	7
54	.18910	.19257	5.1929	.98196	6
55	.18938	.19287	5.1848	.98190	5
56	.18967	.19317	5.1767	.98185	4
57	.18995	.19347	5.1686	.98179	3
58	.19024	.19378	5.1606	.98174	2
59	.19052	.19408	5.1526	.98168	1
60	.19081	.19438	5.1446	.98163	0
′	Cos	Ctn	Tan	Sin	′

11° (191°) (348°) **168°**

′	Sin	Tan	Ctn	Cos	′
0	.19081	.19438	5.1446	.98163	60
1	.19109	.19468	5.1366	.98157	59
2	.19138	.19498	5.1286	.98152	58
3	.19167	.19529	5.1207	.98146	57
4	.19195	.19559	5.1128	.98140	56
5	.19224	.19589	5.1049	.98135	55
6	.19252	.19619	5.0970	.98129	54
7	.19281	.19649	5.0892	.98124	53
8	.19309	.19680	5.0814	.98118	52
9	.19338	.19710	5.0736	.98112	51
10	.19366	.19740	5.0658	.98107	50
11	.19395	.19770	5.0581	.98101	49
12	.19423	.19801	5.0504	.98096	48
13	.19452	.19831	5.0427	.98090	47
14	.19481	.19861	5.0350	.98084	46
15	.19509	.19891	5.0273	.98079	45
16	.19538	.19921	5.0197	.98073	44
17	.19566	.19952	5.0121	.98067	43
18	.19595	.19982	5.0045	.98061	42
19	.19623	.20012	4.9969	.98056	41
20	.19652	.20042	4.9894	.98050	40
21	.19680	.20073	4.9819	.98044	39
22	.19709	.20103	4.9744	.98039	38
23	.19737	.20133	4.9669	.98033	37
24	.19766	.20164	4.9594	.98027	36
25	.19794	.20194	4.9520	.98021	35
26	.19823	.20224	4.9446	.98016	34
27	.19851	.20254	4.9372	.98010	33
28	.19880	.20285	4.9298	.98004	32
29	.19908	.20315	4.9225	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	.19965	.20376	4.9078	.97987	29
32	.19994	.20406	4.9006	.97981	28
33	.20022	.20436	4.8933	.97975	27
34	.20051	.20466	4.8860	.97969	26
35	.20079	.20497	4.8788	.97963	25
36	.20108	.20527	4.8716	.97958	24
37	.20136	.20557	4.8644	.97952	23
38	.20165	.20588	4.8573	.97946	22
39	.20193	.20618	4.8501	.97940	21
40	.20222	.20648	4.8430	.97934	20
41	.20250	.20679	4.8359	.97928	19
42	.20279	.20709	4.8288	.97922	18
43	.20307	.20739	4.8218	.97916	17
44	.20336	.20770	4.8147	.97910	16
45	.20364	.20800	4.8077	.97905	15
46	.20393	.20830	4.8007	.97899	14
47	.20421	.20861	4.7937	.97893	13
48	.20450	.20891	4.7867	.97887	12
49	.20478	.20921	4.7798	.97881	11
50	.20507	.20952	4.7729	.97875	10
51	.20535	.20982	4.7659	.97869	9
52	.20563	.21013	4.7591	.97863	8
53	.20592	.21043	4.7522	.97857	7
54	.20620	.21073	4.7453	.97851	6
55	.20649	.21104	4.7385	.97845	5
56	.20677	.21134	4.7317	.97839	4
57	.20706	.21164	4.7249	.97833	3
58	.20734	.21195	4.7181	.97827	2
59	.20763	.21225	4.7114	.97821	1
60	.20791	.21256	4.7046	.97815	0
′	Cos	Ctn	Tan	Sin	′

100° (280°) (259°) **79°**

101° (281°) (258°) **78°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

12° (192°) (347°) **167°**

′	Sin	Tan	Ctn	Cos	′
0	.20791	.21256	4.7046	.97815	60
1	.20820	.21286	4.6979	.97809	59
2	.20848	.21316	4.6912	.97803	58
3	.20877	.21347	4.6845	.97797	57
4	.20905	.21377	4.6779	.97791	56
5	.20933	.21408	4.6712	.97784	55
6	.20962	.21438	4.6646	.97778	54
7	.20990	.21469	4.6580	.97772	53
8	.21019	.21499	4.6514	.97766	52
9	.21047	.21529	4.6448	.97760	51
10	.21076	.21560	4.6382	.97754	50
11	.21104	.21590	4.6317	.97748	49
12	.21132	.21621	4.6252	.97742	48
13	.21161	.21651	4.6187	.97735	47
14	.21189	.21682	4.6122	.97729	46
15	.21218	.21712	4.6057	.97723	45
16	.21246	.21743	4.5993	.97717	44
17	.21275	.21773	4.5928	.97711	43
18	.21303	.21804	4.5864	.97705	42
19	.21331	.21834	4.5800	.97698	41
20	.21360	.21864	4.5736	.97692	40
21	.21388	.21895	4.5673	.97686	39
22	.21417	.21925	4.5609	.97680	38
23	.21445	.21956	4.5546	.97673	37
24	.21474	.21986	4.5483	.97667	36
25	.21502	.22017	4.5420	.97661	35
26	.21530	.22047	4.5357	.97655	34
27	.21559	.22078	4.5294	.97648	33
28	.21587	.22108	4.5232	.97642	32
29	.21616	.22139	4.5169	.97636	31
30	.21644	.22169	4.5107	.97630	30
31	.21672	.22200	4.5045	.97623	29
32	.21701	.22231	4.4983	.97617	28
33	.21729	.22261	4.4922	.97611	27
34	.21758	.22292	4.4860	.97604	26
35	.21786	.22322	4.4799	.97598	25
36	.21814	.22353	4.4737	.97592	24
37	.21843	.22383	4.4676	.97585	23
38	.21871	.22414	4.4615	.97579	22
39	.21899	.22444	4.4555	.97573	21
40	.21928	.22475	4.4494	.97566	20
41	.21956	.22505	4.4434	.97560	19
42	.21985	.22536	4.4373	.97553	18
43	.22013	.22567	4.4313	.97547	17
44	.22041	.22597	4.4253	.97541	16
45	.22070	.22628	4.4194	.97534	15
46	.22098	.22658	4.4134	.97528	14
47	.22126	.22689	4.4075	.97521	13
48	.22155	.22719	4.4015	.97515	12
49	.22183	.22750	4.3956	.97508	11
50	.22212	.22781	4.3897	.97502	10
51	.22240	.22811	4.3838	.97496	9
52	.22268	.22842	4.3779	.97489	8
53	.22297	.22872	4.3721	.97483	7
54	.22325	.22903	4.3662	.97476	6
55	.22353	.22934	4.3604	.97470	5
56	.22382	.22964	4.3546	.97463	4
57	.22410	.22995	4.3488	.97457	3
58	.22438	.23026	4.3430	.97450	2
59	.22467	.23056	4.3372	.97444	1
60	.22495	.23087	4.3315	.97437	0
′	Cos	Ctn	Tan	Sin	′

102° (282°) (257°) **77°**

13° (193°) (346°) **166°**

′	Sin	Tan	Ctn	Cos	′
0	.22495	.23087	4.3315	.97437	60
1	.22523	.23117	4.3257	.97430	59
2	.22552	.23148	4.3200	.97424	58
3	.22580	.23179	4.3143	.97417	57
4	.22608	.23209	4.3086	.97411	56
5	.22637	.23240	4.3029	.97404	55
6	.22665	.23271	4.2972	.97398	54
7	.22693	.23301	4.2916	.97391	53
8	.22722	.23332	4.2859	.97384	52
9	.22750	.23363	4.2803	.97378	51
10	.22778	.23393	4.2747	.97371	50
11	.22807	.23424	4.2691	.97365	49
12	.22835	.23455	4.2635	.97358	48
13	.22863	.23485	4.2580	.97351	47
14	.22892	.23516	4.2524	.97345	46
15	.22920	.23547	4.2468	.97338	45
16	.22948	.23578	4.2413	.97331	44
17	.22977	.23608	4.2358	.97325	43
18	.23005	.23639	4.2303	.97318	42
19	.23033	.23670	4.2248	.97311	41
20	.23062	.23700	4.2193	.97304	40
21	.23090	.23731	4.2139	.97298	39
22	.23118	.23762	4.2084	.97291	38
23	.23146	.23793	4.2030	.97284	37
24	.23175	.23823	4.1976	.97278	36
25	.23203	.23854	4.1922	.97271	35
26	.23231	.23885	4.1868	.97264	34
27	.23260	.23916	4.1814	.97257	33
28	.23288	.23946	4.1760	.97251	32
29	.23316	.23977	4.1706	.97244	31
30	.23345	.24008	4.1653	.97237	30
31	.23373	.24039	4.1600	.97230	29
32	.23401	.24069	4.1547	.97223	28
33	.23429	.24100	4.1493	.97217	27
34	.23458	.24131	4.1441	.97210	26
35	.23486	.24162	4.1388	.97203	25
36	.23514	.24193	4.1335	.97196	24
37	.23542	.24223	4.1282	.97189	23
38	.23571	.24254	4.1230	.97182	22
39	.23599	.24285	4.1178	.97176	21
40	.23627	.24316	4.1126	.97169	20
41	.23656	.24347	4.1074	.97162	19
42	.23684	.24377	4.1022	.97155	18
43	.23712	.24408	4.0970	.97148	17
44	.23740	.24439	4.0918	.97141	16
45	.23769	.24470	4.0867	.97134	15
46	.23797	.24501	4.0815	.97127	14
47	.23825	.24532	4.0764	.97120	13
48	.23853	.24562	4.0713	.97113	12
49	.23882	.24593	4.0662	.97106	11
50	.23910	.24624	4.0611	.97100	10
51	.23938	.24655	4.0560	.97093	9
52	.23966	.24686	4.0509	.97086	8
53	.23995	.24717	4.0459	.97079	7
54	.24023	.24747	4.0408	.97072	6
55	.24051	.24778	4.0358	.97065	5
56	.24079	.24809	4.0308	.97058	4
57	.24108	.24840	4.0257	.97051	3
58	.24136	.24871	4.0207	.97044	2
59	.24164	.24902	4.0158	.97037	1
60	.24192	.24933	4.0108	.97030	0
′	Cos	Ctn	Tan	Sin	′

103° (283°) (256°) **76°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

14° (194°) (345°) **165°** | **15°** (195°) (344°) **164°**

′	Sin	Tan	Ctn	Cos	′	′	Sin	Tan	Ctn	Cos	′
0	.24192	.24933	4.0108	.97030	60	0	.25882	.26795	3.7321	.96593	60
1	.24220	.24964	4.0058	.97023	59	1	.25910	.26826	3.7277	.96585	59
2	.24249	.24995	4.0009	.97015	58	2	.25938	.26857	3.7234	.96578	58
3	.24277	.25026	3.9959	.97008	57	3	.25966	.26888	3.7191	.96570	57
4	.24305	.25056	3.9910	.97001	56	4	.25994	.26920	3.7148	.96562	56
5	.24333	.25087	3.9861	.96994	55	5	.26022	.26951	3.7105	.96555	55
6	.24362	.25118	3.9812	.96987	54	6	.26050	.26982	3.7062	.96547	54
7	.24390	.25149	3.9763	.96980	53	7	.26079	.27013	3.7019	.96540	53
8	.24418	.25180	3.9714	.96973	52	8	.26107	.27044	3.6976	.96532	52
9	.24446	.25211	3.9665	.96966	51	9	.26135	.27076	3.6933	.96524	51
10	.24474	.25242	3.9617	.96959	50	10	.26163	.27107	3.6891	.96517	50
11	.24503	.25273	3.9568	.96952	49	11	.26191	.27138	3.6848	.96509	49
12	.24531	.25304	3.9520	.96945	48	12	.26219	.27169	3.6806	.96502	48
13	.24559	.25335	3.9471	.96937	47	13	.26247	.27201	3.6764	.96494	47
14	.24587	.25366	3.9423	.96930	46	14	.26275	.27232	3.6722	.96486	46
15	.24615	.25397	3.9375	.96923	45	15	.26303	.27263	3.6680	.96479	45
16	.24644	.25428	3.9327	.96916	44	16	.26331	.27294	3.6638	.96471	44
17	.24672	.25459	3.9279	.96909	43	17	.26359	.27326	3.6596	.96463	43
18	.24700	.25490	3.9232	.96902	42	18	.26387	.27357	3.6554	.96456	42
19	.25728	.25521	3.9184	.96894	41	19	.26415	.27388	3.6512	.96448	41
20	.24756	.25552	3.9136	.96887	40	20	.26443	.27419	3.6470	.96440	40
21	.24784	.25583	3.9089	.96880	39	21	.26471	.27451	3.6429	.96433	39
22	.24813	.25614	3.9042	.96873	38	22	.26500	.27482	3.6387	.96425	38
23	.24841	.25645	3.8995	.96866	37	23	.26528	.27513	3.6346	.96417	37
24	.24869	.25676	3.8947	.96858	36	24	.26556	.27545	3.6305	.96410	36
25	.24897	.25707	3.8900	.96851	35	25	.26584	.27576	3.6264	.96402	35
26	.24925	.25738	3.8854	.96844	34	26	.26612	.27607	3.6222	.96394	34
27	.24954	.25769	3.8807	.96837	33	27	.26640	.27638	3.6181	.96386	33
28	.24982	.25800	3.8760	.96829	32	28	.26668	.27670	3.6140	.96379	32
29	.25010	.25831	3.8714	.96822	31	29	.26696	.27701	3.6100	.96371	31
30	.25038	.25862	3.8667	.96815	30	30	.26724	.27732	3.6059	.96363	30
31	.25066	.25893	3.8621	.96807	29	31	.26752	.27764	3.6018	.96355	29
32	.25094	.25924	3.8575	.96800	28	32	.26780	.27795	3.5978	.96347	28
33	.25122	.25955	3.8528	.96793	27	33	.26808	.27826	3.5937	.96340	27
34	.25151	.25986	3.8482	.96786	26	34	.26836	.27858	3.5897	.96332	26
35	.25179	.26017	3.8436	.96778	25	35	.26864	.27889	3.5856	.96324	25
36	.25207	.26048	3.8391	.96771	24	36	.26892	.27921	3.5816	.96316	24
37	.25235	.26079	3.8345	.96764	23	37	.26920	.27952	3.5776	.96308	23
38	.25263	.26110	3.8299	.96756	22	38	.26948	.27983	3.5736	.96301	22
39	.25291	.26141	3.8254	.96749	21	39	.26976	.28015	3.5696	.96293	21
40	.25320	.26172	3.8208	.96742	20	40	.27004	.28046	3.5656	.96285	20
41	.25348	.26203	3.8163	.96734	19	41	.27032	.28077	3.5616	.96277	19
42	.25376	.26235	3.8118	.96727	18	42	.27060	.28109	3.5576	.96269	18
43	.25404	.26266	3.8073	.96719	17	43	.27088	.28140	3.5536	.96261	17
44	.25432	.26297	3.8028	.96712	16	44	.27116	.28172	3.5497	.96253	16
45	.25460	.26328	3.7983	.96705	15	45	.27144	.28203	3.5457	.96246	15
46	.25488	.26359	3.7938	.96697	14	46	.27172	.28234	3.5418	.96238	14
47	.25516	.26390	3.7893	.96690	13	47	.27200	.28266	3.5379	.96230	13
48	.25545	.26421	3.7848	.96682	12	48	.27228	.28297	3.5339	.96222	12
49	.25573	.26452	3.7804	.96675	11	49	.27256	.28329	3.5300	.96214	11
50	.25601	.26483	3.7760	.96667	10	50	.27284	.28360	3.5261	.96206	10
51	.25629	.26515	3.7715	.96660	9	51	.27312	.28391	3.5222	.96198	9
52	.25657	.26546	3.7671	.96653	8	52	.27340	.28423	3.5183	.96190	8
53	.25685	.26577	3.7627	.96645	7	53	.27368	.28454	3.5144	.96182	7
54	.25713	.26608	3.7583	.96638	6	54	.27396	.28486	3.5105	.96174	6
55	.25741	.26639	3.7539	.96630	5	55	.27424	.28517	3.5067	.96166	5
56	.25769	.26670	3.7495	.96623	4	56	.27452	.28549	3.5028	.96158	4
57	.25798	.26701	3.7451	.96615	3	57	.27480	.28580	3.4989	.96150	3
58	.25826	.26733	3.7408	.96608	2	58	.27508	.28612	3.4951	.96142	2
59	.25854	.26764	3.7364	.96600	1	59	.27536	.28643	3.4912	.96134	1
60	.25882	.26795	3.7321	.96593	0	60	.27564	.28675	3.4874	.96126	0
′	Cos	Ctn	Tan	Sin	′	′	Cos	Ctn	Tan	Sin	′

104° (284°) (255°) **75°** | **105°** (285°) (254°) **74°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

16° (196°) (343°) **163°**

′	Sin	Tan	Ctn	Cos	′
0	.27564	.28675	3.4874	.96126	60
1	.27592	.28706	3.4836	.96118	59
2	.27620	.28738	3.4798	.96110	58
3	.27648	.28769	3.4760	.96102	57
4	.27676	.28801	3.4722	.96094	56
5	.27704	.28832	3.4684	.96086	55
6	.27731	.28864	3.4646	.96078	54
7	.27759	.28895	3.4608	.96070	53
8	.27787	.28927	3.4570	.96062	52
9	.27815	.28958	3.4533	.96054	51
10	.27843	.28990	3.4495	.96046	50
11	.27871	.29021	3.4458	.96037	49
12	.27899	.29053	3.4420	.96029	48
13	.27927	.29084	3.4383	.96021	47
14	.27955	.29116	3.4346	.96013	46
15	.27983	.29147	3.4308	.96005	45
16	.28011	.29179	3.4271	.95997	44
17	.28039	.29210	3.4234	.95989	43
18	.28067	.29242	3.4197	.95981	42
19	.28095	.29274	3.4160	.95972	41
20	.28123	.29305	3.4124	.95964	40
21	.28150	.29337	3.4087	.95956	39
22	.28178	.29368	3.4050	.95948	38
23	.28206	.29400	3.4014	.95940	37
24	.28234	.29432	3.3977	.95931	36
25	.28262	.29463	3.3941	.95923	35
26	.28290	.29495	3.3904	.95915	34
27	.28318	.29526	3.3868	.95907	33
28	.28346	.29558	3.3832	.95898	32
29	.28374	.29590	3.3796	.95890	31
30	.28402	.29621	3.3759	.95882	30
31	.28429	.29653	3.3723	.95874	29
32	.28457	.29685	3.3687	.95865	28
33	.28485	.29716	3.3652	.95857	27
34	.28513	.29748	3.3616	.95849	26
35	.28541	.29780	3.3580	.95841	25
36	.28569	.29811	3.3544	.95832	24
37	.28597	.29843	3.3509	.95824	23
38	.28625	.29875	3.3473	.95816	22
39	.28652	.29906	3.3438	.95807	21
40	.28680	.29938	3.3402	.95799	20
41	.28708	.29970	3.3367	.95791	19
42	.28736	.30001	3.3332	.95782	18
43	.28764	.30033	3.3297	.95774	17
44	.28792	.30065	3.3261	.95766	16
45	.28820	.30097	3.3226	.95757	15
46	.28847	.30128	3.3191	.95749	14
47	.28875	.30160	3.3156	.95740	13
48	.28903	.30192	3.3122	.95732	12
49	.28931	.30224	3.3087	.95724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	.30287	3.3017	.95707	9
52	.29015	.30319	3.2983	.95698	8
53	.29042	.30351	3.2948	.95690	7
54	.29070	.30382	3.2914	.95681	6
55	.29098	.30414	3.2879	.95673	5
56	.29126	.30446	3.2845	.95664	4
57	.29154	.30478	3.2811	.95656	3
58	.29182	.30509	3.2777	.95647	2
59	.29209	.30541	3.2743	.95639	1
60	.29237	.30573	3.2709	.95630	0
′	Cos	Ctn	Tan	Sin	′

106° (286°) (253°) **73°**

17° (197°) (342°) **162°**

′	Sin	Tan	Ctn	Cos	′
0	.29237	.30573	3.2709	.95630	60
1	.29265	.30605	3.2675	.95622	59
2	.29293	.30637	3.2641	.95613	58
3	.29321	.30669	3.2607	.95605	57
4	.29348	.30700	3.2573	.95596	56
5	.29376	.30732	3.2539	.95588	55
6	.29404	.30764	3.2506	.95579	54
7	.29432	.30796	3.2472	.95571	53
8	.29460	.30828	3.2438	.95562	52
9	.29487	.30860	3.2405	.95554	51
10	.29515	.30891	3.2371	.95545	50
11	.29543	.30923	3.2338	.95536	49
12	.29571	.30955	3.2305	.95528	48
13	.29599	.30987	3.2272	.95519	47
14	.29626	.31019	3.2238	.95511	46
15	.29654	.31051	3.2205	.95502	45
16	.29682	.31083	3.2172	.95493	44
17	.29710	.31115	3.2139	.95485	43
18	.29737	.31147	3.2106	.95476	42
19	.29765	.31178	3.2073	.95467	41
20	.29793	.31210	3.2041	.95459	40
21	.29821	.31242	3.2008	.95450	39
22	.29849	.31274	3.1975	.95441	38
23	.29876	.31306	3.1943	.95433	37
24	.29904	.31338	3.1910	.95424	36
25	.29932	.31370	3.1878	.95415	35
26	.29960	.31402	3.1845	.95407	34
27	.29987	.31434	3.1813	.95398	33
28	.30015	.31466	3.1780	.95389	32
29	.30043	.31498	3.1748	.95380	31
30	.30071	.31530	3.1716	.95372	30
31	.30098	.31562	3.1684	.95363	29
32	.30126	.31594	3.1652	.95354	28
33	.30154	.31626	3.1620	.95345	27
34	.30182	.31658	3.1588	.95337	26
35	.30209	.31690	3.1556	.95328	25
36	.30237	.31722	3.1524	.95319	24
37	.30265	.31754	3.1492	.95310	23
38	.30292	.31786	3.1460	.95301	22
39	.30320	.31818	3.1429	.95293	21
40	.30348	.31850	3.1397	.95284	20
41	.30376	.31882	3.1366	.95275	19
42	.30403	.31914	3.1334	.95266	18
43	.30431	.31946	3.1303	.95257	17
44	.30459	.31978	3.1271	.95248	16
45	.30486	.32010	3.1240	.95240	15
46	.30514	.32042	3.1209	.95231	14
47	.30542	.32074	3.1178	.95222	13
48	.30570	.32106	3.1146	.95213	12
49	.30597	.32139	3.1115	.95204	11
50	.30625	.32171	3.1084	.95195	10
51	.30653	.32203	3.1053	.95186	9
52	.30680	.32235	3.1022	.95177	8
53	.30708	.32267	3.0991	.95168	7
54	.30736	.32299	3.0961	.95159	6
55	.30763	.32331	3.0930	.95150	5
56	.30791	.32363	3.0899	.95142	4
57	.30819	.32396	3.0868	.95133	3
58	.30846	.32428	3.0838	.95124	2
59	.30874	.32460	3.0807	.95115	1
60	.30902	.32492	3.0777	.95106	0
′	Cos	Ctn	Tan	Sin	′

107° (287°) (252°) **72°**

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

18° (198°) (341°) **161°**

′	Sin	Tan	Ctn	Cos	′
0	.30902	.32492	3.0777	.95106	60
1	.30929	.32524	3.0746	.95097	59
2	.30957	.32556	3.0716	.95088	58
3	.30985	.32588	3.0686	.95079	57
4	.31012	.32621	3.0655	.95070	56
5	.31040	.32653	3.0625	.95061	55
6	.31068	.32685	3.0595	.95052	54
7	.31095	.32717	3.0565	.95043	53
8	.31123	.32749	3.0535	.95033	52
9	.31151	.32782	3.0505	.95024	51
10	.31178	.32814	3.0475	.95015	50
11	.31206	.32846	3.0445	.95006	49
12	.31233	.32878	3.0415	.94997	48
13	.31261	.32911	3.0385	.94988	47
14	.31289	.32943	3.0356	.94979	46
15	.31316	.32975	3.0326	.94970	45
16	.31344	.33007	3.0296	.94961	44
17	.31372	.33040	3.0267	.94952	43
18	.31399	.33072	3.0237	.94943	42
19	.31427	.33104	3.0208	.94933	41
20	.31454	.33136	3.0178	.94924	40
21	.31482	.33169	3.0149	.94915	39
22	.31510	.33201	3.0120	.94906	38
23	.31537	.33233	3.0090	.94897	37
24	.31565	.33266	3.0061	.94888	36
25	.31593	.33298	3.0032	.94878	35
26	.31620	.33330	3.0003	.94869	34
27	.31648	.33363	2.9974	.94860	33
28	.31675	.33395	2.9945	.94851	32
29	.31703	.33427	2.9916	.94842	31
30	.31730	.33460	2.9887	.94832	30
31	.31758	.33492	2.9858	.94823	29
32	.31786	.33524	2.9829	.94814	28
33	.31813	.33557	2.9800	.94805	27
34	.31841	.33589	2.9772	.94795	26
35	.31868	.33621	2.9743	.94786	25
36	.31896	.33654	2.9714	.94777	24
37	.31923	.33686	2.9686	.94768	23
38	.31951	.33718	2.9657	.94758	22
39	.31979	.33751	2.9629	.94749	21
40	.32006	.33783	2.9600	.94740	20
41	.32034	.33816	2.9572	.94730	19
42	.32061	.33848	2.9544	.94721	18
43	.32089	.33881	2.9515	.94712	17
44	.32116	.33913	2.9487	.94702	16
45	.32144	.33945	2.9459	.94693	15
46	.32171	.33978	2.9431	.94684	14
47	.32199	.34010	2.9403	.94674	13
48	.32227	.34043	2.9375	.94665	12
49	.32254	.34075	2.9347	.94656	11
50	.32282	.34108	2.9319	.94646	10
51	.32309	.34140	2.9291	.94637	9
52	.32337	.34173	2.9263	.94627	8
53	.32364	.34205	2.9235	.94618	7
54	.32392	.34238	2.9208	.94609	6
55	.32419	.34270	2.9180	.94599	5
56	.32447	.34303	2.9152	.94590	4
57	.32474	.34335	2.9125	.94580	3
58	.32502	.34368	2.9097	.94571	2
59	.32529	.34400	2.9070	.94561	1
60	.32557	.34433	2.9042	.94552	0
′	Cos	Ctn	Tan	Sin	′

19° (199°) (340°) **160°**

′	Sin	Tan	Ctn	Cos	′
0	.32557	.34433	2.9042	.94552	60
1	.32584	.34465	2.9015	.94542	59
2	.32612	.34498	2.8987	.94533	58
3	.32639	.34530	2.8960	.94523	57
4	.32667	.34563	2.8933	.94514	56
5	.32694	.34596	2.8905	.94504	55
6	.32722	.34628	2.8878	.94495	54
7	.32749	.34661	2.8851	.94485	53
8	.32777	.34693	2.8824	.94476	52
9	.32804	.34726	2.8797	.94466	51
10	.32832	.34758	2.8770	.94457	50
11	.32859	.34791	2.8743	.94447	49
12	.32887	.34824	2.8716	.94438	48
13	.32914	.34856	2.8689	.94428	47
14	.32942	.34889	2.8662	.94418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	.34954	2.8609	.94399	44
17	.33024	.34987	2.8582	.94390	43
18	.33051	.35020	2.8556	.94380	42
19	.33079	.35052	2.8529	.94370	41
20	.33106	.35085	2.8502	.94361	40
21	.33134	.35118	2.8476	.94351	39
22	.33161	.35150	2.8449	.94342	38
23	.33189	.35183	2.8423	.94332	37
24	.33216	.35216	2.8397	.94322	36
25	.33244	.35248	2.8370	.94313	35
26	.33271	.35281	2.8344	.94303	34
27	.33298	.35314	2.8318	.94293	33
28	.33326	.35346	2.8291	.94284	32
29	.33353	.35379	2.8265	.94274	31
30	.33381	.35412	2.8239	.94264	30
31	.33408	.35445	2.8213	.94254	29
32	.33436	.35477	2.8187	.94245	28
33	.33463	.35510	2.8161	.94235	27
34	.33490	.35543	2.8135	.94225	26
35	.33518	.35576	2.8109	.94215	25
36	.33545	.35608	2.8083	.94206	24
37	.33573	.35641	2.8057	.94196	23
38	.33600	.35674	2.8032	.94186	22
39	.33627	.35707	2.8006	.94176	21
40	.33655	.35740	2.7980	.94167	20
41	.33682	.35772	2.7955	.94157	19
42	.33710	.35805	2.7929	.94147	18
43	.33737	.35838	2.7903	.94137	17
44	.33764	.35871	2.7878	.94127	16
45	.33792	.35904	2.7852	.94118	15
46	.33819	.35937	2.7827	.94108	14
47	.33846	.35969	2.7801	.94098	13
48	.33874	.36002	2.7776	.94088	12
49	.33901	.36035	2.7751	.94078	11
50	.33929	.36068	2.7725	.94068	10
51	.33956	.36101	2.7700	.94058	9
52	.33983	.36134	2.7675	.94049	8
53	.34011	.36167	2.7650	.94039	7
54	.34038	.36199	2.7625	.94029	6
55	.34065	.36232	2.7600	.94019	5
56	.34093	.36265	2.7575	.94009	4
57	.34120	.36298	2.7550	.93999	3
58	.34147	.36331	2.7525	.93989	2
59	.34175	.36364	2.7500	.93979	1
60	.34202	.36397	2.7475	.93969	0
′	Cos	Ctn	Tan	Sin	′

108° (288°) (251°) **71°**

109° (289°) (250°) **70°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

20° (200°) (339°) **159°**

′	Sin	Tan	Ctn	Cos	′
0	.34202	.36397	2.7475	.93969	60
1	.34229	.36430	2.7450	.93959	59
2	.34257	.36463	2.7425	.93949	58
3	.34284	.36496	2.7400	.93939	57
4	.34311	.36529	2.7376	.93929	56
5	.34339	.36562	2.7351	.93919	55
6	.34366	.36595	2.7326	.93909	54
7	.34393	.36628	2.7302	.93899	53
8	.34421	.36661	2.7277	.93889	52
9	.34448	.36694	2.7253	.93879	51
10	.34475	.36727	2.7228	.93869	50
11	.34503	.36760	2.7204	.93859	49
12	.34530	.36793	2.7179	.93849	48
13	.34557	.36826	2.7155	.93839	47
14	.34584	.36859	2.7130	.93829	46
15	.34612	.36892	2.7106	.93819	45
16	.34639	.36925	2.7082	.93809	44
17	.34666	.36958	2.7058	.93799	43
18	.34694	.36991	2.7034	.93789	42
19	.34721	.37024	2.7009	.93779	41
20	.34748	.37057	2.6985	.93769	40
21	.34775	.37090	2.6961	.93759	39
22	.34803	.37123	2.6937	.93748	38
23	.34830	.37157	2.6913	.93738	37
24	.34857	.37190	2.6889	.93728	36
25	.34884	.37223	2.6865	.93718	35
26	.34912	.37256	2.6841	.93708	34
27	.34939	.37289	2.6818	.93698	33
28	.34966	.37322	2.6794	.93688	32
29	.34993	.37355	2.6770	.93677	31
30	.35021	.37388	2.6746	.93667	30
31	.35048	.37422	2.6723	.93657	29
32	.35075	.37455	2.6699	.93647	28
33	.35102	.37488	2.6675	.93637	27
34	.35130	.37521	2.6652	.93626	26
35	.35157	.37554	2.6628	.93616	25
36	.35184	.37588	2.6605	.93606	24
37	.35211	.37621	2.6581	.93596	23
38	.35239	.37654	2.6558	.93585	22
39	.35266	.37687	2.6534	.93575	21
40	.35293	.37720	2.6511	.93565	20
41	.35320	.37754	2.6488	.93555	19
42	.35347	.37787	2.6464	.93544	18
43	.35375	.37820	2.6441	.93534	17
44	.35402	.37853	2.6418	.93524	16
45	.35429	.37887	2.6395	.93514	15
46	.35456	.37920	2.6371	.93503	14
47	.35484	.37953	2.6348	.93493	13
48	.35511	.37986	2.6325	.93483	12
49	.35538	.38020	2.6302	.93472	11
50	.35565	.38053	2.6279	.93462	10
51	.35592	.38086	2.6256	.93452	9
52	.35619	.38120	2.6233	.93441	8
53	.35647	.38153	2.6210	.93431	7
54	.35674	.38186	2.6187	.93420	6
55	.35701	.38220	2.6165	.93410	5
56	.35728	.38253	2.6142	.93400	4
57	.35755	.38286	2.6119	.93389	3
58	.35782	.38320	2.6096	.93379	2
59	.35810	.38353	2.6074	.93368	1
60	.35837	.38386	2.6051	.93358	0
′	Cos	Ctn	Tan	Sin	′

110° (290°) (249°) **69°**

21° (201°) (338°) **158°**

′	Sin	Tan	Ctn	Cos	′
0	.35837	.38386	2.6051	.93358	60
1	.35864	.38420	2.6028	.93348	59
2	.35891	.38453	2.6006	.93337	58
3	.35918	.38487	2.5983	.93327	57
4	.35945	.38520	2.5961	.93316	56
5	.35973	.38553	2.5938	.93306	55
6	.36000	.38587	2.5916	.93295	54
7	.36027	.38620	2.5893	.93285	53
8	.36054	.38654	2.5871	.93274	52
9	.36081	.38687	2.5848	.93264	51
10	.36108	.38721	2.5826	.93253	50
11	.36135	.38754	2.5804	.93243	49
12	.36162	.38787	2.5782	.93232	48
13	.36190	.38821	2.5759	.93222	47
14	.36217	.38854	2.5737	.93211	46
15	.36244	.38888	2.5715	.93201	45
16	.36271	.38921	2.5693	.93190	44
17	.36298	.38955	2.5671	.93180	43
18	.36325	.38988	2.5649	.93169	42
19	.36352	.39022	2.5627	.93159	41
20	.36379	.39055	2.5605	.93148	40
21	.36406	.39089	2.5583	.93137	39
22	.36434	.39122	2.5561	.93127	38
23	.36461	.39156	2.5539	.93116	37
24	.36488	.39190	2.5517	.93106	36
25	.36515	.39223	2.5495	.93095	35
26	.36542	.39257	2.5473	.93084	34
27	.36569	.39290	2.5452	.93074	33
28	.36596	.39324	2.5430	.93063	32
29	.36623	.39357	2.5408	.93052	31
30	.36650	.39391	2.5386	.93042	30
31	.36677	.39425	2.5365	.93031	29
32	.36704	.39458	2.5343	.93020	28
33	.36731	.39492	2.5322	.93010	27
34	.36758	.39526	2.5300	.92999	26
35	.36785	.39559	2.5279	.92988	25
36	.36812	.39593	2.5257	.92978	24
37	.36839	.39626	2.5236	.92967	23
38	.36867	.39660	2.5214	.92956	22
39	.36894	.39694	2.5193	.92945	21
40	.36921	.39727	2.5172	.92935	20
41	.36948	.39761	2.5150	.92924	19
42	.36975	.39795	2.5129	.92913	18
43	.37002	.39829	2.5108	.92902	17
44	.37029	.39862	2.5086	.92892	16
45	.37056	.39896	2.5065	.92881	15
46	.37083	.39930	2.5044	.92870	14
47	.37110	.39963	2.5023	.92859	13
48	.37137	.39997	2.5002	.92849	12
49	.37164	.40031	2.4981	.92838	11
50	.37191	.40065	2.4960	.92827	10
51	.37218	.40098	2.4939	.92816	9
52	.37245	.40132	2.4918	.92805	8
53	.37272	.40166	2.4897	.92794	7
54	.37299	.40200	2.4876	.92784	6
55	.37326	.40234	2.4855	.92773	5
56	.37353	.40267	2.4834	.92762	4
57	.37380	.40301	2.4813	.92751	3
58	.37407	.40335	2.4792	.92740	2
59	.37434	.40369	2.4772	.92729	1
60	.37461	.40403	2.4751	.92718	0
′	Cos	Ctn	Tan	Sin	′

111° (291°) (248°) **68°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

22° (202°) (337°) **157°** | **23°** (203°) (336°) **156°**

′	Sin	Tan	Ctn	Cos	′	′	Sin	Tan	Ctn	Cos	′
0	.37461	.40403	2.4751	.92718	60	0	.39073	.42447	2.3559	.92050	60
1	.37488	.40436	2.4730	.92707	59	1	.39100	.42482	2.3539	.92039	59
2	.37515	.40470	2.4709	.92697	58	2	.39127	.42516	2.3520	.92028	58
3	.37542	.40504	2.4689	.92686	57	3	.39153	.42551	2.3501	.92016	57
4	.37569	.40538	2.4668	.92675	56	4	.39180	.42585	2.3483	.92005	56
5	.37595	.40572	2.4648	.92664	55	5	.39207	.42619	2.3464	.91994	55
6	.37622	.40606	2.4627	.92653	54	6	.39234	.42654	2.3445	.91982	54
7	.37649	.40640	2.4606	.92642	53	7	.39260	.42688	2.3426	.91971	53
8	.37676	.40674	2.4586	.92631	52	8	.39287	.42722	2.3407	.91959	52
9	.37703	.40707	2.4566	.92620	51	9	.39314	.42757	2.3388	.91948	51
10	.37730	.40741	2.4545	.92609	50	10	.39341	.42791	2.3369	.91936	50
11	.37757	.40775	2.4525	.92598	49	11	.39367	.42826	2.3351	.91925	49
12	.37784	.40809	2.4504	.92587	48	12	.39394	.42860	2.3332	.91914	48
13	.37811	.40843	2.4484	.92576	47	13	.39421	.42894	2.3313	.91902	47
14	.37838	.40877	2.4464	.92565	46	14	.39448	.42929	2.3294	.91891	46
15	.37865	.40911	2.4443	.92554	45	15	.39474	.42963	2.3276	.91879	45
16	.37892	.40945	2.4423	.92543	44	16	.39501	.42998	2.3257	.91868	44
17	.37919	.40979	2.4403	.92532	43	17	.39528	.43032	2.3238	.91856	43
18	.37946	.41013	2.4383	.92521	42	18	.39555	.43067	2.3220	.91845	42
19	.37973	.41047	2.4362	.92510	41	19	.39581	.43101	2.3201	.91833	41
20	.37999	.41081	2.4342	.92499	40	20	.39608	.43136	2.3183	.91822	40
21	.38026	.41115	2.4322	.92488	39	21	.39635	.43170	2.3164	.91810	39
22	.38053	.41149	2.4302	.92477	38	22	.39661	.43205	2.3146	.91799	38
23	.38080	.41183	2.4282	.92466	37	23	.39688	.43239	2.3127	.91787	37
24	.38107	.41217	2.4262	.92455	36	24	.39715	.43274	2.3109	.91775	36
25	.38134	.41251	2.4242	.92444	35	25	.39741	.43308	2.3090	.91764	35
26	.38161	.41285	2.4222	.92432	34	26	.39768	.43343	2.3072	.91752	34
27	.38188	.41319	2.4202	.92421	33	27	.39795	.43378	2.3053	.91741	33
28	.38215	.41353	2.4182	.92410	32	28	.39822	.43412	2.3035	.91729	32
29	.38241	.41387	2.4162	.92399	31	29	.39848	.43447	2.3017	.91718	31
30	.38268	.41421	2.4142	.92388	30	30	.39875	.43481	2.2998	.91706	30
31	.38295	.41455	2.4122	.92377	29	31	.39902	.43516	2.2980	.91694	29
32	.38322	.41490	2.4102	.92366	28	32	.39928	.43550	2.2962	.91683	28
33	.38349	.41524	2.4083	.92355	27	33	.39955	.43585	2.2944	.91671	27
34	.38376	.41558	2.4063	.92343	26	34	.39982	.43620	2.2925	.91660	26
35	.38403	.41592	2.4043	.92332	25	35	.40008	.43654	2.2907	.91648	25
36	.38430	.41626	2.4023	.92321	24	36	.40035	.43689	2.2889	.91636	24
37	.38456	.41660	2.4004	.92310	23	37	.40062	.43724	2.2871	.91625	23
38	.38483	.41694	2.3984	.92299	22	38	.40088	.43758	2.2853	.91613	22
39	.38510	.41728	2.3964	.92287	21	39	.40115	.43793	2.2835	.91601	21
40	.38537	.41763	2.3945	.92276	20	40	.40141	.43828	2.2817	.91590	20
41	.38564	.41797	2.3925	.92265	19	41	.40168	.43862	2.2799	.91578	19
42	.38591	.41831	2.3906	.92254	18	42	.40195	.43897	2.2781	.91566	18
43	.38617	.41865	2.3886	.92243	17	43	.40221	.43932	2.2763	.91555	17
44	.38644	.41899	2.3867	.92231	16	44	.40248	.43966	2.2745	.91543	16
45	.38671	.41933	2.3847	.92220	15	45	.40275	.44001	2.2727	.91531	15
46	.38698	.41968	2.3828	.92209	14	46	.40301	.44036	2.2709	.91519	14
47	.38725	.42002	2.3808	.92198	13	47	.40328	.44071	2.2691	.91508	13
48	.38752	.42036	2.3789	.92186	12	48	.40355	.44105	2.2673	.91496	12
49	.38778	.42070	2.3770	.92175	11	49	.40381	.44140	2.2655	.91484	11
50	.38805	.42105	2.3750	.92164	10	50	.40408	.44175	2.2637	.91472	10
51	.38832	.42139	2.3731	.92152	9	51	.40434	.44210	2.2620	.91461	9
52	.38859	.42173	2.3712	.92141	8	52	.40461	.44244	2.2602	.91449	8
53	.38886	.42207	2.3693	.92130	7	53	.40488	.44279	2.2584	.91437	7
54	.38912	.42242	2.3673	.92119	6	54	.40514	.44314	2.2566	.91425	6
55	.38939	.42276	2.3654	.92107	5	55	.40541	.44349	2.2549	.91414	5
56	.38966	.42310	2.3635	.92096	4	56	.40567	.44384	2.2531	.91402	4
57	.38993	.42345	2.3616	.92085	3	57	.40594	.44418	2.2513	.91390	3
58	.39020	.42379	2.3597	.92073	2	58	.40621	.44453	2.2496	.91378	2
59	.39046	.42413	2.3578	.92062	1	59	.40647	.44488	2.2478	.91366	1
60	.39073	.42447	2.3559	.92050	0	60	.40674	.44523	2.2460	.91355	0
′	Cos	Ctn	Tan	Sin	′	′	Cos	Ctn	Tan	Sin	′

112° (292°) (247°) **67°** | **113°** (293°) (246°) **66°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

24° (204°) (335°) **155°**

′	Sin	Tan	Ctn	Cos	′
0	.40674	.44523	2.2460	.91355	60
1	.40700	.44558	2.2443	.91343	59
2	.40727	.44593	2.2425	.91331	58
3	.40753	.44627	2.2408	.91319	57
4	.40780	.44662	2.2390	.91307	56
5	.40806	.44697	2.2373	.91295	55
6	.40833	.44732	2.2355	.91283	54
7	.40860	.44767	2.2338	.91272	53
8	.40886	.44802	2.2320	.91260	52
9	.40913	.44837	2.2303	.91248	51
10	.40939	.44872	2.2286	.91236	50
11	.40966	.44907	2.2268	.91224	49
12	.40992	.44942	2.2251	.91212	48
13	.41019	.44977	2.2234	.91200	47
14	.41045	.45012	2.2216	.91188	46
15	.41072	.45047	2.2199	.91176	45
16	.41098	.45082	2.2182	.91164	44
17	.41125	.45117	2.2165	.91152	43
18	.41151	.45152	2.2148	.91140	42
19	.41178	.45187	2.2130	.91128	41
20	.41204	.45222	2.2113	.91116	40
21	.41231	.45257	2.2096	.91104	39
22	.41257	.45292	2.2079	.91092	38
23	.41284	.45327	2.2062	.91080	37
24	.41310	.45362	2.2045	.91068	36
25	.41337	.45397	2.2028	.91056	35
26	.41363	.45432	2.2011	.91044	34
27	.41390	.45467	2.1994	.91032	33
28	.41416	.45502	2.1977	.91020	32
29	.41443	.45538	2.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	.41496	.45608	2.1926	.90984	29
32	.41522	.45643	2.1909	.90972	28
33	.41549	.45678	2.1892	.90960	27
34	.41575	.45713	2.1876	.90948	26
35	.41602	.45748	2.1859	.90936	25
36	.41628	.45784	2.1842	.90924	24
37	.41655	.45819	2.1825	.90911	23
38	.41681	.45854	2.1808	.90899	22
39	.41707	.45889	2.1792	.90887	21
40	.41734	.45924	2.1775	.90875	20
41	.41760	.45960	2.1758	.90863	19
42	.41787	.45995	2.1742	.90851	18
43	.41813	.46030	2.1725	.90839	17
44	.41840	.46065	2.1708	.90826	16
45	.41866	.46101	2.1692	.90814	15
46	.41892	.46136	2.1675	.90802	14
47	.41919	.46171	2.1659	.90790	13
48	.41945	.46206	2.1642	.90778	12
49	.41972	.46242	2.1625	.90766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	.46312	2.1592	.90741	9
52	.42051	.46348	2.1576	.90729	8
53	.42077	.46383	2.1560	.90717	7
54	.42104	.46418	2.1543	.90704	6
55	.42130	.46454	2.1527	.90692	5
56	.42156	.46489	2.1510	.90680	4
57	.42183	.46525	2.1494	.90668	3
58	.42209	.46560	2.1478	.90655	2
59	.42235	.46595	2.1461	.90643	1
60	.42262	.46631	2.1445	.90631	0
′	Cos	Ctn	Tan	Sin	′

114° (294°) (245°) **65°**

25° (205°) (334°) **154°**

′	Sin	Tan	Ctn	Cos	′
0	.42262	.46631	2.1445	.90631	60
1	.42288	.46666	2.1429	.90618	59
2	.42315	.46702	2.1413	.90606	58
3	.42341	.46737	2.1396	.90594	57
4	.42367	.46772	2.1380	.90582	56
5	.42394	.46808	2.1364	.90569	55
6	.42420	.46843	2.1348	.90557	54
7	.42446	.46879	2.1332	.90545	53
8	.42473	.46914	2.1315	.90532	52
9	.42499	.46950	2.1299	.90520	51
10	.42525	.46985	2.1283	.90507	50
11	.42552	.47021	2.1267	.90495	49
12	.42578	.47056	2.1251	.90483	48
13	.42604	.47092	2.1235	.90470	47
14	.42631	.47128	2.1219	.90458	46
15	.42657	.47163	2.1203	.90446	45
16	.42683	.47199	2.1187	.90433	44
17	.42709	.47234	2.1171	.90421	43
18	.42736	.47270	2.1155	.90408	42
19	.42762	.47305	2.1139	.90396	41
20	.42788	.47341	2.1123	.90383	40
21	.42815	.47377	2.1107	.90371	39
22	.42841	.47412	2.1092	.90358	38
23	.42867	.47448	2.1076	.90346	37
24	.42894	.47483	2.1060	.90334	36
25	.42920	.47519	2.1044	.90321	35
26	.42946	.47555	2.1028	.90309	34
27	.42972	.47590	2.1013	.90296	33
28	.42999	.47626	2.0997	.90284	32
29	.43025	.47662	2.0981	.90271	31
30	.43051	.47698	2.0965	.90259	30
31	.43077	.47733	2.0950	.90246	29
32	.43104	.47769	2.0934	.90233	28
33	.43130	.47805	2.0918	.90221	27
34	.43156	.47840	2.0903	.90208	26
35	.43182	.47876	2.0887	.90196	25
36	.43209	.47912	2.0872	.90183	24
37	.43235	.47948	2.0856	.90171	23
38	.43261	.47984	2.0840	.90158	22
39	.43287	.48019	2.0825	.90146	21
40	.43313	.48055	2.0809	.90133	20
41	.43340	.48091	2.0794	.90120	19
42	.43366	.48127	2.0778	.90108	18
43	.43392	.48163	2.0763	.90095	17
44	.43418	.48198	2.0748	.90082	16
45	.43445	.48234	2.0732	.90070	15
46	.43471	.48270	2.0717	.90057	14
47	.43497	.48306	2.0701	.90045	13
48	.43523	.48342	2.0686	.90032	12
49	.43549	.48378	2.0671	.90019	11
50	.43575	.48414	2.0655	.90007	10
51	.43602	.48450	2.0640	.89994	9
52	.43628	.48486	2.0625	.89981	8
53	.43654	.48521	2.0609	.89968	7
54	.43680	.48557	2.0594	.89956	6
55	.43706	.48593	2.0579	.89943	5
56	.43733	.48629	2.0564	.89930	4
57	.43759	.48665	2.0549	.89918	3
58	.43785	.48701	2.0533	.89905	2
59	.43811	.48737	2.0518	.89892	1
60	.43837	.48773	2.0503	.89879	0
′	Cos	Ctn	Tan	Sin	′

115° (295°) (244°) **64°**

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

26° (206°) (333°) **153°**

′	Sin	Tan	Ctn	Cos	′
0	.43837	.48773	2.0503	.89879	60
1	.43863	.48809	2.0488	.89867	59
2	.43889	.48845	2.0473	.89854	58
3	.43916	.48881	2.0458	.89841	57
4	.43942	.48917	2.0443	.89828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	2.0413	.89803	54
7	.44020	.49026	2.0398	.89790	53
8	.44046	.49062	2.0383	.89777	52
9	.44072	.49098	2.0368	.89764	51
10	.44098	.49134	2.0353	.89752	50
11	.44124	.49170	2.0338	.89739	49
12	.44151	.49206	2.0323	.89726	48
13	.44177	.49242	2.0308	.89713	47
14	.44203	.49278	2.0293	.89700	46
15	.44229	.49315	2.0278	.89687	45
16	.44255	.49351	2.0263	.89674	44
17	.44281	.49387	2.0248	.89662	43
18	.44307	.49423	2.0233	.89649	42
19	.44333	.49459	2.0219	.89636	41
20	.44359	.49495	2.0204	.89623	40
21	.44385	.49532	2.0189	.89610	39
22	.44411	.49568	2.0174	.89597	38
23	.44437	.49604	2.0160	.89584	37
24	.44464	.49640	2.0145	.89571	36
25	.44490	.49677	2.0130	.89558	35
26	.44516	.49713	2.0115	.89545	34
27	.44542	.49749	2.0101	.89532	33
28	.44568	.49786	2.0086	.89519	32
29	.44594	.49822	2.0072	.89506	31
30	.44620	.49858	2.0057	.89493	30
31	.44646	.49894	2.0042	.89480	29
32	.44672	.49931	2.0028	.89467	28
33	.44698	.49967	2.0013	.89454	27
34	.44724	.50004	1.9999	.89441	26
35	.44750	.50040	1.9984	.89428	25
36	.44776	.50076	1.9970	.89415	24
37	.44802	.50113	1.9955	.89402	23
38	.44828	.50149	1.9941	.89389	22
39	.44854	.50185	1.9926	.89376	21
40	.44880	.50222	1.9912	.89363	20
41	.44906	.50258	1.9897	.89350	19
42	.44932	.50295	1.9883	.89337	18
43	.44958	.50331	1.9868	.89324	17
44	.44984	.50368	1.9854	.89311	16
45	.45010	.50404	1.9840	.89298	15
46	.45036	.50441	1.9825	.89285	14
47	.45062	.50477	1.9811	.89272	13
48	.45088	.50514	1.9797	.89259	12
49	.45114	.50550	1.9782	.89245	11
50	.45140	.50587	1.9768	.89232	10
51	.45166	.50623	1.9754	.89219	9
52	.45192	.50660	1.9740	.89206	8
53	.45218	.50696	1.9725	.89193	7
54	.45243	.50733	1.9711	.89180	6
55	.45269	.50769	1.9697	.89167	5
56	.45295	.50806	1.9683	.89153	4
57	.45321	.50843	1.9669	.89140	3
58	.45347	.50879	1.9654	.89127	2
59	.45373	.50916	1.9640	.89114	1
60	.45399	.50953	1.9626	.89101	0
′	Cos	Ctn	Tan	Sin	′

116° (296°) (243°) **63°**

27° (207°) (332°) **152°**

′	Sin	Tan	Ctn	Cos	′
0	.45399	.50953	1.9626	.89101	60
1	.45425	.50989	1.9612	.89087	59
2	.45451	.51026	1.9598	.89074	58
3	.45477	.51063	1.9584	.89061	57
4	.45503	.51099	1.9570	.89048	56
5	.45529	.51136	1.9556	.89035	55
6	.45554	.51173	1.9542	.89021	54
7	.45580	.51209	1.9528	.89008	53
8	.45606	.51246	1.9514	.88995	52
9	.45632	.51283	1.9500	.88981	51
10	.45658	.51319	1.9486	.88968	50
11	.45684	.51356	1.9472	.88955	49
12	.45710	.51393	1.9458	.88942	48
13	.45736	.51430	1.9444	.88928	47
14	.45762	.51467	1.9430	.88915	46
15	.45787	.51503	1.9416	.88902	45
16	.45813	.51540	1.9402	.88888	44
17	.45839	.51577	1.9388	.88875	43
18	.45865	.51614	1.9375	.88862	42
19	.45891	.51651	1.9361	.88848	41
20	.45917	.51688	1.9347	.88835	40
21	.45942	.51724	1.9333	.88822	39
22	.45968	.51761	1.9319	.88808	38
23	.45994	.51798	1.9306	.88795	37
24	.46020	.51835	1.9292	.88782	36
25	.46046	.51872	1.9278	.88768	35
26	.46072	.51909	1.9265	.88755	34
27	.46097	.51946	1.9251	.88741	33
28	.46123	.51983	1.9237	.88728	32
29	.46149	.52020	1.9223	.88715	31
30	.46175	.52057	1.9210	.88701	30
31	.46201	.52094	1.9196	.88688	29
32	.46226	.52131	1.9183	.88674	28
33	.46252	.52168	1.9169	.88661	27
34	.46278	.52205	1.9155	.88647	26
35	.46304	.52242	1.9142	.88634	25
36	.46330	.52279	1.9128	.88620	24
37	.46355	.52316	1.9115	.88607	23
38	.46381	.52353	1.9101	.88593	22
39	.46407	.52390	1.9088	.88580	21
40	.46433	.52427	1.9074	.88566	20
41	.46458	.52464	1.9061	.88553	19
42	.46484	.52501	1.9047	.88539	18
43	.46510	.52538	1.9034	.88526	17
44	.46536	.52575	1.9020	.88512	16
45	.46561	.52613	1.9007	.88499	15
46	.46587	.52650	1.8993	.88485	14
47	.46613	.52687	1.8980	.88472	13
48	.46639	.52724	1.8967	.88458	12
49	.46664	.52761	1.8953	.88445	11
50	.46690	.52798	1.8940	.88431	10
51	.46716	.52836	1.8927	.88417	9
52	.46742	.52873	1.8913	.88404	8
53	.46767	.52910	1.8900	.88390	7
54	.46793	.52947	1.8887	.88377	6
55	.46819	.52985	1.8873	.88363	5
56	.46844	.53022	1.8860	.88349	4
57	.46870	.53059	1.8847	.88336	3
58	.46896	.53096	1.8834	.88322	2
59	.46921	.53134	1.8820	.88308	1
60	.46947	.53171	1.8807	.88295	0
′	Cos	Ctn	Tan	Sin	′

117° (297°) (242°) **62°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

28° (208°) (331°) **151°**

′	Sin	Tan	Ctn	Cos	′
0	.46947	.53171	1.8807	.88295	60
1	.46973	.53208	1.8794	.88281	59
2	.46999	.53246	1.8781	.88267	58
3	.47024	.53283	1.8768	.88254	57
4	.47050	.53320	1.8755	.88240	56
5	.47076	.53358	1.8741	.88226	55
6	.47101	.53395	1.8728	.88213	54
7	.47127	.53432	1.8715	.88199	53
8	.47153	.53470	1.8702	.88185	52
9	.47178	.53507	1.8689	.88172	51
10	.47204	.53545	1.8676	.88158	50
11	.47229	.53582	1.8663	.88144	49
12	.47255	.53620	1.8650	.88130	48
13	.47281	.53657	1.8637	.88117	47
14	.47306	.53694	1.8624	.88103	46
15	.47332	.53732	1.8611	.88089	45
16	.47358	.53769	1.8598	.88075	44
17	.47383	.53807	1.8585	.88062	43
18	.47409	.53844	1.8572	.88048	42
19	.47434	.53882	1.8559	.88034	41
20	.47460	.53920	1.8546	.88020	40
21	.47486	.53957	1.8533	.88006	39
22	.47511	.53995	1.8520	.87993	38
23	.47537	.54032	1.8507	.87979	37
24	.47562	.54070	1.8495	.87965	36
25	.47588	.54107	1.8482	.87951	35
26	.47614	.54145	1.8469	.87937	34
27	.47639	.54183	1.8456	.87923	33
28	.47665	.54220	1.8443	.87909	32
29	.47690	.54258	1.8430	.87896	31
30	.47716	.54296	1.8418	.87882	30
31	.47741	.54333	1.8405	.87868	29
32	.47767	.54371	1.8392	.87854	28
33	.47793	.54409	1.8379	.87840	27
34	.47818	.54446	1.8367	.87826	26
35	.47844	.54484	1.8354	.87812	25
36	.47869	.54522	1.8341	.87798	24
37	.47895	.54560	1.8329	.87784	23
38	.47920	.54597	1.8316	.87770	22
39	.47946	.54635	1.8303	.87756	21
40	.47971	.54673	1.8291	.87743	20
41	.47997	.54711	1.8278	.87729	19
42	.48022	.54748	1.8265	.87715	18
43	.48048	.54786	1.8253	.87701	17
44	.48073	.54824	1.8240	.87687	16
45	.48099	.54862	1.8228	.87673	15
46	.48124	.54900	1.8215	.87659	14
47	.48150	.54938	1.8202	.87645	13
48	.48175	.54975	1.8190	.87631	12
49	.48201	.55013	1.8177	.87617	11
50	.48226	.55051	1.8165	.87603	10
51	.48252	.55089	1.8152	.87589	9
52	.48277	.55127	1.8140	.87575	8
53	.48303	.55165	1.8127	.87561	7
54	.48328	.55203	1.8115	.87546	6
55	.48354	.55241	1.8103	.87532	5
56	.48379	.55279	1.8090	.87518	4
57	.48405	.55317	1.8078	.87504	3
58	.48430	.55355	1.8065	.87490	2
59	.48456	.55393	1.8053	.87476	1
60	.48481	.55431	1.8040	.87462	0
′	Cos	Ctn	Tan	Sin	′

29° (209°) (330°) **150°**

′	Sin	Tan	Ctn	Cos	′
0	.48481	.55431	1.8040	.87462	60
1	.48506	.55469	1.8028	.87448	59
2	.48532	.55507	1.8016	.87434	58
3	.48557	.55545	1.8003	.87420	57
4	.48583	.55583	1.7991	.87406	56
5	.48608	.55621	1.7979	.87391	55
6	.48634	.55659	1.7966	.87377	54
7	.48659	.55697	1.7954	.87363	53
8	.48684	.55736	1.7942	.87349	52
9	.48710	.55774	1.7930	.87335	51
10	.48735	.55812	1.7917	.87321	50
11	.48761	.55850	1.7905	.87306	49
12	.48786	.55888	1.7893	.87292	48
13	.48811	.55926	1.7881	.87278	47
14	.48837	.55964	1.7868	.87264	46
15	.48862	.56003	1.7856	.87250	45
16	.48888	.56041	1.7844	.87235	44
17	.48913	.56079	1.7832	.87221	43
18	.48938	.56117	1.7820	.87207	42
19	.48964	.56156	1.7808	.87193	41
20	.48989	.56194	1.7796	.87178	40
21	.49014	.56232	1.7783	.87164	39
22	.49040	.56270	1.7771	.87150	38
23	.49065	.56309	1.7759	.87136	37
24	.49090	.56347	1.7747	.87121	36
25	.49116	.56385	1.7735	.87107	35
26	.49141	.56424	1.7723	.87093	34
27	.49166	.56462	1.7711	.87079	33
28	.49192	.56501	1.7699	.87064	32
29	.49217	.56539	1.7687	.87050	31
30	.49242	.56577	1.7675	.87036	30
31	.49268	.56616	1.7663	.87021	29
32	.49293	.56654	1.7651	.87007	28
33	.49318	.56693	1.7639	.86993	27
34	.49344	.56731	1.7627	.86978	26
35	.49369	.56769	1.7615	.86964	25
36	.49394	.56808	1.7603	.86949	24
37	.49419	.56846	1.7591	.86935	23
38	.49445	.56885	1.7579	.86921	22
39	.49470	.56923	1.7567	.86906	21
40	.49495	.56962	1.7556	.86892	20
41	.49521	.57000	1.7544	.86878	19
42	.49546	.57039	1.7532	.86863	18
43	.49571	.57078	1.7520	.86849	17
44	.49596	.57116	1.7508	.86834	16
45	.49622	.57155	1.7496	.86820	15
46	.49647	.57193	1.7485	.86805	14
47	.49672	.57232	1.7473	.86791	13
48	.49697	.57271	1.7461	.86777	12
49	.49723	.57309	1.7449	.86762	11
50	.49748	.57348	1.7437	.86748	10
51	.49773	.57386	1.7426	.86733	9
52	.49798	.57425	1.7414	.86719	8
53	.49824	.57464	1.7402	.86704	7
54	.49849	.57503	1.7391	.86690	6
55	.49874	.57541	1.7379	.86675	5
56	.49899	.57580	1.7367	.86661	4
57	.49924	.57619	1.7355	.86646	3
58	.49950	.57657	1.7344	.86632	2
59	.49975	.57696	1.7332	.86617	1
60	.50000	.57735	1.7321	.86603	0
′	Cos	Ctn	Tan	Sin	′

118° (298°) (241°) **61°**

119° (299°) (240°) **60°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

30° (210°) (329°) **149°**

′	Sin	Tan	Ctn	Cos	′
0	.50000	.57735	1.7321	.86603	60
1	.50025	.57774	1.7309	.86588	59
2	.50050	.57813	1.7297	.86573	58
3	.50076	.57851	1.7286	.86559	57
4	.50101	.57890	1.7274	.86544	56
5	.50126	.57929	1.7262	.86530	55
6	.50151	.57968	1.7251	.86515	54
7	.50176	.58007	1.7239	.86501	53
8	.50201	.58046	1.7228	.86486	52
9	.50227	.58085	1.7216	.86471	51
10	.50252	.58124	1.7205	.86457	50
11	.50277	.58162	1.7193	.86442	49
12	.50302	.48201	1.7182	.86427	48
13	.50327	.58240	1.7170	.86413	47
14	.50352	.58279	1.7159	.86398	46
15	.50377	.58318	1.7147	.86384	45
16	.50403	.58357	1.7136	.86369	44
17	.50428	.58396	1.7124	.86354	43
18	.50453	.58435	1.7113	.86340	42
19	.50478	.58474	1.7102	.86325	41
20	.50503	.58513	1.7090	.86310	40
21	.50528	.58552	1.7079	.86295	39
22	.50553	.58591	1.7067	.86281	38
23	.50578	.58631	1.7056	.86266	37
24	.50603	.58670	1.7045	.86251	36
25	.50628	.58709	1.7033	.86237	35
26	.50654	.58748	1.7022	.86222	34
27	.50679	.58787	1.7011	.86207	33
28	.50704	.58826	1.6999	.86192	32
29	.50729	.58865	1.6988	.86178	31
30	.50754	.58905	1.6977	.86163	30
31	.50779	.58944	1.6965	.86148	29
32	.50804	.58983	1.6954	.86133	28
33	.50829	.59022	1.6943	.86119	27
34	.50854	.59061	1.6932	.86104	26
35	.50879	.59101	1.6920	.86089	25
36	.50904	.59140	1.6909	.86074	24
37	.50929	.59179	1.6898	.86059	23
38	.50954	.59218	1.6887	.86045	22
39	.50979	.59258	1.6875	.86030	21
40	.51004	.59297	1.6864	.86015	20
41	.51029	.59336	1.6853	.86000	19
42	.51054	.59376	1.6842	.85985	18
43	.51079	.59415	1.6831	.85970	17
44	.51104	.59454	1.6820	.85956	16
45	.51129	.59494	1.6808	.85941	15
46	.51154	.59533	1.6797	.85926	14
47	.51179	.59573	1.6786	.85911	13
48	.51204	.59612	1.6775	.85896	12
49	.51229	.59651	1.6764	.85881	11
50	.51254	.59691	1.6753	.85866	10
51	.51279	.59730	1.6742	.85851	9
52	.51304	.59770	1.6731	.85836	8
53	.51329	.59809	1.6720	.85821	7
54	.51354	.59849	1.6709	.85806	6
55	.51379	.59888	1.6698	.85792	5
56	.51404	.59928	1.6687	.85777	4
57	.51429	.59967	1.6676	.85762	3
58	.51454	.60007	1.6665	.85747	2
59	.51479	.60046	1.6654	.85732	1
60	.51504	.60086	1.6643	.85717	0
′	Cos	Ctn	Tan	Sin	′

120° (300°) (239°) **59°**

31° (211°) (328°) **148°**

′	Sin	Tan	Ctn	Cos	′
0	.51504	.60086	1.6643	.85717	60
1	.51529	.60126	1.6632	.85702	59
2	.51554	.60165	1.6621	.85687	58
3	.51579	.60205	1.6610	.85672	57
4	.51604	.60245	1.6599	.85657	56
5	.51628	.60284	1.6588	.85642	55
6	.51653	.60324	1.6577	.85627	54
7	.51678	.60364	1.6566	.85612	53
8	.51703	.60403	1.6555	.85597	52
9	.51728	.60443	1.6545	.85582	51
10	.51753	.60483	1.6534	.85567	50
11	.51778	.60522	1.6523	.85551	49
12	.51803	.60562	1.6512	.85536	48
13	.51828	.60602	1.6501	.85521	47
14	.51852	.60642	1.6490	.85506	46
15	.51877	.60681	1.6479	.85491	45
16	.51902	.60721	1.6469	.85476	44
17	.51927	.60761	1.6458	.85461	43
18	.51952	.60801	1.6447	.85446	42
19	.51977	.60841	1.6436	.85431	41
20	.52002	.60881	1.6426	.85416	40
21	.52026	.60921	1.6415	.85401	39
22	.52051	.60960	1.6404	.85385	38
23	.52076	.61000	1.6393	.85370	37
24	.52101	.61040	1.6383	.85355	36
25	.52126	.61080	1.6372	.85340	35
26	.52151	.61120	1.6361	.85325	34
27	.52175	.61160	1.6351	.85310	33
28	.52200	.61200	1.6340	.85294	32
29	.52225	.61240	1.6329	.85279	31
30	.52250	.61280	1.6319	.85264	30
31	.52275	.61320	1.6308	.85249	29
32	.52299	.61360	1.6297	.85234	28
33	.52324	.61400	1.6287	.85218	27
34	.52349	.61440	1.6276	.85203	26
35	.52374	.61480	1.6265	.85188	25
36	.52399	.61520	1.6255	.85173	24
37	.52423	.61561	1.6244	.85157	23
38	.52448	.61601	1.6234	.85142	22
39	.52473	.61641	1.6223	.85127	21
40	.52498	.61681	1.6212	.85112	20
41	.52522	.61721	1.6202	.85096	19
42	.52547	.61761	1.6191	.85081	18
43	.52572	.61801	1.6181	.85066	17
44	.52597	.61842	1.6170	.85051	16
45	.52621	.61882	1.6160	.85035	15
46	.52646	.61922	1.6149	.85020	14
47	.52671	.61962	1.6139	.85005	13
48	.52696	.62003	1.6128	.84989	12
49	.52720	.62043	1.6118	.84974	11
50	.52745	.62083	1.6107	.84959	10
51	.52770	.62124	1.6097	.84943	9
52	.52794	.62164	1.6087	.84928	8
53	.52819	.62204	1.6076	.84913	7
54	.52844	.62245	1.6066	.84897	6
55	.52869	.62285	1.6055	.84882	5
56	.52893	.62325	1.6045	.84866	4
57	.52918	.62366	1.6034	.84851	3
58	.52943	.62406	1.6024	.84836	2
59	.52967	.62446	1.6014	.84820	1
60	.52992	.62487	1.6003	.84805	0
′	Cos	Ctn	Tan	Sin	′

121° (301°) (238°) **58°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

32° (212°) (327°) **147°**

′	Sin	Tan	Ctn	Cos	′
0	.52992	.62487	1.6003	.84805	60
1	.53017	.62527	1.5993	.84789	59
2	.53041	.62568	1.5983	.84774	58
3	.53066	.62608	1.5972	.84759	57
4	.53091	.62649	1.5962	.84743	56
5	.53115	.62689	1.5952	.84728	55
6	.53140	.62730	1.5941	.84712	54
7	.53164	.62770	1.5931	.84697	53
8	.53189	.62811	1.5921	.84681	52
9	.53214	.62852	1.5911	.84666	51
10	.53238	.62892	1.5900	.84650	50
11	.53263	.62933	1.5890	.84635	49
12	.53288	.62973	1.5880	.84619	48
13	.53312	.63014	1.5869	.84604	47
14	.53337	.63055	1.5859	.84588	46
15	.53361	.63095	1.5849	.84573	45
16	.53386	.63136	1.5839	.84557	44
17	.53411	.63177	1.5829	.84542	43
18	.53435	.63217	1.5818	.84526	42
19	.53460	.63258	1.5808	.84511	41
20	.53484	.63299	1.5798	.84495	40
21	.53509	.63340	1.5788	.84480	39
22	.53534	.63380	1.5778	.84464	38
23	.53558	.63421	1.5768	.84448	37
24	.53583	.63462	1.5757	.84433	36
25	.53607	.63503	1.5747	.84417	35
26	.53632	.63544	1.5737	.84402	34
27	.53656	.63584	1.5727	.84386	33
28	.53681	.63625	1.5717	.84370	32
29	.53705	.63666	1.5707	.84355	31
30	.53730	.63707	1.5697	.84339	30
31	.53754	.63748	1.5687	.84324	29
32	.53779	.63789	1.5677	.84308	28
33	.53804	.63830	1.5667	.84292	27
34	.53828	.63871	1.5657	.84277	26
35	.53853	.63912	1.5647	.84261	25
36	.53877	.63953	1.5637	.84245	24
37	.53902	.63994	1.5627	.84230	23
38	.53926	.64035	1.5617	.84214	22
39	.53951	.64076	1.5607	.84198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	.64158	1.5587	.84167	19
42	.54024	.64199	1.5577	.84151	18
43	.54049	.64240	1.5567	.84135	17
44	.54073	.64281	1.5557	.84120	16
45	.54097	.64322	1.5547	.84104	15
46	.54122	.64363	1.5537	.84088	14
47	.54146	.64404	1.5527	.84072	13
48	.54171	.64446	1.5517	.84057	12
49	.54195	.64487	1.5507	.84041	11
50	.54220	.64528	1.5497	.84025	10
51	.54244	.64569	1.5487	.84009	9
52	.54269	.64610	1.5477	.83994	8
53	.54293	.64652	1.5468	.83978	7
54	.54317	.64693	1.5458	.83962	6
55	.54342	.64734	1.5448	.83946	5
56	.54366	.64775	1.5438	.83930	4
57	.54391	.64817	1.5428	.83915	3
58	.54415	.64858	1.5418	.83899	2
59	.54440	.64899	1.5408	.83883	1
60	.54464	.64941	1.5399	.83867	0
′	Cos	Ctn	Tan	Sin	′

33° (213°) (326°) **146°**

′	Sin	Tan	Ctn	Cos	′
0	.54464	.64941	1.5399	.83867	60
1	.54488	.64982	1.5389	.83851	59
2	.54513	.65024	1.5379	.83835	58
3	.54537	.65065	1.5369	.83819	57
4	.54561	.65106	1.5359	.83804	56
5	.54586	.65148	1.5350	.83788	55
6	.54610	.65189	1.5340	.83772	54
7	.54635	.65231	1.5330	.83756	53
8	.54659	.65272	1.5320	.83740	52
9	.54683	.65314	1.5311	.83724	51
10	.54708	.65355	1.5301	.83708	50
11	.54732	.65397	1.5291	.83692	49
12	.54756	.65438	1.5282	.83676	48
13	.54781	.65480	1.5272	.83660	47
14	.54805	.65521	1.5262	.83645	46
15	.54829	.65563	1.5253	.83629	45
16	.54854	.65604	1.5243	.83613	44
17	.54878	.65646	1.5233	.83597	43
18	.54902	.65688	1.5224	.83581	42
19	.54927	.65729	1.5214	.83565	41
20	.54951	.65771	1.5204	.83549	40
21	.54975	.65813	1.5195	.83533	39
22	.54999	.65854	1.5185	.83517	38
23	.55024	.65896	1.5175	.83501	37
24	.55048	.65938	1.5166	.83485	36
25	.55072	.65980	1.5156	.83469	35
26	.55097	.66021	1.5147	.83453	34
27	.55121	.66063	1.5137	.83437	33
28	.55145	.66105	1.5127	.83421	32
29	.55169	.66147	1.5118	.83405	31
30	.55194	.66189	1.5108	.83389	30
31	.55218	.66230	1.5099	.83373	29
32	.55242	.66272	1.5089	.83356	28
33	.55266	.66314	1.5080	.83340	27
34	.55291	.66356	1.5070	.83324	26
35	.55315	.66398	1.5061	.83308	25
36	.55339	.66440	1.5051	.83292	24
37	.55363	.66482	1.5042	.83276	23
38	.55388	.66524	1.5032	.83260	22
39	.55412	.66566	1.5023	.83244	21
40	.55436	.66608	1.5013	.83228	20
41	.55460	.66650	1.5004	.83212	19
42	.55484	.66692	1.4994	.83195	18
43	.55509	.66734	1.4985	.83179	17
44	.55533	.66776	1.4975	.83163	16
45	.55557	.66818	1.4966	.83147	15
46	.55581	.66860	1.4957	.83131	14
47	.55605	.66902	1.4947	.83115	13
48	.55630	.66944	1.4938	.83098	12
49	.55654	.66986	1.4928	.83082	11
50	.55678	.67028	1.4919	.83066	10
51	.55702	.67071	1.4910	.83050	9
52	.55726	.67113	1.4900	.83034	8
53	.55750	.67155	1.4891	.83017	7
54	.55775	.67197	1.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	.55823	.67282	1.4863	.82969	4
57	.55847	.67324	1.4854	.82953	3
58	.55871	.67366	1.4844	.82936	2
59	.55895	.67409	1.4835	.82920	1
60	.55919	.67451	1.4826	.82904	0
′	Cos	Ctn	Tan	Sin	′

122° (302°) (237°) **57°**

123° (303°) (236°) **56°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

34° (214°) (325°) **145°**

′	Sin	Tan	Ctn	Cos	′
0	.55919	.67451	1.4826	.82904	60
1	.55943	.67493	1.4816	.82887	59
2	.55968	.67536	1.4807	.82871	58
3	.55992	.67578	1.4798	.82855	57
4	.56016	.67620	1.4788	.82839	56
5	.56040	.67663	1.4779	.82822	55
6	.56064	.67705	1.4770	.82806	54
7	.56088	.67748	1.4761	.82790	53
8	.56112	.67790	1.4751	.82773	52
9	.56136	.67832	1.4742	.82757	51
10	.56160	.67875	1.4733	.82741	50
11	.56184	.67917	1.4724	.82724	49
12	.56208	.67960	1.4715	.82708	48
13	.56232	.68002	1.4705	.82692	47
14	.56256	.68045	1.4696	.82675	46
15	.56280	.68088	1.4687	.82659	45
16	.56305	.68130	1.4678	.82643	44
17	.56329	.68173	1.4669	.82626	43
18	.56353	.68215	1.4659	.82610	42
19	.56377	.68258	1.4650	.82593	41
20	.56401	.68301	1.4641	.82577	40
21	.56425	.68343	1.4632	.82561	39
22	.56449	.68386	1.4623	.82544	38
23	.56473	.68429	1.4614	.82528	37
24	.56497	.68471	1.4605	.82511	36
25	.56521	.68514	1.4596	.82495	35
26	.56545	.68557	1.4586	.82478	34
27	.56569	.68600	1.4577	.82462	33
28	.56593	.68642	1.4568	.82446	32
29	.56617	.68685	1.4559	.82429	31
30	.56641	.68728	1.4550	.82413	30
31	.56665	.68771	1.4541	.82396	29
32	.56689	.68814	1.4532	.82380	28
33	.56713	.68857	1.4523	.82363	27
34	.56736	.68900	1.4514	.82347	26
35	.56760	.68942	1.4505	.82330	25
36	.56784	.68985	1.4496	.82314	24
37	.56808	.69028	1.4487	.82297	23
38	.56832	.69071	1.4478	.82281	22
39	.56856	.69114	1.4469	.82264	21
40	.56880	.69157	1.4460	.82248	20
41	.56904	.69200	1.4451	.82231	19
42	.56928	.69243	1.4442	.82214	18
43	.56952	.69286	1.4433	.82198	17
44	.56976	.69329	1.4424	.82181	16
45	.57000	.69372	1.4415	.82165	15
46	.57024	.69416	1.4406	.82148	14
47	.57047	.69459	1.4397	.82132	13
48	.57071	.69502	1.4388	.82115	12
49	.57095	.69545	1.4379	.82098	11
50	.57119	.69588	1.4370	.82082	10
51	.57143	.69631	1.4361	.82065	9
52	.57167	.69675	1.4352	.82048	8
53	.57191	.69718	1.4344	.82032	7
54	.57215	.69761	1.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	.47262	.69847	1.4317	.81982	4
57	.57286	.69891	1.4308	.81965	3
58	.57310	.69934	1.4299	.81949	2
59	.57334	.69977	1.4290	.81932	1
60	.57358	.70021	1.4281	.81915	0
′	Cos	Ctn	Tan	Sin	′

124° (304°) (235°) **55°**

35° (215°) (324°) **144°**

′	Sin	Tan	Ctn	Cos	′
0	.57358	.70021	1.4281	.81915	60
1	.57381	.70064	1.4273	.81899	59
2	.57405	.70107	1.4264	.81882	58
3	.57429	.70151	1.4255	.81865	57
4	.57453	.70194	1.4246	.81848	56
5	.57477	.70238	1.4237	.81832	55
6	.57501	.70281	1.4229	.81815	54
7	.57524	.70325	1.4220	.81798	53
8	.57548	.70368	1.4211	.81782	52
9	.57572	.70412	1.4202	.81765	51
10	.57596	.70455	1.4193	.81748	50
11	.57619	.70499	1.4185	.81731	49
12	.57643	.70542	1.4176	.81714	48
13	.57667	.70586	1.4167	.81698	47
14	.57691	.70629	1.4158	.81681	46
15	.57715	.70673	1.4150	.81664	45
16	.57738	.70717	1.4141	.81647	44
17	.57762	.70760	1.4132	.81631	43
18	.57786	.70804	1.4124	.81614	42
19	.57810	.70848	1.4115	.81597	41
20	.57833	.70891	1.4106	.81580	40
21	.57857	.70935	1.4097	.81563	39
22	.57881	.70979	1.4089	.81546	38
23	.57904	.71023	1.4080	.81530	37
24	.57928	.71066	1.4071	.81513	36
25	.57952	.71110	1.4063	.81496	35
26	.57976	.71154	1.4054	.81479	34
27	.57999	.71198	1.4045	.81462	33
28	.58023	.71242	1.4037	.81445	32
29	.58047	.71285	1.4028	.81428	31
30	.58070	.71329	1.4019	.81412	30
31	.58094	.71373	1.4011	.81395	29
32	.58118	.71417	1.4002	.81378	28
33	.58141	.71461	1.3994	.81361	27
34	.58165	.71505	1.3985	.81344	26
35	.58189	.71549	1.3976	.81327	25
36	.58212	.71593	1.3968	.81310	24
37	.58236	.71637	1.3959	.81293	23
38	.58260	.71681	1.3951	.81276	22
39	.58283	.71725	1.3942	.81259	21
40	.58307	.71769	1.3934	.81242	20
41	.58330	.71813	1.3925	.81225	19
42	.58354	.71857	1.3916	.81208	18
43	.58378	.71901	1.3908	.81191	17
44	.58401	.71946	1.3899	.81174	16
45	.58425	.71990	1.3891	.81157	15
46	.58449	.72034	1.3882	.81140	14
47	.58472	.72078	1.3874	.81123	13
48	.58496	.72122	1.3865	.81106	12
49	.58519	.72167	1.3857	.81089	11
50	.58543	.72211	1.3848	.81072	10
51	.58567	.72255	1.3840	.81055	9
52	.58590	.72299	1.3831	.81038	8
53	.58614	.72344	1.3823	.81021	7
54	.58637	.72388	1.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	.58684	.72477	1.3798	.80970	4
57	.58708	.72521	1.3789	.80953	3
58	.58731	.72565	1.3781	.80936	2
59	.58755	.72610	1.3772	.80919	1
60	.58779	.72654	1.3764	.80902	0
′	Cos	Ctn	Tan	Sin	′

125° (305°) (234°) **54°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

36° (216°) (323°) **143°**

′	Sin	Tan	Ctn	Cos	′
0	.58779	.72654	1.3764	.80902	60
1	.58802	.72699	1.3755	.80885	59
2	.58826	.72743	1.3747	.80867	58
3	.58849	.72788	1.3739	.80850	57
4	.58873	.72832	1.3730	.80833	56
5	.58896	.72877	1.3722	.80816	55
6	.58920	.72921	1.3713	.80799	54
7	.58943	.72966	1.3705	.80782	53
8	.58967	.73010	1.3697	.80765	52
9	.58990	.73055	1.3688	.80748	51
10	.59014	.73100	1.3680	.80730	50
11	.59037	.73144	1.3672	.80713	49
12	.59061	.73189	1.3663	.80696	48
13	.59084	.73234	1.3655	.80679	47
14	.59108	.73278	1.3647	.80662	46
15	.59131	.73323	1.3638	.80644	45
16	.59154	.73368	1.3630	.80627	44
17	.59178	.73413	1.3622	.80610	43
18	.59201	.73457	1.3613	.80593	42
19	.59225	.73502	1.3605	.80576	41
20	.59248	.73547	1.3597	.80558	40
21	.59272	.73592	1.3588	.80541	39
22	.59295	.73637	1.3580	.80524	38
23	.59318	.73681	1.3572	.80507	37
24	.59342	.73726	1.3564	.80489	36
25	.59365	.73771	1.3555	.80472	35
26	.59389	.73816	1.3547	.80455	34
27	.59412	.73861	1.3539	.80438	33
28	.59436	.73906	1.3531	.80420	32
29	.59459	.73951	1.3522	.80403	31
30	.59482	.73996	1.3514	.80386	30
31	.59506	.74041	1.3506	.80368	29
32	.59529	.74086	1.3498	.80351	28
33	.59552	.74131	1.3490	.80334	27
34	.59576	.74176	1.3481	.80316	26
35	.59599	.74221	1.3473	.80299	25
36	.59622	.74267	1.3465	.80282	24
37	.59646	.74312	1.3457	.80264	23
38	.59669	.74357	1.3449	.80247	22
39	.59693	.74402	1.3440	.80230	21
40	.59716	.74447	1.3432	.80212	20
41	.59739	.74492	1.3424	.80195	19
42	.59763	.74538	1.3416	.80178	18
43	.59786	.74583	1.3408	.80160	17
44	.59809	.74628	1.3400	.80143	16
45	.59832	.74674	1.3392	.80125	15
46	.59856	.74719	1.3384	.80108	14
47	.59879	.74764	1.3375	.80091	13
48	.59902	.74810	1.3367	.80073	12
49	.59926	.74855	1.3359	.80056	11
50	.59949	.74900	1.3351	.80038	10
51	.59972	.74946	1.3343	.80021	9
52	.59995	.74991	1.3335	.80003	8
53	.60019	.75037	1.3327	.79986	7
54	.60042	.75082	1.3319	.79968	6
55	.60065	.75128	1.3311	.79951	5
56	.60089	.75173	1.3303	.79934	4
57	.60112	.75219	1.3295	.79916	3
58	.60135	.75264	1.3287	.79899	2
59	.60158	.75310	1.3278	.79881	1
60	.60182	.75355	1.3270	.79864	0
′	Cos	Ctn	Tan	Sin	′

126° (306°) (233°) **53°**

37° (217°) (322°) **142°**

′	Sin	Tan	Ctn	Cos	′
0	.60182	.75355	1.3270	.79864	60
1	.60205	.75401	1.3262	.79846	59
2	.60228	.75447	1.3254	.79829	58
3	.60251	.75492	1.3246	.79811	57
4	.60274	.75538	1.3238	.79793	56
5	.60298	.75584	1.3230	.79776	55
6	.60321	.75629	1.3222	.79758	54
7	.60344	.75675	1.3214	.79741	53
8	.60367	.75721	1.3206	.79723	52
9	.60390	.75767	1.3198	.79706	51
10	.60414	.75812	1.3190	.79688	50
11	.60437	.75858	1.3182	.79671	49
12	.60460	.75904	1.3175	.79653	48
13	.60483	.75950	1.3167	.79635	47
14	.60506	.75996	1.3159	.79618	46
15	.60529	.76042	1.3151	.79600	45
16	.60553	.76088	1.3143	.79583	44
17	.60576	.76134	1.3135	.79565	43
18	.60599	.76180	1.3127	.79547	42
19	.60622	.76226	1.3119	.79530	41
20	.60645	.76272	1.3111	.79512	40
21	.60668	.76318	1.3103	.79494	39
22	.60691	.76364	1.3095	.79477	38
23	.60714	.76410	1.3087	.79459	37
24	.60738	.76456	1.3079	.79441	36
25	.60761	.76502	1.3072	.79424	35
26	.60784	.76548	1.3064	.79406	34
27	.60807	.76594	1.3056	.79388	33
28	.60830	.76640	1.3048	.79371	32
29	.60853	.76686	1.3040	.79353	31
30	.60876	.76733	1.3032	.79335	30
31	.60899	.76779	1.3024	.79318	29
32	.60922	.76825	1.3017	.79300	28
33	.60945	.76871	1.3009	.79282	27
34	.60968	.76918	1.3001	.79264	26
35	.60991	.76964	1.2993	.79247	25
36	.61015	.77010	1.2985	.79229	24
37	.61038	.77057	1.2977	.79211	23
38	.61061	.77103	1.2970	.79193	22
39	.61084	.77149	1.2962	.79176	21
40	.61107	.77196	1.2954	.79158	20
41	.61130	.77242	1.2946	.79140	19
42	.61153	.77289	1.2938	.79122	18
43	.61176	.77335	1.2931	.79105	17
44	.61199	.77382	1.2923	.79087	16
45	.61222	.77428	1.2915	.79069	15
46	.61245	.77475	1.2907	.79051	14
47	.61268	.77521	1.2900	.79033	13
48	.61291	.77568	1.2892	.79016	12
49	.61314	.77615	1.2884	.78998	11
50	.61337	.77661	1.2876	.78980	10
51	.61360	.77708	1.2869	.78962	9
52	.61383	.77754	1.2861	.78944	8
53	.61406	.77801	1.2853	.78926	7
54	.61429	.77848	1.2846	.78908	6
55	.61451	.77895	1.2838	.78891	5
56	.61474	.77941	1.2830	.78873	4
57	.61497	.77988	1.2822	.78855	3
58	.61520	.78035	1.2815	.78837	2
59	.61543	.78082	1.2807	.78819	1
60	.61566	.78129	1.2799	.78801	0
′	Cos	Ctn	Tan	Sin	′

127° (307°) (232°) **52°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

38° (218°) (321°) **141°**

′	Sin	Tan	Ctn	Cos	′
0	.61566	.78129	1.2799	.78801	60
1	.61589	.78175	1.2792	.78783	59
2	.61612	.78222	1.2784	.78765	58
3	.61635	.78269	1.2776	.78747	57
4	.61658	.78316	1.2769	.78729	56
5	.61681	.78363	1.2761	.78711	55
6	.61704	.78410	1.2753	.78694	54
7	.61726	.78457	1.2746	.78676	53
8	.61749	.78504	1.2738	.78658	52
9	.61772	.78551	1.2731	.78640	51
10	.61795	.78598	1.2723	.78622	50
11	.61818	.78645	1.2715	.78604	49
12	.61841	.78692	1.2708	.78586	48
13	.61864	.78739	1.2700	.78568	47
14	.61887	.78786	1.2693	.78550	46
15	.61909	.78834	1.2685	.78532	45
16	.61932	.78881	1.2677	.78514	44
17	.61955	.78928	1.2670	.78496	43
18	.61978	.78975	1.2662	.78478	42
19	.62001	.79022	1.2655	.78460	41
20	.62024	.79070	1.2647	.78442	40
21	.62046	.79117	1.2640	.78424	39
22	.62069	.79164	1.2632	.78405	38
23	.62092	.79212	1.2624	.78387	37
24	.62115	.79259	1.2617	.78369	36
25	.62138	.79306	1.2609	.78351	35
26	.62160	.79354	1.2602	.78333	34
27	.62183	.79401	1.2594	.78315	33
28	.62206	.79449	1.2587	.78297	32
29	.62229	.79496	1.2579	.78279	31
30	.62251	.79544	1.2572	.78261	30
31	.62274	.79591	1.2564	.78243	29
32	.62297	.79639	1.2557	.78225	28
33	.62320	.79686	1.2549	.78206	27
34	.62342	.79734	1.2542	.78188	26
35	.62365	.79781	1.2534	.78170	25
36	.62388	.79829	1.2527	.78152	24
37	.62411	.79877	1.2519	.78134	23
38	.62433	.79924	1.2512	.78116	22
39	.62456	.79972	1.2504	.78098	21
40	.62479	.80020	1.2497	.78079	20
41	.62502	.80067	1.2489	.78061	19
42	.62524	.80115	1.2482	.78043	18
43	.62547	.80163	1.2475	.78025	17
44	.62570	.80211	1.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	.62615	.80306	1.2452	.77970	14
47	.62638	.80354	1.2445	.77952	13
48	.62660	.80402	1.2437	.77934	12
49	.62683	.80450	1.2430	.77916	11
50	.62706	.80498	1.2423	.77897	10
51	.62728	.80546	1.2415	.77879	9
52	.62751	.80594	1.2408	.77861	8
53	.62774	.80642	1.2401	.77843	7
54	.62796	.80690	1.2393	.77824	6
55	.62819	.80738	1.2386	.77806	5
56	.62842	.80786	1.2378	.77788	4
57	.62864	.80834	1.2371	.77769	3
58	.62887	.80882	1.2364	.77751	2
59	.62909	.80930	1.2356	.77733	1
60	.62932	.80978	1.2349	.77715	0
′	Cos	Ctn	Tan	Sin	′

128° (308°) (231°) **51°**

39° (219°) (320°) **140°**

′	Sin	Tan	Ctn	Cos	′
0	.62932	.80978	1.2349	.77715	60
1	.62955	.81027	1.2342	.77696	59
2	.62977	.81075	1.2334	.77678	58
3	.63000	.81123	1.2327	.77660	57
4	.63022	.81171	1.2320	.77641	56
5	.63045	.81220	1.2312	.77623	55
6	.63068	.81268	1.2305	.77605	54
7	.63090	.81316	1.2298	.77586	53
8	.63113	.81364	1.2290	.77568	52
9	.63135	.81413	1.2283	.77550	51
10	.63158	.81461	1.2276	.77531	50
11	.63180	.81510	1.2268	.77513	49
12	.63203	.81558	1.2261	.77494	48
13	.63225	.81606	1.2254	.77476	47
14	.63248	.81655	1.2247	.77458	46
15	.63271	.81703	1.2239	.77439	45
16	.63293	.81752	1.2232	.77421	44
17	.63316	.81800	1.2225	.77402	43
18	.63338	.81849	1.2218	.77384	42
19	.63361	.81898	1.2210	.77366	41
20	.63383	.81946	1.2203	.77347	40
21	.63406	.81995	1.2196	.77329	39
22	.63428	.82044	1.2189	.77310	38
23	.63451	.82092	1.2181	.77292	37
24	.63473	.82141	1.2174	.77273	36
25	.63496	.82190	1.2167	.77255	35
26	.63518	.82238	1.2160	.77236	34
27	.63540	.82287	1.2153	.77218	33
28	.63563	.82336	1.2145	.77199	32
29	.63585	.82385	1.2138	.77181	31
30	.63608	.82434	1.2131	.77162	30
31	.63630	.82483	1.2124	.77144	29
32	.63653	.82531	1.2117	.77125	28
33	.63675	.82580	1.2109	.77107	27
34	.63698	.82629	1.2102	.77088	26
35	.63720	.82678	1.2095	.77070	25
36	.63742	.82727	1.2088	.77051	24
37	.63765	.82776	1.2081	.77033	23
38	.63787	.82825	1.2074	.77014	22
39	.63810	.82874	1.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	.63854	.82972	1.2052	.76959	19
42	.63877	.83022	1.2045	.76940	18
43	.63899	.83071	1.2038	.76921	17
44	.63922	.83120	1.2031	.76903	16
45	.63944	.83169	1.2024	.76884	15
46	.63966	.83218	1.2017	.76866	14
47	.63989	.83268	1.2009	.76847	13
48	.64011	.83317	1.2002	.76828	12
49	.64033	.83366	1.1995	.76810	11
50	.64056	.83415	1.1988	.76791	10
51	.64078	.83465	1.1981	.76772	9
52	.64100	.83514	1.1974	.76754	8
53	.64123	.83564	1.1967	.76735	7
54	.64145	.83613	1.1960	.76717	6
55	.64167	.83662	1.1953	.76698	5
56	.64190	.83712	1.1946	.76679	4
57	.64212	.83761	1.1939	.76661	3
58	.64234	.83811	1.1932	.76642	2
59	.64256	.83860	1.1925	.76623	1
60	.64279	.83910	1.1918	.76604	0
′	Cos	Ctn	Tan	Sin	′

129° (309°) (230°) **50°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

40° (220°) (319°) **139°**

′	Sin	Tan	Ctn	Cos	′
0	.64279	.83910	1.1918	.76604	60
1	.64301	.83960	1.1910	.76586	59
2	.64323	.84009	1.1903	.76567	58
3	.64346	.84059	1.1896	.76548	57
4	.64368	.84108	1.1889	.76530	56
5	.64390	.84158	1.1882	.76511	55
6	.64412	.84208	1.1875	.76492	54
7	.64435	.84258	1.1868	.76473	53
8	.64457	.84307	1.1861	.76455	52
9	.64479	.84357	1.1854	.76436	51
10	.64501	.84407	1.1847	.76417	50
11	.64524	.84457	1.1840	.76398	49
12	.64546	.84507	1.1833	.76380	48
13	.64568	.84556	1.1826	.76361	47
14	.64590	.84606	1.1819	.76342	46
15	.64612	.84656	1.1812	.76323	45
16	.64635	.84706	1.1806	.76304	44
17	.64657	.84756	1.1799	.76286	43
18	.64679	.84806	1.1792	.76267	42
19	.64701	.84856	1.1785	.76248	41
20	.64723	.84906	1.1778	.76229	40
21	.64746	.84956	1.1771	.76210	39
22	.64768	.85006	1.1764	.76192	38
23	.64790	.85057	1.1757	.76173	37
24	.64812	.85107	1.1750	.76154	36
25	.64834	.85157	1.1743	.76135	35
26	.64856	.85207	1.1736	.76116	34
27	.64878	.85257	1.1729	.76097	33
28	.64901	.85308	1.1722	.76078	32
29	.64923	.85358	1.1715	.76059	31
30	.64945	.85408	1.1708	.76041	30
31	.64967	.85458	1.1702	.76022	29
32	.64989	.85509	1.1695	.76003	28
33	.65011	.85559	1.1688	.75984	27
34	.65033	.85609	1.1681	.75965	26
35	.65055	.85660	1.1674	.75946	25
36	.65077	.85710	1.1667	.75927	24
37	.65100	.85761	1.1660	.75908	23
38	.65122	.85811	1.1653	.75889	22
39	.65144	.85862	1.1647	.75870	21
40	.65166	.85912	1.1640	.75851	20
41	.65188	.85963	1.1633	.75832	19
42	.65210	.86014	1.1626	.75813	18
43	.65232	.86064	1.1619	.75794	17
44	.65254	.86115	1.1612	.75775	16
45	.65276	.86166	1.1606	.75756	15
46	.65298	.86216	1.1599	.75738	14
47	.65320	.86267	1.1592	.75719	13
48	.65342	.86318	1.1585	.75700	12
49	.65364	.86368	1.1578	.75680	11
50	.65386	.86419	1.1571	.75661	10
51	.65408	.86470	1.1565	.75642	9
52	.65430	.86521	1.1558	.75623	8
53	.65452	.86572	1.1551	.75604	7
54	.65474	.86623	1.1544	.75585	6
55	.65496	.86674	1.1538	.75566	5
56	.65518	.86725	1.1531	.75547	4
57	.65540	.86776	1.1524	.75528	3
58	.65562	.86827	1.1517	.75509	2
59	.65584	.86878	1.1510	.75490	1
60	.65606	.86929	1.1504	.75471	0
′	Cos	Ctn	Tan	Sin	′

130° (310°) (229°) **49°**

41° (221°) (318°) **138°**

′	Sin	Tan	Ctn	Cos	′
0	.65606	.86929	1.1504	.75471	60
1	.65628	.86980	1.1497	.75452	59
2	.65650	.87031	1.1490	.75433	58
3	.65672	.87082	1.1483	.75414	57
4	.65694	.87133	1.1477	.75395	56
5	.65716	.87184	1.1470	.75375	55
6	.65738	.87236	1.1463	.75356	54
7	.65759	.87287	1.1456	.75337	53
8	.65781	.87338	1.1450	.75318	52
9	.65803	.87389	1.1443	.75299	51
10	.65825	.87441	1.1436	.75280	50
11	.65847	.87492	1.1430	.75261	49
12	.65869	.87543	1.1423	.75241	48
13	.65891	.87595	1.1416	.75222	47
14	.65913	.87646	1.1410	.75203	46
15	.65935	.87698	1.1403	.75184	45
16	.65956	.87749	1.1396	.75165	44
17	.65978	.87801	1.1389	.75146	43
18	.66000	.87852	1.1383	.75126	42
19	.66022	.87904	1.1376	.75107	41
20	.66044	.87955	1.1369	.75088	40
21	.66066	.88007	1.1363	.75069	39
22	.66088	.88059	1.1356	.75050	38
23	.66109	.88110	1.1349	.75030	37
24	.66131	.88162	1.1343	.75011	36
25	.66153	.88214	1.1336	.74992	35
26	.66175	.88265	1.1329	.74973	34
27	.66197	.88317	1.1323	.74953	33
28	.66218	.88369	1.1316	.74934	32
29	.66240	.88421	1.1310	.74915	31
30	.66262	.88473	1.1303	.74896	30
31	.66284	.88524	1.1296	.74876	29
32	.66306	.88576	1.1290	.74857	28
33	.66327	.88628	1.1283	.74838	27
34	.66349	.88680	1.1276	.74818	26
35	.66371	.88732	1.1270	.74799	25
36	.66393	.88784	1.1263	.74780	24
37	.66414	.88836	1.1257	.74760	23
38	.66436	.88888	1.1250	.74741	22
39	.66458	.88940	1.1243	.74722	21
40	.66480	.88992	1.1237	.74703	20
41	.66501	.89045	1.1230	.74683	19
42	.66523	.89097	1.1224	.74664	18
43	.66545	.89149	1.1217	.74644	17
44	.66566	.89201	1.1211	.74625	16
45	.66588	.89253	1.1204	.74606	15
46	.66610	.89306	1.1197	.74586	14
47	.66632	.89358	1.1191	.74567	13
48	.66653	.89410	1.1184	.74548	12
49	.66675	.89463	1.1178	.74528	11
50	.66697	.89515	1.1171	.74509	10
51	.66718	.89567	1.1165	.74489	9
52	.66740	.89620	1.1158	.74470	8
53	.66762	.89672	1.1152	.74451	7
54	.66783	.89725	1.1145	.74431	6
55	.66805	.89777	1.1139	.74412	5
56	.66827	.89830	1.1132	.74392	4
57	.66848	.89883	1.1126	.74373	3
58	.66870	.89935	1.1119	.74353	2
59	.66891	.89988	1.1113	.74334	1
60	.66913	.90040	1.1106	.74314	0
′	Cos	Ctn	Tan	Sin	′

131° (311°) (228°) **48°**

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

42° (222°)				(317°) 137°		43° (223°)				(316°) 136°	
′	Sin	Tan	Ctn	Cos	′	′	Sin	Tan	Ctn	Cos	′
0	.66913	.90040	1.1106	.74314	60	0	.68200	.93252	1.0724	.73135	60
1	.66935	.90093	1.1100	.74295	59	1	.68221	.93306	1.0717	.73116	59
2	.66956	.90146	1.1093	.74276	58	2	.68242	.93360	1.0711	.73096	58
3	.66978	.90199	1.1087	.74256	57	3	.68264	.93415	1.0705	.73076	57
4	.66999	.90251	1.1080	.74237	56	4	.68285	.93469	1.0699	.73056	56
5	.67021	.90304	1.1074	.74217	55	5	.68306	.93524	1.0692	.73036	55
6	.67043	.90357	1.1067	.74198	54	6	.68327	.93578	1.0686	.73016	54
7	.67064	.90410	1.1061	.74178	53	7	.68349	.93633	1.0680	.72996	53
8	.67086	.90463	1.1054	.74159	52	8	.68370	.93688	1.0674	.72976	52
9	.67107	.90516	1.1048	.74139	51	9	.68391	.93742	1.0668	.72957	51
10	.67129	.90569	1.1041	.74120	50	10	.68412	.93797	1.0661	.72937	50
11	.67151	.90621	1.1035	.74100	49	11	.68434	.93852	1.0655	.72917	49
12	.67172	.90674	1.1028	.74080	48	12	.68455	.93906	1.0649	.72897	48
13	.67194	.90727	1.1022	.74061	47	13	.68476	.93961	1.0643	.72877	47
14	.67215	.90781	1.1016	.74041	46	14	.68497	.94016	1.0637	.72857	46
15	.67237	.90834	1.1009	.74022	45	15	.68518	.94071	1.0630	.72837	45
16	.67258	.90887	1.1003	.74002	44	16	.68539	.94125	1.0624	.72817	44
17	.67280	.90940	1.0996	.73983	43	17	.68561	.94180	1.0618	.72797	43
18	.67301	.90993	1.0990	.73963	42	18	.68582	.94235	1.0612	.72777	42
19	.67323	.91046	1.0983	.73944	41	19	.68603	.94290	1.0606	.72757	41
20	.67344	.91099	1.0977	.73924	40	20	.68624	.94345	1.0599	.72737	40
21	.67366	.91153	1.0971	.73904	39	21	.68645	.94400	1.0593	.72717	39
22	.67387	.91206	1.0964	.73885	38	22	.68666	.94455	1.0587	.72697	38
23	.67409	.91259	1.0958	.73865	37	23	.68688	.94510	1.0581	.72677	37
24	.67430	.91313	1.0951	.73846	36	24	.68709	.94565	1.0575	.72657	36
25	.67452	.91366	1.0945	.73826	35	25	.68730	.94620	1.0569	.72637	35
26	.67473	.91419	1.0939	.73806	34	26	.68751	.94676	1.0562	.72617	34
27	.67495	.91473	1.0932	.73787	33	27	.68772	.94731	1.0556	.72597	33
28	.67516	.91526	1.0926	.73767	32	28	.68793	.94786	1.0550	.72577	32
29	.67538	.91580	1.0919	.73747	31	29	.68814	.94841	1.0544	.72557	31
30	.67559	.91633	1.0913	.73728	30	30	.68835	.94896	1.0538	.72537	30
31	.67580	.91687	1.0907	.73708	29	31	.68857	.94952	1.0532	.72517	29
32	.67602	.91740	1.0900	.73688	28	32	.68878	.95007	1.0526	.72497	28
33	.67623	.91794	1.0894	.73669	27	33	.68899	.95062	1.0519	.72477	27
34	.67645	.91847	1.0888	.73649	26	34	.68920	.95118	1.0513	.72457	26
35	.67666	.91901	1.0881	.73629	25	35	.68941	.95173	1.0507	.72437	25
36	.67688	.91955	1.0875	.73610	24	36	.68962	.95229	1.0501	.72417	24
37	.67709	.92008	1.0869	.73590	23	37	.68983	.95284	1.0495	.72397	23
38	.67730	.92062	1.0862	.73570	22	38	.69004	.95340	1.0489	.72377	22
39	.67752	.92116	1.0856	.73551	21	39	.69025	.95395	1.0483	.72357	21
40	.67773	.92170	1.0850	.73531	20	40	.69046	.95451	1.0477	.72337	20
41	.67795	.92224	1.0843	.73511	19	41	.69067	.95506	1.0470	.72317	19
42	.67816	.92277	1.0837	.73491	18	42	.69088	.95562	1.0464	.72297	18
43	.67837	.92331	1.0831	.73472	17	43	.69109	.95618	1.0458	.72277	17
44	.67859	.92385	1.0824	.73452	16	44	.69130	.95673	1.0452	.72257	16
45	.67880	.92439	1.0818	.73432	15	45	.69151	.95729	1.0446	.72236	15
46	.67901	.92493	1.0812	.73413	14	46	.69172	.95785	1.0440	.72216	14
47	.67923	.92547	1.0805	.73393	13	47	.69193	.95841	1.0434	.72196	13
48	.67944	.92601	1.0799	.73373	12	48	.69214	.95897	1.0428	.72176	12
49	.67965	.92655	1.0793	.73353	11	49	.69235	.95952	1.0422	.72156	11
50	.67987	.92709	1.0786	.73333	10	50	.69256	.96008	1.0416	.72136	10
51	.68008	.92763	1.0780	.73314	9	51	.69277	.96064	1.0410	.72116	9
52	.68029	.92817	1.0774	.73294	8	52	.69298	.96120	1.0404	.72095	8
53	.68051	.92872	1.0768	.73274	7	53	.69319	.96176	1.0398	.72075	7
54	.68072	.92926	1.0761	.73254	6	54	.69340	.96232	1.0392	.72055	6
55	.68093	.92980	1.0755	.73234	5	55	.69361	.96288	1.0385	.72035	5
56	.68115	.93034	1.0749	.73215	4	56	.69382	.96344	1.0379	.72015	4
57	.68136	.93088	1.0742	.73195	3	57	.69403	.96400	1.0373	.71995	3
58	.68157	.93143	1.0736	.73175	2	58	.69424	.96457	1.0367	.71974	2
59	.68179	.93197	1.0730	.73155	1	59	.69445	.96513	1.0361	.71954	1
60	.68200	.93252	1.0724	.73135	0	60	.69466	.96569	1.0355	.71934	0
′	Cos	Ctn	Tan	Sin	′	′	Cos	Ctn	Tan	Sin	′
132° (312°)				(227°) 47°		133° (313°)				(226°) 46°	

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

44° (224°) (315°) **135°**

′	Sin	Tan	Ctn	Cos	′
0	.69466	.96569	1.0355	.71934	60
1	.69487	.96625	1.0349	.71914	59
2	.69508	.96681	1.0343	.71894	58
3	.69529	.96738	1.0337	.71873	57
4	.69549	.96794	1.0331	.71853	56
5	.69570	.96850	1.0325	.71833	55
6	.69591	.96907	1.0319	.71813	54
7	.69612	.96963	1.0313	.71792	53
8	.69633	.97020	1.0307	.71772	52
9	.69654	.97076	1.0301	.71752	51
10	.69675	.97133	1.0295	.71732	50
11	.69696	.97189	1.0289	.71711	49
12	.69717	.97246	1.0283	.71691	48
13	.69737	.97302	1.0277	.71671	47
14	.69758	.97359	1.0271	.71650	46
15	.69779	.97416	1.0265	.71630	45
16	.69800	.97472	1.0259	.71610	44
17	.69821	.97529	1.0253	.71590	43
18	.69842	.97586	1.0247	.71569	42
19	.69862	.97643	1.0241	.71549	41
20	.69883	.97700	1.0235	.71529	40
21	.69904	.97756	1.0230	.71508	39
22	.69925	.97813	1.0224	.71488	38
23	.69946	.97870	1.0218	.71468	37
24	.69966	.97927	1.0212	.71447	36
25	.69987	.97984	1.0206	.71427	35
26	.70008	.98041	1.0200	.71407	34
27	.70029	.98098	1.0194	.71386	33
28	.70049	.98155	1.0188	.71366	32
29	.70070	.98213	1.0182	.71345	31
30	.70091	.98270	1.0176	.71325	30
31	.70112	.98327	1.0170	.71305	29
32	.70132	.98384	1.0164	.71284	28
33	.70153	.98441	1.0158	.71264	27
34	.70174	.98499	1.0152	.71243	26
35	.70195	.98556	1.0147	.71223	25
36	.70215	.98613	1.0141	.71203	24
37	.70236	.98671	1.0135	.71182	23
38	.70257	.98728	1.0129	.71162	22
39	.70277	.98786	1.0123	.71141	21
40	.70298	.98843	1.0117	.71121	20
41	.70319	.98901	1.0111	.71100	19
42	.70339	.98958	1.0105	.71080	18
43	.70360	.99016	1.0099	.71059	17
44	.70381	.99073	1.0094	.71039	16
45	.70401	.99131	1.0088	.71019	15
46	.70422	.99189	1.0082	.70998	14
47	.70443	.99247	1.0076	.70978	13
48	.70463	.99304	1.0070	.70957	12
49	.70484	.99362	1.0064	.70937	11
50	.70505	.99420	1.0058	.70916	10
51	.70525	.99478	1.0052	.70896	9
52	.70546	.99536	1.0047	.70875	8
53	.70567	.99594	1.0041	.70855	7
54	.70587	.99652	1.0035	.70834	6
55	.70608	.99710	1.0029	.70813	5
56	.70628	.99768	1.0023	.70793	4
57	.70649	.99826	1.0017	.70772	3
58	.70670	.99884	1.0012	.70752	2
59	.70690	.99942	1.0006	.70731	1
60	.70711	1.0000	1.0000	.70711	0
′	Cos	Ctn	Tan	Sin	′

134° (314°) (225°) **45°**

Table 6.5(d)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

0° (180°)		(359°) 179°	
′	Sec	Csc	′
0	1.0000		60
1	1.0000	3437.7	59
2	1.0000	1718.9	58
3	1.0000	1145.9	57
4	1.0000	859.44	56
5	1.0000	687.55	55
6	1.0000	572.96	54
7	1.0000	491.11	53
8	1.0000	429.72	52
9	1.0000	381.97	51
10	1.0000	343.78	50
11	1.0000	312.52	49
12	1.0000	286.48	48
13	1.0000	264.44	47
14	1.0000	245.55	46
15	1.0000	229.18	45
16	1.0000	214.86	44
17	1.0000	202.22	43
18	1.0000	190.99	42
19	1.0000	180.93	41
20	1.0000	171.89	40
21	1.0000	163.70	39
22	1.0000	156.26	38
23	1.0000	149.47	37
24	1.0000	143.24	36
25	1.0000	137.51	35
26	1.0000	132.22	34
27	1.0000	127.33	33
28	1.0000	122.78	32
29	1.0000	118.54	31
30	1.0000	114.59	30
31	1.0000	110.90	29
32	1.0000	107.43	28
33	1.0000	104.18	27
34	1.0000	101.11	26
35	1.0001	98.223	25
36	1.0001	95.495	24
37	1.0001	92.914	23
38	1.0001	90.469	22
39	1.0001	88.149	21
40	1.0001	85.946	20
41	1.0001	83.849	19
42	1.0001	81.853	18
43	1.0001	79.950	17
44	1.0001	78.133	16
45	1.0001	76.397	15
46	1.0001	74.736	14
47	1.0001	73.146	13
48	1.0001	71.622	12
49	1.0001	70.160	11
50	1.0001	68.757	10
51	1.0001	67.409	9
52	1.0001	66.113	8
53	1.0001	64.866	7
54	1.0001	63.665	6
55	1.0001	62.507	5
56	1.0001	61.391	4
57	1.0001	60.314	3
58	1.0001	59.274	2
59	1.0001	58.270	1
60	1.0002	57.299	0
′	Csc	Sec	′
90° (270°)		(269°) 89°	

1° (181°)		(358°) 178°	
′	Sec	Csc	′
0	1.0002	57.299	60
1	1.0002	56.359	59
2	1.0002	55.451	58
3	1.0002	54.570	57
4	1.0002	53.718	56
5	1.0002	52.892	55
6	1.0002	52.090	54
7	1.0002	51.313	53
8	1.0002	50.558	52
9	1.0002	49.826	51
10	1.0002	49.114	50
11	1.0002	48.422	49
12	1.0002	47.750	48
13	1.0002	47.096	47
14	1.0002	46.460	46
15	1.0002	45.840	45
16	1.0002	45.237	44
17	1.0003	44.650	43
18	1.0003	44.077	42
19	1.0003	43.520	41
20	1.0003	42.976	40
21	1.0003	42.445	39
22	1.0003	41.928	38
23	1.0003	41.423	37
24	1.0003	40.930	36
25	1.0003	40.448	35
26	1.0003	39.978	34
27	1.0003	39.519	33
28	1.0003	39.070	32
29	1.0003	38.631	31
30	1.0003	38.202	30
31	1.0004	37.782	29
32	1.0004	37.371	28
33	1.0004	36.970	27
34	1.0004	36.576	26
35	1.0004	36.191	25
36	1.0004	35.815	24
37	1.0004	35.445	23
38	1.0004	35.084	22
39	1.0004	34.730	21
40	1.0004	34.382	20
41	1.0004	34.042	19
42	1.0004	33.708	18
43	1.0004	33.381	17
44	1.0005	33.060	16
45	1.0005	32.746	15
46	1.0005	32.437	14
47	1.0005	32.134	13
48	1.0005	31.836	12
49	1.0005	31.544	11
50	1.0005	31.258	10
51	1.0005	30.976	9
52	1.0005	30.700	8
53	1.0005	30.428	7
54	1.0006	30.161	6
55	1.0006	29.899	5
56	1.0006	29.641	4
57	1.0006	29.388	3
58	1.0006	29.139	2
59	1.0006	28.894	1
60	1.0006	28.654	0
′	Csc	Sec	′
91° (271°)		(268°) 88°	

2° (182°)		(357°) 177°	
′	Sec	Csc	′
0	1.0006	28.654	60
1	1.0006	28.417	59
2	1.0006	28.184	58
3	1.0006	27.955	57
4	1.0007	27.730	56
5	1.0007	27.508	55
6	1.0007	27.290	54
7	1.0007	27.075	53
8	1.0007	26.864	52
9	1.0007	26.655	51
10	1.0007	26.451	50
11	1.0007	26.249	49
12	1.0007	26.050	48
13	1.0007	25.854	47
14	1.0008	25.661	46
15	1.0008	25.471	45
16	1.0008	25.284	44
17	1.0008	25.100	43
18	1.0008	24.918	42
19	1.0008	24.739	41
20	1.0008	24.562	40
21	1.0008	24.388	39
22	1.0009	24.216	38
23	1.0009	24.047	37
24	1.0009	23.880	36
25	1.0009	23.716	35
26	1.0009	23.553	34
27	1.0009	23.393	33
28	1.0009	23.235	32
29	1.0009	23.079	31
30	1.0010	22.926	30
31	1.0010	22.774	29
32	1.0010	22.624	28
33	1.0010	22.476	27
34	1.0010	22.330	26
35	1.0010	22.187	25
36	1.0010	22.044	24
37	1.0010	21.904	23
38	1.0011	21.766	22
39	1.0011	21.629	21
40	1.0011	21.494	20
41	1.0011	21.360	19
42	1.0011	21.229	18
43	1.0011	21.098	17
44	1.0011	20.970	16
45	1.0012	20.843	15
46	1.0012	20.717	14
47	1.0012	20.593	13
48	1.0012	20.471	12
49	1.0012	20.350	11
50	1.0012	20.230	10
51	1.0012	20.112	9
52	1.0013	19.995	8
53	1.0013	19.880	7
54	1.0013	19.766	6
55	1.0013	19.653	5
56	1.0013	19.541	4
57	1.0013	19.431	3
58	1.0013	19.322	2
59	1.0014	19.214	1
60	1.0014	19.107	0
′	Csc	Sec	′
92° (272°)		(267°) 87°	

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

3° (183°) (356°) **176°**

′	Sec	Csc	′
0	1.0014	19.107	60
1	1.0014	19.002	59
2	1.0014	18.898	58
3	1.0014	18.794	57
4	1.0014	18.692	56
5	1.0014	18.591	55
6	1.0015	18.492	54
7	1.0015	18.393	53
8	1.0015	18.295	52
9	1.0015	18.198	51
10	1.0015	18.103	50
11	1.0015	18.008	49
12	1.0016	17.914	48
13	1.0016	17.822	47
14	1.0016	17.730	46
15	1.0016	17.639	45
16	1.0016	17.549	44
17	1.0016	17.460	43
18	1.0017	17.372	42
19	1.0017	17.285	41
20	1.0017	17.198	40
21	1.0017	17.113	39
22	1.0017	17.028	38
23	1.0017	16.945	37
24	1.0018	16.862	36
25	1.0018	16.779	35
26	1.0018	16.698	34
27	1.0018	16.618	33
28	1.0018	16.538	32
29	1.0019	16.459	31
30	1.0019	16.380	30
31	1.0019	16.303	29
32	1.0019	16.226	28
33	1.0019	16.150	27
34	1.0019	16.075	26
35	1.0020	16.000	25
36	1.0020	15.926	24
37	1.0020	15.853	23
38	1.0020	15.780	22
39	1.0020	15.708	21
40	1.0021	15.637	20
41	1.0021	15.566	19
42	1.0021	15.496	18
43	1.0021	15.427	17
44	1.0021	15.358	16
45	1.0021	15.290	15
46	1.0022	15.222	14
47	1.0022	15.155	13
48	1.0022	15.089	12
49	1.0022	15.023	11
50	1.0022	14.958	10
51	1.0023	14.893	9
52	1.0023	14.829	8
53	1.0023	14.766	7
54	1.0023	14.703	6
55	1.0023	14.640	5
56	1.0024	14.578	4
57	1.0024	14.517	3
58	1.0024	14.456	2
59	1.0024	14.395	1
60	1.0024	14.336	0
′	Csc	Sec	′

93° (273°) (266°) **86°**

4° (184°) (355°) **175°**

′	Sec	Csc	′
0	1.0024	14.336	60
1	1.0025	14.276	59
2	1.0025	14.217	58
3	1.0025	14.159	57
4	1.0025	14.101	56
5	1.0025	14.044	55
6	1.0026	13.987	54
7	1.0026	13.930	53
8	1.0026	13.874	52
9	1.0026	13.818	51
10	1.0027	13.763	50
11	1.0027	13.708	49
12	1.0027	13.654	48
13	1.0027	13.600	47
14	1.0027	13.547	46
15	1.0028	13.494	45
16	1.0028	13.441	44
17	1.0028	13.389	43
18	1.0028	13.337	42
19	1.0028	13.286	41
20	1.0029	13.235	40
21	1.0029	13.184	39
22	1.0029	13.134	38
23	1.0029	13.084	37
24	1.0030	13.035	36
25	1.0030	12.985	35
26	1.0030	12.937	34
27	1.0030	12.888	33
28	1.0030	12.840	32
29	1.0031	12.793	31
30	1.0031	12.745	30
31	1.0031	12.699	29
32	1.0031	12.652	28
33	1.0032	12.606	27
34	1.0032	12.560	26
35	1.0032	12.514	25
36	1.0032	12.469	24
37	1.0033	12.424	23
38	1.0033	12.379	22
39	1.0033	12.335	21
40	1.0033	12.291	20
41	1.0034	12.248	19
42	1.0034	12.204	18
43	1.0034	12.161	17
44	1.0034	12.119	16
45	1.0034	12.076	15
46	1.0035	12.034	14
47	1.0035	11.992	13
48	1.0035	11.951	12
49	1.0035	11.909	11
50	1.0036	11.868	10
51	1.0036	11.828	9
52	1.0036	11.787	8
53	1.0036	11.747	7
54	1.0037	11.707	6
55	1.0037	11.668	5
56	1.0037	11.628	4
57	1.0037	11.589	3
58	1.0038	11.551	2
59	1.0038	11.512	1
60	1.0038	11.474	0
′	Csc	Sec	′

94° (274°) (265°) **85°**

5° (185°) (354°) **174°**

′	Sec	Csc	′
0	1.0038	11.474	60
1	1.0038	11.436	59
2	1.0039	11.398	58
3	1.0039	11.360	57
4	1.0039	11.323	56
5	1.0039	11.286	55
6	1.0040	11.249	54
7	1.0040	11.213	53
8	1.0040	11.176	52
9	1.0041	11.140	51
10	1.0041	11.105	50
11	1.0041	11.069	49
12	1.0041	11.034	48
13	1.0042	10.998	47
14	1.0042	10.963	46
15	1.0042	10.929	45
16	1.0042	10.894	44
17	1.0043	10.860	43
18	1.0043	10.826	42
19	1.0043	10.792	41
20	1.0043	10.758	40
21	1.0044	10.725	39
22	1.0044	10.692	38
23	1.0044	10.659	37
24	1.0045	10.626	36
25	1.0045	10.593	35
26	1.0045	10.561	34
27	1.0045	10.529	33
28	1.0046	10.497	32
29	1.0046	10.465	31
30	1.0046	10.433	30
31	1.0047	10.402	29
32	1.0047	10.371	28
33	1.0047	10.340	27
34	1.0047	10.309	26
35	1.0048	10.278	25
36	1.0048	10.248	24
37	1.0048	10.217	23
38	1.0049	10.187	22
39	1.0049	10.157	21
40	1.0049	10.128	20
41	1.0049	10.098	19
42	1.0050	10.068	18
43	1.0050	10.039	17
44	1.0050	10.010	16
45	1.0051	9.9812	15
46	1.0051	9.9525	14
47	1.0051	9.9239	13
48	1.0051	9.8955	12
49	1.0052	9.8672	11
50	1.0052	9.8391	10
51	1.0052	9.8112	9
52	1.0053	9.7834	8
53	1.0053	9.7558	7
54	1.0053	9.7283	6
55	1.0054	9.7010	5
56	1.0054	9.6739	4
57	1.0054	9.6469	3
58	1.0054	9.6200	2
59	1.0055	9.5933	1
60	1.0055	9.5668	0
′	Csc	Sec	′

95° (275°) (264°) **84°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

6° (186°) (353°) **173°**

′	Sec	Csc	′
0	1.0055	9.5668	60
1	1.0055	9.5404	59
2	1.0056	9.5141	58
3	1.0056	9.4880	57
4	1.0056	9.4620	56
5	1.0057	9.4362	55
6	1.0057	9.4105	54
7	1.0057	9.3850	53
8	1.0058	9.3596	52
9	1.0058	9.3343	51
10	1.0058	9.3092	50
11	1.0059	9.2842	49
12	1.0059	9.2593	48
13	1.0059	9.2346	47
14	1.0059	9.2100	46
15	1.0060	9.1855	45
16	1.0060	9.1612	44
17	1.0060	9.1370	43
18	1.0061	9.1129	42
19	1.0061	9.0890	41
20	1.0061	9.0652	40
21	1.0062	9.0415	39
22	1.0062	9.0179	38
23	1.0062	8.9944	37
24	1.0063	8.9711	36
25	1.0063	8.9479	35
26	1.0063	8.9248	34
27	1.0064	8.9019	33
28	1.0064	8.8790	32
29	1.0064	8.8563	31
30	1.0065	8.8337	30
31	1.0065	8.8112	29
32	1.0065	8.7888	28
33	1.0066	8.7665	27
34	1.0066	8.7444	26
35	1.0066	8.7223	25
36	1.0067	8.7004	24
37	1.0067	8.6786	23
38	1.0067	8.6569	22
39	1.0068	8.6353	21
40	1.0068	8.6138	20
41	1.0068	8.5924	19
42	1.0069	8.5711	18
43	1.0069	8.5500	17
44	1.0069	8.5289	16
45	1.0070	8.5079	15
46	1.0070	8.4871	14
47	1.0070	8.4663	13
48	1.0071	8.4457	12
49	1.0071	8.4251	11
50	1.0072	8.4047	10
51	1.0072	8.3843	9
52	1.0072	8.3641	8
53	1.0073	8.3439	7
54	1.0073	8.3238	6
55	1.0073	8.3039	5
56	1.0074	8.2840	4
57	1.0074	8.2642	3
58	1.0074	8.2446	2
59	1.0075	8.2250	1
60	1.0075	8.2055	0
′	Csc	Sec	′

7° (187°) (352°) **172°**

′	Sec	Csc	′
0	1.0075	8.2055	60
1	1.0075	8.1861	59
2	1.0076	8.1668	58
3	1.0076	8.1476	57
4	1.0077	8.1285	56
5	1.0077	8.1095	55
6	1.0077	8.0905	54
7	1.0078	8.0717	53
8	1.0078	8.0529	52
9	1.0078	8.0342	51
10	1.0079	8.0156	50
11	1.0079	7.9971	49
12	1.0079	7.9787	48
13	1.0080	7.9604	47
14	1.0080	7.9422	46
15	1.0081	7.9240	45
16	1.0081	7.9059	44
17	1.0081	7.8879	43
18	1.0082	7.8700	42
19	1.0082	7.8522	41
20	1.0082	7.8344	40
21	1.0083	7.8168	39
22	1.0083	7.7992	38
23	1.0084	7.7817	37
24	1.0084	7.7642	36
25	1.0084	7.7469	35
26	1.0085	7.7296	34
27	1.0085	7.7124	33
28	1.0086	7.6953	32
29	1.0086	7.6783	31
30	1.0086	7.6613	30
31	1.0087	7.6444	29
32	1.0087	7.6276	28
33	1.0087	7.6109	27
34	1.0088	7.5942	26
35	1.0088	7.5776	25
36	1.0089	7.5611	24
37	1.0089	7.5446	23
38	1.0089	7.5282	22
39	1.0090	7.5119	21
40	1.0090	7.4957	20
41	1.0091	7.4795	19
42	1.0091	7.4635	18
43	1.0091	7.4474	17
44	1.0092	7.4315	16
45	1.0092	7.4156	15
46	1.0093	7.3998	14
47	1.0093	7.3840	13
48	1.0093	7.3684	12
49	1.0094	7.3527	11
50	1.0094	7.3372	10
51	1.0095	7.3217	9
52	1.0095	7.3063	8
53	1.0095	7.2909	7
54	1.0096	7.2757	6
55	1.0096	7.2604	5
56	1.0097	7.2453	4
57	1.0097	7.2302	3
58	1.0097	7.2152	2
59	1.0098	7.2002	1
60	1.0098	7.1853	0
′	Csc	Sec	′

8° (188°) (351°) **171°**

′	Sec	Csc	′
0	1.0098	7.1853	60
1	1.0099	7.1705	59
2	1.0099	7.1557	58
3	1.0100	7.1410	57
4	1.0100	7.1263	56
5	1.0100	7.1117	55
6	1.0101	7.0972	54
7	1.0101	7.0827	53
8	1.0102	7.0683	52
9	1.0102	7.0539	51
10	1.0102	7.0396	50
11	1.0103	7.0254	49
12	1.0103	7.0112	48
13	1.0104	6.9971	47
14	1.0104	6.9830	46
15	1.0105	6.9690	45
16	1.0105	6.9550	44
17	1.0105	6.9411	43
18	1.0106	6.9273	42
19	1.0106	6.9135	41
20	1.0107	6.8998	40
21	1.0107	6.8861	39
22	1.0108	6.8725	38
23	1.0108	6.8589	37
24	1.0108	6.8454	36
25	1.0109	6.8320	35
26	1.0109	6.8186	34
27	1.0110	6.8052	33
28	1.0110	6.7919	32
29	1.0111	6.7787	31
30	1.0111	6.7655	30
31	1.0112	6.7523	29
32	1.0112	6.7392	28
33	1.0112	6.7262	27
34	1.0113	6.7132	26
35	1.0113	6.7003	25
36	1.0114	6.6874	24
37	1.0114	6.6745	23
38	1.0115	6.6618	22
39	1.0115	6.6490	21
40	1.0116	6.6363	20
41	1.0016	6.6237	19
42	1.0116	6.6111	18
43	1.0117	6.5986	17
44	1.0117	6.5861	16
45	1.0118	6.5736	15
46	1.0118	6.5612	14
47	1.0119	6.5489	13
48	1.0119	6.5366	12
49	1.0120	6.5243	11
50	1.0120	6.5121	10
51	1.0120	6.4999	9
52	1.0121	6.4878	8
53	1.0121	6.4757	7
54	1.0122	6.4637	6
55	1.0122	6.4517	5
56	1.0123	6.4398	4
57	1.0123	6.4279	3
58	1.0124	6.4160	2
59	1.0124	6.4042	1
60	1.0125	6.3925	0
′	Csc	Sec	′

96° (276°) (263°) **83°**

97° (277°) (262°) **82°**

98° (278°) (261°) **81°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

9° (189°) (350°) **170°**

′	Sec	Csc	′
0	1.0125	6.3925	60
1	1.0125	6.3807	59
2	1.0126	6.3691	58
3	1.0126	6.3574	57
4	1.0127	6.3458	56
5	1.0127	6.3343	55
6	1.0127	6.3228	54
7	1.0128	6.3113	53
8	1.0128	6.2999	52
9	1.0129	6.2885	51
10	1.0129	6.2772	50
11	1.0130	6.2659	49
12	1.0130	6.2546	48
13	1.0131	6.2434	47
14	1.0131	6.2323	46
15	1.0132	6.2211	45
16	1.0132	6.2100	44
17	1.0133	6.1990	43
18	1.0133	6.1880	42
19	1.0134	6.1770	41
20	1.0134	6.1661	40
21	1.0135	6.1552	39
22	1.0135	6.1443	38
23	1.0136	6.1335	37
24	1.0136	6.1227	36
25	1.0137	6.1120	35
26	1.0137	6.1013	34
27	1.0138	6.0906	33
28	1.0138	6.0800	32
29	1.0139	6.0694	31
30	1.0139	6.0589	30
31	1.0140	6.0483	29
32	1.0140	6.0379	28
33	1.0141	6.0274	27
34	1.0141	6.0170	26
35	1.0142	6.0067	25
36	1.0142	5.9963	24
37	1.0143	5.9860	23
38	1.0143	5.9758	22
39	1.0144	5.9656	21
40	1.0144	5.9554	20
41	1.0145	5.9452	19
42	1.0145	5.9351	18
43	1.0146	5.9250	17
44	1.0146	5.9150	16
45	1.0147	5.9049	15
46	1.0147	5.8950	14
47	1.0148	5.8850	13
48	1.0148	5.8751	12
49	1.0149	5.8652	11
50	1.0149	5.8554	10
51	1.0150	5.8456	9
52	1.0150	5.8358	8
53	1.0151	5.8261	7
54	1.0151	5.8164	6
55	1.0152	5.8067	5
56	1.0152	5.7970	4
57	1.0153	5.7874	3
58	1.0153	5.7778	2
59	1.0154	5.7683	1
60	1.0154	5.7588	0
′	Csc	Sec	′

99° (279°) (260°) **80°**

10° (190°) (349°) **169°**

′	Sec	Csc	′
0	1.0154	5.7588	60
1	1.0155	5.7493	59
2	1.0155	5.7398	58
3	1.0156	5.7304	57
4	1.0156	5.7210	56
5	1.0157	5.7117	55
6	1.0157	5.7023	54
7	1.0158	5.6930	53
8	1.0158	5.6838	52
9	1.0159	5.6745	51
10	1.0160	5.6653	50
11	1.0160	5.6562	49
12	1.0161	5.6470	48
13	1.0161	5.6379	47
14	1.0162	5.6288	46
15	1.0162	5.6198	45
16	1.0163	5.6107	44
17	1.0163	5.6017	43
18	1.0164	5.5928	42
19	1.0164	5.5838	41
20	1.0165	5.5749	40
21	1.0165	5.5660	39
22	1.0166	5.5572	38
23	1.0166	5.5484	37
24	1.0167	5.5396	36
25	1.0168	5.5308	35
26	1.0168	5.5221	34
27	1.0169	5.5134	33
28	1.0169	5.5047	32
29	1.0170	5.4960	31
30	1.0170	5.4874	30
31	1.0171	5.4788	29
32	1.0171	5.4702	28
33	1.0172	5.4617	27
34	1.0173	5.4532	26
35	1.0173	5.4447	25
36	1.0174	5.4362	24
37	1.0174	5.4278	23
38	1.0175	5.4194	22
39	1.0175	5.4110	21
40	1.0176	5.4026	20
41	1.0176	5.3943	19
42	1.0177	5.3860	18
43	1.0178	5.3777	17
44	1.0178	5.3695	16
45	1.0179	5.3612	15
46	1.0179	5.3530	14
47	1.0180	5.3449	13
48	1.0180	5.3367	12
49	1.0181	5.3286	11
50	1.0181	5.3205	10
51	1.0182	5.3124	9
52	1.0183	5.3044	8
53	1.0183	5.2963	7
54	1.0184	5.2883	6
55	1.0184	5.2804	5
56	1.0185	5.2724	4
57	1.0185	5.2645	3
58	1.0186	5.2566	2
59	1.0187	5.2487	1
60	1.0187	5.2408	0
′	Csc	Sec	′

100° (280°) (259°) **79°**

11° (191°) (348°) **168°**

′	Sec	Csc	′
0	1.0187	5.2408	60
1	1.0188	5.2330	59
2	1.0188	5.2252	58
3	1.0189	5.2174	57
4	1.0189	5.2097	56
5	1.0190	5.2019	55
6	1.0191	5.1942	54
7	1.0191	5.1865	53
8	1.0192	5.1789	52
9	1.0192	5.1712	51
10	1.0193	5.1636	50
11	1.0194	5.1560	49
12	1.0194	5.1484	48
13	1.0195	5.1409	47
14	1.0195	5.1333	46
15	1.0196	5.1258	45
16	1.0197	5.1183	44
17	1.0197	5.1109	43
18	1.0198	5.1034	42
19	1.0198	5.0960	41
20	1.0199	5.0886	40
21	1.0199	5.0813	39
22	1.0200	5.0739	38
23	1.0201	5.0666	37
24	1.0201	5.0593	36
25	1.0202	5.0520	35
26	1.0202	5.0447	34
27	1.0203	5.0375	33
28	1.0204	5.0302	32
29	1.0204	5.0230	31
30	1.0205	5.0159	30
31	1.0205	5.0087	29
32	1.0206	5.0016	28
33	1.0207	4.9944	27
34	1.0207	4.9873	26
35	1.0208	4.9803	25
36	1.0209	4.9732	24
37	1.0209	4.9662	23
38	1.0210	4.9591	22
39	1.0210	4.9521	21
40	1.0211	4.9452	20
41	1.0212	4.9382	19
42	1.0212	4.9313	18
43	1.0213	4.9244	17
44	1.0213	4.9175	16
45	1.0214	4.9106	15
46	1.0215	4.9037	14
47	1.0215	4.8969	13
48	1.0216	4.8901	12
49	1.0217	4.8833	11
50	1.0217	4.8765	10
51	1.0218	4.8697	9
52	1.0218	4.8630	8
53	1.0219	4.8563	7
54	1.0220	4.8496	6
55	1.0220	4.8429	5
56	1.0221	4.8362	4
57	1.0222	4.8296	3
58	1.0222	4.8229	2
59	1.0223	4.8163	1
60	1.0223	4.8097	0
′	Csc	Sec	′

101° (281°) (258°) **78°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

12° (192°) (347°) **167°**

′	Sec	Csc	′
0	1.0223	4.8097	60
1	1.0224	4.8032	59
2	1.0225	4.7966	58
3	1.0225	4.7901	57
4	1.0226	4.7836	56
5	1.0227	4.7771	55
6	1.0227	4.7706	54
7	1.0228	4.7641	53
8	1.0228	4.7577	52
9	1.0229	4.7512	51
10	1.0230	4.7448	50
11	1.0230	4.7384	49
12	1.0231	4.7321	48
13	1.0232	4.7257	47
14	1.0232	4.7194	46
15	1.0233	4.7130	45
16	1.0234	4.7067	44
17	1.0234	4.7004	43
18	1.0235	4.6942	42
19	1.0236	4.6879	41
20	1.0236	4.6817	40
21	1.0237	4.6755	39
22	1.0238	4.6693	38
23	1.0238	4.6631	37
24	1.0239	4.6569	36
25	1.0240	4.6507	35
26	1.0240	4.6446	34
27	1.0241	4.6385	33
28	1.0241	4.6324	32
29	1.0242	4.6263	31
30	1.0243	4.6202	30
31	1.0243	4.6142	29
32	1.0244	4.6081	28
33	1.0245	4.6021	27
34	1.0245	4.5961	26
35	1.0246	4.5901	25
36	1.0247	4.5841	24
37	1.0247	4.5782	23
38	1.0248	4.5722	22
39	1.0249	4.5663	21
40	1.0249	4.5604	20
41	1.0250	4.5545	19
42	1.0251	4.5486	18
43	1.0251	4.5428	17
44	1.0252	4.5369	16
45	1.0253	4.5311	15
46	1.0253	4.5253	14
47	1.0254	4.5195	13
48	1.0255	4.5137	12
49	1.0256	4.5079	11
50	1.0256	4.5022	10
51	1.0257	4.4964	9
52	1.0258	4.4907	8
53	1.0258	4.4850	7
54	1.0259	4.4793	6
55	1.0260	4.4736	5
56	1.0260	4.4679	4
57	1.0261	4.4623	3
58	1.0262	4.4566	2
59	1.0262	4.4510	1
60	1.0263	4.4454	0
′	Csc	Sec	′

102° (282°) (257°) **77°**

13° (193°) (346°) **166°**

′	Sec	Csc	′
0	1.0263	4.4454	60
1	1.0264	4.4398	59
2	1.0264	4.4342	58
3	1.0265	4.4287	57
4	1.0266	4.4231	56
5	1.0266	4.4176	55
6	1.0267	4.4121	54
7	1.0268	4.4066	53
8	1.0269	4.4011	52
9	1.0269	4.3956	51
10	1.0270	4.3901	50
11	1.0271	4.3847	49
12	1.0271	4.3792	48
13	1.0272	4.3738	47
14	1.0273	4.3684	46
15	1.0273	4.3630	45
16	1.0274	4.3576	44
17	1.0275	4.3522	43
18	1.0276	4.3469	42
19	1.0276	4.3415	41
20	1.0277	4.3362	40
21	1.0278	4.3309	39
22	1.0278	4.3256	38
23	1.0279	4.3203	37
24	1.0280	4.3150	36
25	1.0281	4.3098	35
26	1.0281	4.3045	34
27	1.0282	4.2993	33
28	1.0283	4.2941	32
29	1.0283	4.2889	31
30	1.0284	4.2837	30
31	1.0285	4.2785	29
32	1.0286	4.2733	28
33	1.0286	4.2681	27
34	1.0287	4.2630	26
35	1.0288	4.2579	25
36	1.0288	4.2527	24
37	1.0289	4.2476	23
38	1.0290	4.2425	22
39	1.0291	4.2375	21
40	1.0291	4.2324	20
41	1.0292	4.2273	19
42	1.0293	4.2223	18
43	1.0294	4.2173	17
44	1.0294	4.2122	16
45	1.0295	4.2072	15
46	1.0296	4.2022	14
47	1.0297	4.1973	13
48	1.0297	4.1923	12
49	1.0298	4.1873	11
50	1.0299	4.1824	10
51	1.0299	4.1774	9
52	1.0300	4.1725	8
53	1.0301	4.1676	7
54	1.0302	4.1627	6
55	1.0302	4.1578	5
56	1.0303	4.1529	4
57	1.0304	4.1481	3
58	1.0305	4.1432	2
59	1.0305	4.1384	1
60	1.0306	4.1336	0
′	Csc	Sec	′

103° (283°) (256°) **76°**

14° (194°) (345°) **165°**

′	Sec	Csc	′
0	1.0306	4.1336	60
1	1.0307	4.1287	59
2	1.0308	4.1239	58
3	1.0308	4.1191	57
4	1.0309	4.1144	56
5	1.0310	4.1096	55
6	1.0311	4.1048	54
7	1.0311	4.1001	53
8	1.0312	4.0954	52
9	1.0313	4.0906	51
10	1.0314	4.0859	50
11	1.0314	4.0812	49
12	1.0315	4.0765	48
13	1.0316	4.0718	47
14	1.0317	4.0672	46
15	1.0317	4.0625	45
16	1.0318	4.0579	44
17	1.0319	4.0532	43
18	1.0320	4.0486	42
19	1.0321	4.0440	41
20	1.0321	4.0394	40
21	1.0322	4.0348	39
22	1.0323	4.0302	38
23	1.0324	4.0256	37
24	1.0324	4.0211	36
25	1.0325	4.0165	35
26	1.0326	4.0120	34
27	1.0327	4.0075	33
28	1.0327	4.0029	32
29	1.0328	3.9984	31
30	1.0329	3.9939	30
31	1.0330	3.9894	29
32	1.0331	3.9850	28
33	1.0331	3.9805	27
34	1.0332	3.9760	26
35	1.0333	3.9716	25
36	1.0334	3.9672	24
37	1.0334	3.9627	23
38	1.0335	3.9583	22
39	1.0336	3.9539	21
40	1.0337	3.9495	20
41	1.0338	3.9451	19
42	1.0338	3.9408	18
43	1.0339	3.9364	17
44	1.0340	3.9320	16
45	1.0341	3.9277	15
46	1.0342	3.9234	14
47	1.0342	3.9190	13
48	1.0343	3.9147	12
49	1.0344	3.9104	11
50	1.0345	3.9061	10
51	1.0346	3.9018	9
52	1.0346	3.8976	8
53	1.0347	3.8933	7
54	1.0348	3.8890	6
55	1.0349	3.8848	5
56	1.0350	3.8806	4
57	1.0350	3.8763	3
58	1.0351	3.8721	2
59	1.0352	3.8679	1
60	1.0353	3.8637	0
′	Csc	Sec	′

104° (284°) (255°) **75°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

15° (195°) (344°) **164°**

′	Sec	Csc	′
0	1.0353	3.8637	60
1	1.0354	3.8595	59
2	1.0354	3.8553	58
3	1.0355	3.8512	57
4	1.0356	3.8470	56
5	1.0357	3.8428	55
6	1.0358	3.8387	54
7	1.0358	3.8346	53
8	1.0359	3.8304	52
9	1.0360	3.8263	51
10	1.0361	3.8222	50
11	1.0362	3.8181	49
12	1.0363	3.8140	48
13	1.0363	3.8100	47
14	1.0364	3.8059	46
15	1.0365	3.8018	45
16	1.0366	3.7978	44
17	1.0367	3.7937	43
18	1.0367	3.7897	42
19	1.0368	3.7857	41
20	1.0369	3.7817	40
21	1.0370	3.7777	39
22	1.0371	3.7737	38
23	1.0372	3.7697	37
24	1.0372	3.7657	36
25	1.0373	3.7617	35
26	1.0374	3.7577	34
27	1.0375	3.7538	33
28	1.0376	3.7498	32
29	1.0377	3.7459	31
30	1.0377	3.7420	30
31	1.0378	3.7381	29
32	1.0379	3.7341	28
33	1.0380	3.7302	27
34	1.0381	3.7263	26
35	1.0382	3.7225	25
36	1.0382	3.7186	24
37	1.0383	3.7147	23
38	1.0384	3.7108	22
39	1.0385	3.7070	21
40	1.0386	3.7032	20
41	1.0387	3.6993	19
42	1.0388	3.6955	18
43	1.0388	3.6917	17
44	1.0389	3.6879	16
45	1.0390	3.6840	15
46	1.0391	3.6803	14
47	1.0392	3.6765	13
48	1.0393	3.6727	12
49	1.0394	3.6689	11
50	1.0394	3.6652	10
51	1.0395	3.6614	9
52	1.0396	3.6576	8
53	1.0397	3.6539	7
54	1.0398	3.6502	6
55	1.0399	3.6465	5
56	1.0400	3.6427	4
57	1.0400	3.6390	3
58	1.0401	3.6353	2
59	1.0402	3.6316	1
60	1.0403	3.6280	0
′	Csc	Sec	′

105° (285°) (254°) **74°**

16° (196°) (343°) **163°**

′	Sec	Csc	′
0	1.0403	3.6280	60
1	1.0404	3.6243	59
2	1.0405	3.6206	58
3	1.0406	3.6169	57
4	1.0406	3.6133	56
5	1.0407	3.6097	55
6	1.0408	3.6060	54
7	1.0409	3.6024	53
8	1.0410	3.5988	52
9	1.0411	3.5951	51
10	1.0412	3.5915	50
11	1.0413	3.5879	49
12	1.0413	3.5843	48
13	1.0414	3.5808	47
14	1.0415	3.5772	46
15	1.0416	3.5736	45
16	1.0417	3.5700	44
17	1.0418	3.5665	43
18	1.0419	3.5629	42
19	1.0420	3.5594	41
20	1.0421	3.5559	40
21	1.0421	3.5523	39
22	1.0422	3.5488	38
23	1.0423	3.5453	37
24	1.0424	3.5418	36
25	1.0425	3.5383	35
26	1.0426	3.5348	34
27	1.0427	3.5313	33
28	1.0428	3.5279	32
29	1.0429	3.5244	31
30	1.0429	3.5209	30
31	1.0430	3.5175	29
32	1.0431	3.5140	28
33	1.0432	3.5106	27
34	1.0433	3.5072	26
35	1.0434	3.5037	25
36	1.0435	3.5003	24
37	1.0436	3.4969	23
38	1.0437	3.4935	22
39	1.0438	3.4901	21
40	1.0439	3.4867	20
41	1.0439	3.4833	19
42	1.0440	3.4799	18
43	1.0441	3.4766	17
44	1.0442	3.4732	16
45	1.0443	3.4699	15
46	1.0444	3.4665	14
47	1.0445	3.4632	13
48	1.0446	3.4598	12
49	1.0447	3.4565	11
50	1.0448	3.4532	10
51	1.0449	3.4499	9
52	1.0450	3.4465	8
53	1.0450	3.4432	7
54	1.0451	3.4399	6
55	1.0452	3.4367	5
56	1.0453	3.4334	4
57	1.0454	3.4301	3
58	1.0455	3.4268	2
59	1.0456	3.4236	1
60	1.0457	3.4203	0
′	Csc	Sec	′

106° (286°) (253°) **73°**

17° (197°) (342°) **162°**

′	Sec	Csc	′
0	1.0457	3.4203	60
1	1.0458	3.4171	59
2	1.0459	3.4138	58
3	1.0460	3.4106	57
4	1.0461	3.4073	56
5	1.0462	3.4041	55
6	1.0463	3.4009	54
7	1.0463	3.3977	53
8	1.0464	3.3945	52
9	1.0465	3.3913	51
10	1.0466	3.3881	50
11	1.0467	3.3849	49
12	1.0468	3.3817	48
13	1.0469	3.3785	47
14	1.0470	3.3754	46
15	1.0471	3.3722	45
16	1.0472	3.3691	44
17	1.0473	3.3659	43
18	1.0474	3.3628	42
19	1.0475	3.3596	41
20	1.0476	3.3565	40
21	1.0477	3.3534	39
22	1.0478	3.3502	38
23	1.0479	3.3471	37
24	1.0480	3.3440	36
25	1.0480	3.3409	35
26	1.0481	3.3378	34
27	1.0482	3.3347	33
28	1.0483	3.3317	32
29	1.0484	3.3286	31
30	1.0485	3.3255	30
31	1.0486	3.3224	29
32	1.0487	3.3194	28
33	1.0488	3.3163	27
34	1.0489	3.3133	26
35	1.0490	3.3102	25
36	1.0491	3.3072	24
37	1.0492	3.3042	23
38	1.0493	3.3012	22
39	1.0494	3.2981	21
40	1.0495	3.2951	20
41	1.0496	3.2921	19
42	1.0497	3.2891	18
43	1.0498	3.2861	17
44	1.0499	3.2831	16
45	1.0500	3.2801	15
46	1.0501	3.2772	14
47	1.0502	3.2742	13
48	1.0503	3.2712	12
49	1.0504	3.2683	11
50	1.0505	3.2653	10
51	1.0506	3.2624	9
52	1.0507	3.2594	8
53	1.0508	3.2565	7
54	1.0509	3.2535	6
55	1.0510	3.2506	5
56	1.0511	3.2477	4
57	1.0512	3.2448	3
58	1.0513	3.2419	2
59	1.0514	3.2390	1
60	1.0515	3.2361	0
′	Csc	Sec	′

107° (287°) (252°) **72°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

18° (198^g) (341°) **161°** | **19°** (199°) (340°) **160°** | **20°** (200°) (339°) **159°**

′	Sec	Csc	′	′	Sec	Csc	′	′	Sec	Csc	′
0	1.0515	3.2361	60	0	1.0576	3.0716	60	0	1.0642	2.9238	60
1	1.0516	3.2332	59	1	1.0577	3.0690	59	1	1.0643	2.9215	59
2	1.0517	3.2303	58	2	1.0578	3.0664	58	2	1.0644	2.9191	58
3	1.0518	3.2274	57	3	1.0579	3.0638	57	3	1.0645	2.9168	57
4	1.0519	3.2245	56	4	1.0580	3.0612	56	4	1.0646	2.9145	56
5	1.0520	3.2217	55	5	1.0582	3.0586	55	5	1.0647	2.9122	55
6	1.0521	3.2188	54	6	1.0583	3.0561	54	6	1.0649	2.9099	54
7	1.0522	3.2159	53	7	1.0584	3.0535	53	7	1.0650	2.9075	53
8	1.0523	3.2131	52	8	1.0585	3.0509	52	8	1.0651	2.9052	52
9	1.0524	3.2102	51	9	1.0586	3.0484	51	9	1.0652	2.9029	51
10	1.0525	3.2074	50	10	1.0587	3.0458	50	10	1.0653	2.9006	50
11	1.0526	3.2045	49	11	1.0588	3.0433	49	11	1.0654	2.8983	49
12	1.0527	3.2017	48	12	1.0589	3.0407	48	12	1.0655	2.8960	48
13	1.0528	3.1989	47	13	1.0590	3.0382	47	13	1.0657	2.8938	47
14	1.0529	3.1960	46	14	1.0591	3.0357	46	14	1.0658	2.8915	46
15	1.0530	3.1932	45	15	1.0592	3.0331	45	15	1.0659	2.8892	45
16	1.0531	3.1904	44	16	1.0593	3.0306	44	16	1.0660	2.8869	44
17	1.0532	3.1876	43	17	1.0594	3.0281	43	17	1.0661	2.8846	43
18	1.0533	3.1848	42	18	1.0595	3.0256	42	18	1.0662	2.8824	42
19	1.0534	3.1820	41	19	1.0597	3.0231	41	19	1.0663	2.8801	41
20	1.0535	3.1792	40	20	1.0598	3.0206	40	20	1.0665	2.8779	40
21	1.0536	3.1764	39	21	1.0599	3.0181	39	21	1.0666	2.8756	39
22	1.0537	3.1736	38	22	1.0600	3.0156	38	22	1.0667	2.8733	38
23	1.0538	3.1708	37	23	1.0601	3.0131	37	23	1.0668	2.8711	37
24	1.0539	3.1681	36	24	1.0602	3.0106	36	24	1.0669	2.8688	36
25	1.0540	3.1653	35	25	1.0603	3.0081	35	25	1.0670	2.8666	35
26	1.0541	3.1625	34	26	1.0604	3.0056	34	26	1.0671	2.8644	34
27	1.0542	3.1598	33	27	1.0605	3.0031	33	27	1.0673	2.8621	33
28	1.0543	3.1570	32	28	1.0606	3.0007	32	28	1.0674	2.8599	32
29	1.0544	3.1543	31	29	1.0607	2.9982	31	29	1.0675	2.8577	31
30	1.0545	3.1515	30	30	1.0608	2.9957	30	30	1.0676	2.8555	30
31	1.0546	3.1488	29	31	1.0610	2.9933	29	31	1.0677	2.8532	29
32	1.0547	3.1461	28	32	1.0611	2.9908	28	32	1.0678	2.8510	28
33	1.0548	3.1433	27	33	1.0612	2.9884	27	33	1.0680	2.8488	27
34	1.0549	3.1406	26	34	1.0613	2.9859	26	34	1.0681	2.8466	26
35	1.0550	3.1379	25	35	1.0614	2.9835	25	35	1.0682	2.8444	25
36	1.0551	3.1352	24	36	1.0615	2.9811	24	36	1.0683	2.8422	24
37	1.0552	3.1325	23	37	1.0616	2.9786	23	37	1.0684	2.8400	23
38	1.0553	3.1298	22	38	1.0617	2.9762	22	38	1.0685	2.8378	22
39	1.0554	3.1271	21	39	1.0618	2.9738	21	39	1.0687	2.8356	21
40	1.0555	3.1244	20	40	1.0619	2.9713	20	40	1.0688	2.8334	20
41	1.0556	3.1217	19	41	1.0621	2.9689	19	41	1.0689	2.8312	19
42	1.0557	3.1190	18	42	1.0622	2.9665	18	42	1.0690	2.8291	18
43	1.0558	3.1163	17	43	1.0623	2.9641	17	43	1.0691	2.8269	17
44	1.0559	3.1137	16	44	1.0624	2.9617	16	44	1.0692	2.8247	16
45	1.0560	3.1110	15	45	1.0625	2.9593	15	45	1.0694	2.8225	15
46	1.0561	3.1083	14	46	1.0626	2.9569	14	46	1.0695	2.8204	14
47	1.0563	3.1057	13	47	1.0627	2.9545	13	47	1.0696	2.8182	13
48	1.0564	3.1030	12	48	1.0628	2.9521	12	48	1.0697	2.8161	12
49	1.0565	3.1004	11	49	1.0629	2.9498	11	49	1.0698	2.8139	11
50	1.0566	3.0977	10	50	1.0631	2.9474	10	50	1.0700	2.8117	10
51	1.0567	3.0951	9	51	1.0632	2.9450	9	51	1.0701	2.8096	9
52	1.0568	3.0925	8	52	1.0633	2.9426	8	52	1.0702	2.8075	8
53	1.0569	3.0898	7	53	1.0634	2.9403	7	53	1.0703	2.8053	7
54	1.0570	3.0872	6	54	1.0635	2.9379	6	54	1.0704	2.8032	6
55	1.0571	3.0846	5	55	1.0636	2.9355	5	55	1.0705	2.8010	5
56	1.0572	3.0820	4	56	1.0637	2.9332	4	56	1.0707	2.7989	4
57	1.0573	3.0794	3	57	1.0638	2.9308	3	57	1.0708	2.7968	3
58	1.0574	3.0768	2	58	1.0640	2.9285	2	58	1.0709	2.7947	2
59	1.0575	3.0742	1	59	1.0641	2.9261	1	59	1.0710	2.7925	1
60	1.0576	3.0716	0	60	1.0642	2.9238	0	60	1.0711	2.7904	0
′	Csc	Sec	′	′	Csc	Sec	′	′	Csc	Sec	′

108° (288°) (251°) **71°** | **109°** (289°) (250°) **70°** | **110°** (290°) (249°) **69°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

21° (201°) (338°) **158°**

′	Sec	Csc	′
0	1.0711	2.7904	60
1	1.0713	2.7883	59
2	1.0714	2.7862	58
3	1.0715	2.7841	57
4	1.0716	2.7820	56
5	1.0717	2.7799	55
6	1.0719	2.7778	54
7	1.0720	2.7757	53
8	1.0721	2.7736	52
9	1.0722	2.7715	51
10	1.0723	2.7695	50
11	1.0725	2.7674	49
12	1.0726	2.7653	48
13	1.0727	2.7632	47
14	1.0728	2.7612	46
15	1.0730	2.7591	45
16	1.0731	2.7570	44
17	1.0732	2.7550	43
18	1.0733	2.7529	42
19	1.0734	2.7509	41
20	1.0736	2.7488	40
21	1.0737	2.7468	39
22	1.0738	2.7447	38
23	1.0739	2.7427	37
24	1.0740	2.7407	36
25	1.0742	2.7386	35
26	1.0743	2.7366	34
27	1.0744	2.7346	33
28	1.0745	2.7325	32
29	1.0747	2.7305	31
30	1.0748	2.7285	30
31	1.0749	2.7265	29
32	1.0750	2.7245	28
33	1.0752	2.7225	27
34	1.0753	2.7205	26
35	1.0754	2.7185	25
36	1.0755	2.7165	24
37	1.0757	2.7145	23
38	1.0758	2.7125	22
39	1.0759	2.7105	21
40	1.0760	2.7085	20
41	1.0761	2.7065	19
42	1.0763	2.7046	18
43	1.0764	2.7026	17
44	1.0765	2.7006	16
45	1.0766	2.6986	15
46	1.0768	2.6967	14
47	1.0769	2.6947	13
48	1.0770	2.6927	12
49	1.0771	2.6908	11
50	1.0773	2.6888	10
51	1.0774	2.6869	9
52	1.0775	2.6849	8
53	1.0777	2.6830	7
54	1.0778	2.6811	6
55	1.0779	2.6791	5
56	1.0780	2.6772	4
57	1.0782	2.6752	3
58	1.0783	2.6733	2
59	1.0784	2.6714	1
60	1.0785	2.6695	0
′	Csc	Sec	′

111° (291°) (248°) **68°**

22° (202°) (337°) **157°**

′	Sec	Csc	′
0	1.0785	2.6695	60
1	1.0787	2.6675	59
2	1.0788	2.6656	58
3	1.0789	2.6637	57
4	1.0790	2.6618	56
5	1.0792	2.6599	55
6	1.0793	2.6580	54
7	1.0794	2.6561	53
8	1.0796	2.6542	52
9	1.0797	2.6523	51
10	1.0798	2.6504	50
11	1.0799	2.6485	49
12	1.0801	2.6466	48
13	1.0802	2.6447	47
14	1.0803	2.6429	46
15	1.0804	2.6410	45
16	1.0806	2.6391	44
17	1.0807	2.6372	43
18	1.0808	2.6354	42
19	1.0810	2.6335	41
20	1.0811	2.6316	40
21	1.0812	2.6298	39
22	1.0814	2.6279	38
23	1.0815	2.6260	37
24	1.0816	2.6242	36
25	1.0817	2.6223	35
26	1.0819	2.6205	34
27	1.0820	2.6186	33
28	1.0821	2.6168	32
29	1.0823	2.6150	31
30	1.0824	2.6131	30
31	1.0825	2.6113	29
32	1.0827	2.6095	28
33	1.0828	2.6076	27
34	1.0829	2.6058	26
35	1.0830	2.6040	25
36	1.0832	2.6022	24
37	1.0833	2.6003	23
38	1.0834	2.5985	22
39	1.0836	2.5967	21
40	1.0837	2.5949	20
41	1.0838	2.5931	19
42	1.0840	2.5913	18
43	1.0841	2.5895	17
44	1.0842	2.5877	16
45	1.0844	2.5859	15
46	1.0845	2.5841	14
47	1.0846	2.5823	13
48	1.0843	2.5805	12
49	1.0849	2.5788	11
50	1.0850	2.5770	10
51	1.0852	2.5752	9
52	1.0853	2.5734	8
53	1.0854	2.5716	7
54	1.0856	2.5699	6
55	1.0857	2.5681	5
56	1.0858	2.5663	4
57	1.0860	2.5646	3
58	1.0861	2.5628	2
59	1.0862	2.5611	1
60	1.0864	2.5593	0
′	Csc	Sec	′

112° (292°) (247°) **67°**

23° (203°) (336°) **156°**

′	Sec	Csc	′
0	1.0864	2.5593	60
1	1.0865	2.5576	59
2	1.0866	2.5558	58
3	1.0868	2.5541	57
4	1.0869	2.5523	56
5	1.0870	2.5506	55
6	1.0872	2.5488	54
7	1.0873	2.5471	53
8	1.0874	2.5454	52
9	1.0876	2.5436	51
10	1.0877	2.5419	50
11	1.0878	2.5402	49
12	1.0880	2.5384	48
13	1.0881	2.5367	47
14	1.0883	2.5350	46
15	1.0884	2.5333	45
16	1.0885	2.5316	44
17	1.0887	2.5299	43
18	1.0888	2.5282	42
19	1.0889	2.5264	41
20	1.0891	2.5247	40
21	1.0892	2.5230	39
22	1.0893	2.5213	38
23	1.0895	2.5196	37
24	1.0896	2.5180	36
25	1.0898	2.5163	35
26	1.0899	2.5146	34
27	1.0900	2.5129	33
28	1.0902	2.5112	32
29	1.0903	2.5095	31
30	1.0904	2.5078	30
31	1.0906	2.5062	29
32	1.0907	2.5045	28
33	1.0909	2.5028	27
34	1.0910	2.5012	26
35	1.0911	2.4995	25
36	1.0913	2.4978	24
37	1.0914	2.4962	23
38	1.0915	2.4945	22
39	1.0917	2.4928	21
40	1.0918	2.4912	20
41	1.0920	2.4895	19
42	1.0921	2.4879	18
43	1.0922	2.4862	17
44	1.0924	2.4846	16
45	1.0925	2.4830	15
46	1.0927	2.4813	14
47	1.0928	2.4797	13
48	1.0929	2.4780	12
49	1.0931	2.4764	11
50	1.0932	2.4748	10
51	1.0934	2.4731	9
52	1.0935	2.4715	8
53	1.0936	2.4699	7
54	1.0938	2.4683	6
55	1.0939	2.4667	5
56	1.0941	2.4650	4
57	1.0942	2.4634	3
58	1.0944	2.4618	2
59	1.0945	2.4602	1
60	1.0946	2.4586	0
′	Csc	Sec	′

113° (293°) (246°) **66°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

24° (204°)		(335°) 155°		25° (205°)		(334°) 154°		26° (206°)		(333°) 153°	
′	Sec	Csc	′	′	Sec	Csc	′	′	Sec	Csc	′
0	1.0946	2.4586	60	0	1.1034	2.3662	60	0	1.1126	2.2812	60
1	1.0948	2.4570	59	1	1.1035	2.3647	59	1	1.1128	2.2798	59
2	1.0949	2.4554	58	2	1.1037	2.3633	58	2	1.1129	2.2785	58
3	1.0951	2.4538	57	3	1.1038	2.3618	57	3	1.1131	2.2771	57
4	1.0952	2.4522	56	4	1.1040	2.3603	56	4	1.1132	2.2757	56
5	1.0953	2.4506	55	5	1.1041	2.3588	55	5	1.1134	2.2744	55
6	1.0955	2.4490	54	6	1.1043	2.3574	54	6	1.1136	2.2730	54
7	1.0956	2.4474	53	7	1.1044	2.3559	53	7	1.1137	2.2717	53
8	1.0958	2.4458	52	8	1.1046	2.3545	52	8	1.1139	2.2703	52
9	1.0959	2.4442	51	9	1.1047	2.3530	51	9	1.1140	2.2690	51
10	1.0961	2.4426	50	10	1.1049	2.3515	50	10	1.1142	2.2677	50
11	1.0962	2.4411	49	11	1.1050	2.3501	49	11	1.1143	2.2663	49
12	1.0963	2.4395	48	12	1.1052	2.3486	48	12	1.1145	2.2650	48
13	1.0965	2.4379	47	13	1.1053	2.3472	47	13	1.1147	2.2636	47
14	1.0966	2.4363	46	14	1.1055	2.3457	46	14	1.1148	2.2623	46
15	1.0968	2.4348	45	15	1.1056	2.3443	45	15	1.1150	2.2610	45
16	1.0969	2.4332	44	16	1.1058	2.3428	44	16	1.1151	2.2596	44
17	1.0971	2.4316	43	17	1.1059	2.3414	43	17	1.1153	2.2583	43
18	1.0972	2.4300	42	18	1.1061	2.3400	42	18	1.1155	2.2570	42
19	1.0974	2.4285	41	19	1.1062	2.3385	41	19	1.1156	2.2556	41
20	1.0975	2.4269	40	20	1.1064	2.3371	40	20	1.1158	2.2543	40
21	1.0976	2.4254	39	21	1.1066	2.3356	39	21	1.1159	2.2530	39
22	1.0978	2.4238	38	22	1.1067	2.3342	38	22	1.1161	2.2517	38
23	1.0979	2.4222	37	23	1.1069	2.3328	37	23	1.1163	2.2504	37
24	1.0981	2.4207	36	24	1.1070	2.3314	36	24	1.1164	2.2490	36
25	1.0982	2.4191	35	25	1.1072	2.3299	35	25	1.1166	2.2477	35
26	1.0984	2.4176	34	26	1.1073	2.3285	34	26	1.1168	2.2464	34
27	1.0985	2.4160	33	27	1.1075	2.3271	33	27	1.1169	2.2451	33
28	1.0987	2.4145	32	28	1.1076	2.3257	32	28	1.1171	2.2438	32
29	1.0988	2.4130	31	29	1.1078	2.3242	31	29	1.1172	2.2425	31
30	1.0989	2.4114	30	30	1.1079	2.3228	30	30	1.1174	2.2412	30
31	1.0991	2.4099	29	31	1.1081	2.3214	29	31	1.1176	2.2399	29
32	1.0992	2.4083	28	32	1.1082	2.3200	28	32	1.1177	2.2385	28
33	1.0994	2.4068	27	33	1.1084	2.3186	27	33	1.1179	2.2372	27
34	1.0995	2.4053	26	34	1.1085	2.3172	26	34	1.1180	2.2359	26
35	1.0997	2.4038	25	35	1.1087	2.3158	25	35	1.1182	2.2346	25
36	1.0998	2.4022	24	36	1.1089	2.3144	24	36	1.1184	2.2333	24
37	1.1000	2.4007	23	37	1.1090	2.3130	23	37	1.1185	2.2320	23
38	1.1001	2.3992	22	38	1.1092	2.3115	22	38	1.1187	2.2308	22
39	1.1003	2.3977	21	39	1.1093	2.3101	21	39	1.1189	2.2295	21
40	1.1004	2.3961	20	40	1.1095	2.3088	20	40	1.1190	2.2282	20
41	1.1006	2.3946	19	41	1.1096	2.3074	19	41	1.1192	2.2269	19
42	1.1007	2.3931	18	42	1.1098	2.3060	18	42	1.1194	2.2256	18
43	1.1009	2.3916	17	43	1.1099	2.3046	17	43	1.1195	2.2243	17
44	1.1010	2.3901	16	44	1.1101	2.3032	16	44	1.1197	2.2230	16
45	1.1011	2.3886	15	45	1.1102	2.3018	15	45	1.1198	2.2217	15
46	1.1013	2.3871	14	46	1.1104	2.3004	14	46	1.1200	2.2205	14
47	1.1014	2.3856	13	47	1.1106	2.2990	13	47	1.1202	2.2192	13
48	1.1016	2.3841	12	48	1.1107	2.2976	12	48	1.1203	2.2179	12
49	1.1017	2.3826	11	49	1.1109	2.2962	11	49	1.1205	2.2166	11
50	1.1019	2.3811	10	50	1.1110	2.2949	10	50	1.1207	2.2153	10
51	1.1020	2.3796	9	51	1.1112	2.2935	9	51	1.1208	2.2141	9
52	1.1022	2.3781	8	52	1.1113	2.2921	8	52	1.1210	2.2128	8
53	1.1023	2.3766	7	53	1.1115	2.2907	7	53	1.1212	2.2115	7
54	1.1025	2.3751	6	54	1.1117	2.2894	6	54	1.1213	2.2103	6
55	1.1026	2.3736	5	55	1.1118	2.2880	5	55	1.1215	2.2090	5
56	1.1028	2.3721	4	56	1.1120	2.2866	4	56	1.1217	2.2077	4
57	1.1029	2.3706	3	57	1.1121	2.2853	3	57	1.1218	2.2065	3
58	1.1031	2.3692	2	58	1.1123	2.2839	2	58	1.1220	2.2052	2
59	1.1032	2.3677	1	59	1.1124	2.2825	1	59	1.1222	2.2039	1
60	1.1034	2.3662	0	60	1.1126	2.2812	0	60	1.1223	2.2027	0
′	Csc	Sec	′	′	Csc	Sec	′	′	Csc	Sec	′
114° (294°)		(245°) **65°**		**115°** (295°)		(244°) **64°**		**116°** (296°)		(243°) **63°**	

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

27° (207°) (332°) **152°**

′	Sec	Csc	′
0	1.1223	2.2027	60
1	1.1225	2.2014	59
2	1.1227	2.2002	58
3	1.1228	2.1989	57
4	1.1230	2.1977	56
5	1.1232	2.1964	55
6	1.1233	2.1952	54
7	1.1235	2.1939	53
8	1.1237	2.1927	52
9	1.1238	2.1914	51
10	1.1240	2.1902	50
11	1.1242	2.1890	49
12	1.1243	2.1877	48
13	1.1245	2.1865	47
14	1.1247	2.1852	46
15	1.1248	2.1840	45
16	1.1250	2.1828	44
17	1.1252	2.1815	43
18	1.1253	2.1803	42
19	1.1255	2.1791	41
20	1.1257	2.1779	40
21	1.1259	2.1766	39
22	1.1260	2.1754	38
23	1.1262	2.1742	37
24	1.1264	2.1730	36
25	1.1265	2.1718	35
26	1.1267	2.1705	34
27	1.1269	2.1693	33
28	1.1270	2.1681	32
29	1.1272	2.1669	31
30	1.1274	2.1657	30
31	1.1276	2.1645	29
32	1.1277	2.1633	28
33	1.1279	2.1621	27
34	1.1281	2.1609	26
35	1.1282	2.1596	25
36	1.1284	2.1584	24
37	1.1286	2.1572	23
38	1.1288	2.1560	22
39	1.1289	2.1549	21
40	1.1291	2.1537	20
41	1.1293	2.1525	19
42	1.1294	2.1513	18
43	1.1296	2.1501	17
44	1.1298	2.1489	16
45	1.1300	2.1477	15
46	1.1301	2.1465	14
47	1.1303	2.1453	13
48	1.1305	2.1441	12
49	1.1307	2.1430	11
50	1.1308	2.1418	10
51	1.1310	2.1406	9
52	1.1312	2.1394	8
53	1.1313	2.1382	7
54	1.1315	2.1371	6
55	1.1317	2.1359	5
56	1.1319	2.1347	4
57	1.1320	2.1336	3
58	1.1322	2.1324	2
59	1.1324	2.1312	1
60	1.1326	2.1301	0
′	Csc	Sec	′

117° (297°) (242°) **62°**

28° (208°) (331°) **151°**

′	Sec	Csc	′
0	1.1326	2.1301	60
1	1.1327	2.1289	59
2	1.1329	2.1277	58
3	1.1331	2.1266	57
4	1.1333	2.1254	56
5	1.1334	2.1242	55
6	1.1336	2.1231	54
7	1.1338	2.1219	53
8	1.1340	2.1208	52
9	1.1342	2.1196	51
10	1.1343	2.1185	50
11	1.1345	2.1173	49
12	1.1347	2.1162	48
13	1.1349	2.1150	47
14	1.1350	2.1139	46
15	1.1352	2.1127	45
16	1.1354	2.1116	44
17	1.1356	2.1105	43
18	1.1357	2.1093	42
19	1.1359	2.1082	41
20	1.1361	2.1070	40
21	1.1363	2.1059	39
22	1.1365	2.1048	38
23	1.1366	2.1036	37
24	1.1368	2.1025	36
25	1.1370	2.1014	35
26	1.1372	2.1002	34
27	1.1374	2.0991	33
28	1.1375	2.0980	32
29	1.1377	2.0969	31
30	1.1379	2.0957	30
31	1.1381	2.0946	29
32	1.1383	2.0935	28
33	1.1384	2.0924	27
34	1.1386	2.0913	26
35	1.1388	2.0901	25
36	1.1390	2.0890	24
37	1.1392	2.0879	23
38	1.1393	2.0868	22
39	1.1395	2.0857	21
40	1.1397	2.0846	20
41	1.1399	2.0835	19
42	1.1401	2.0824	18
43	1.1402	2.0813	17
44	1.1404	2.0802	16
45	1.1406	2.0791	15
46	1.1408	2.0779	14
47	1.1410	2.0768	13
48	1.1412	2.0757	12
49	1.1413	2.0747	11
50	1.1415	2.0736	10
51	1.1417	2.0725	9
52	1.1419	2.0714	8
53	1.1421	2.0703	7
54	1.1423	2.0692	6
55	1.1424	2.0681	5
56	1.1426	2.0670	4
57	1.1428	2.0659	3
58	1.1430	2.0648	2
59	1.1432	2.0637	1
60	1.1434	2.0627	0
′	Csc	Sec	′

118° (298°) (241°) **61°**

29° (209°) (330°) **150°**

′	Sec	Csc	′
0	1.1434	2.0627	60
1	1.1435	2.0616	59
2	1.1437	2.0605	58
3	1.1439	2.0594	57
4	1.1441	2.0583	56
5	1.1443	2.0573	55
6	1.1445	2.0562	54
7	1.1446	2.0551	53
8	1.1448	2.0540	52
9	1.1450	2.0530	51
10	1.1452	2.0519	50
11	1.1454	2.0508	49
12	1.1456	2.0498	48
13	1.1458	2.0487	47
14	1.1460	2.0476	46
15	1.1461	2.0466	45
16	1.1463	2.0455	44
17	1.1465	2.0445	43
18	1.1467	2.0434	42
19	1.1469	2.0423	41
20	1.1471	2.0413	40
21	1.1473	2.0402	39
22	1.1474	2.0392	38
23	1.1476	2.0381	37
24	1.1478	2.0371	36
25	1.1480	2.0360	35
26	1.1482	2.0350	34
27	1.1484	2.0339	33
28	1.1486	2.0329	32
29	1.1488	2.0318	31
30	1.1490	2.0308	30
31	1.1491	2.0297	29
32	1.1493	2.0287	28
33	1.1495	2.0276	27
34	1.1497	2.0266	26
35	1.1499	2.0256	25
36	1.1501	2.0245	24
37	1.1503	2.0235	23
38	1.1505	2.0225	22
39	1.1507	2.0214	21
40	1.1509	2.0204	20
41	1.1510	2.0194	19
42	1.1512	2.0183	18
43	1.1514	2.0173	17
44	1.1516	2.0163	16
45	1.1518	2.0152	15
46	1.1520	2.0142	14
47	1.1522	2.0132	13
48	1.1524	2.0122	12
49	1.1526	2.0112	11
50	1.1528	2.0101	10
51	1.1530	2.0091	9
52	1.1532	2.0081	8
53	1.1533	2.0071	7
54	1.1535	2.0061	6
55	1.1537	2.0051	5
56	1.1539	2.0040	4
57	1.1541	2.0030	3
58	1.1543	2.0020	2
59	1.1545	2.0010	1
60	1.1547	2.0000	0
′	Csc	Sec	′

119° (299°) (240°) **60°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

30° (210°) (329°) **149°**

′	Sec	Csc	′
0	1.1547	2.0000	60
1	1.1549	1.9990	59
2	1.1551	1.9980	58
3	1.1553	1.9970	57
4	1.1555	1.9960	56
5	1.1557	1.9950	55
6	1.1559	1.9940	54
7	1.1561	1.9930	53
8	1.1563	1.9920	52
9	1.1565	1.9910	51
10	1.1566	1.9900	50
11	1.1568	1.9890	49
12	1.1570	1.9880	48
13	1.1572	1.9870	47
14	1.1574	1.9860	46
15	1.1576	1.9850	45
16	1.1578	1.9840	44
17	1.1580	1.9830	43
18	1.1582	1.9821	42
19	1.1584	1.9811	41
20	1.1586	1.9801	40
21	1.1588	1.9791	39
22	1.1590	1.9781	38
23	1.1592	1.9771	37
24	1.1594	1.9762	36
25	1.1596	1.9752	35
26	1.1598	1.9742	34
27	1.1600	1.9732	33
28	1.1602	1.9722	32
29	1.1604	1.9713	31
30	1.1606	1.9703	30
31	1.1608	1.9693	29
32	1.1610	1.9684	28
33	1.1612	1.9674	27
34	1.1614	1.9664	26
35	1.1616	1.9654	25
36	1.1618	1.9645	24
37	1.1620	1.9635	23
38	1.1622	1.9625	22
39	1.1624	1.9616	21
40	1.1626	1.9606	20
41	1.1628	1.9597	19
42	1.1630	1.9587	18
43	1.1632	1.9577	17
44	1.1634	1.9568	16
45	1.1636	1.9558	15
46	1.1638	1.9549	14
47	1.1640	1.9539	13
48	1.1642	1.9530	12
49	1.1644	1.9520	11
50	1.1646	1.9511	10
51	1.1648	1.9501	9
52	1.1650	1.9492	8
53	1.1652	1.9482	7
54	1.1654	1.9473	6
55	1.1656	1.9463	5
56	1.1658	1.9454	4
57	1.1660	1.9444	3
58	1.1662	1.9435	2
59	1.1664	1.9425	1
60	1.1666	1.9416	0
′	Csc	Sec	′

120° (300°) (239°) **59°**

31° (211°) (328°) **148°**

′	Sec	Csc	′
0	1.1666	1.9416	60
1	1.1668	1.9407	59
2	1.1670	1.9397	58
3	1.1672	1.9388	57
4	1.1675	1.9379	56
5	1.1677	1.9369	55
6	1.1679	1.9360	54
7	1.1681	1.9351	53
8	1.1683	1.9341	52
9	1.1685	1.9332	51
10	1.1687	1.9323	50
11	1.1689	1.9313	49
12	1.1691	1.9304	48
13	1.1693	1.9295	47
14	1.1695	1.9285	46
15	1.1697	1.9276	45
16	1.1699	1.9267	44
17	1.1701	1.9258	43
18	1.1703	1.9249	42
19	1.1705	1.9239	41
20	1.1707	1.9230	40
21	1.1710	1.9221	39
22	1.1712	1.9212	38
23	1.1714	1.9203	37
24	1.1716	1.9194	36
25	1.1718	1.9184	35
26	1.1720	1.9175	34
27	1.1722	1.9166	33
28	1.1724	1.9157	32
29	1.1726	1.9148	31
30	1.1728	1.9139	30
31	1.1730	1.9130	29
32	1.1732	1.9121	28
33	1.1735	1.9112	27
34	1.1737	1.9103	26
35	1.1739	1.9094	25
36	1.1741	1.9084	24
37	1.1743	1.9075	23
38	1.1745	1.9066	22
39	1.1747	1.9057	21
40	1.1749	1.9048	20
41	1.1751	1.9039	19
42	1.1753	1.9031	18
43	1.1756	1.9022	17
44	1.1758	1.9013	16
45	1.1760	1.9004	15
46	1.1762	1.8995	14
47	1.1764	1.8986	13
48	1.1766	1.8977	12
49	1.1768	1.8968	11
50	1.1770	1.8959	10
51	1.1773	1.8950	9
52	1.1775	1.8941	8
53	1.1777	1.8933	7
54	1.1779	1.8924	6
55	1.1781	1.8915	5
56	1.1783	1.8906	4
57	1.1785	1.8897	3
58	1.1788	1.8888	2
59	1.1790	1.8880	1
60	1.1792	1.8871	0
′	Csc	Sec	′

121° (301°) (238°) **58°**

32° (212°) (327°) **147°**

′	Sec	Csc	′
0	1.1792	1.8871	60
1	1.1794	1.8862	59
2	1.1796	1.8853	58
3	1.1798	1.8844	57
4	1.1800	1.8836	56
5	1.1803	1.8827	55
6	1.1805	1.8818	54
7	1.1807	1.8810	53
8	1.1809	1.8801	52
9	1.1811	1.8792	51
10	1.1813	1.8783	50
11	1.1815	1.8775	49
12	1.1818	1.8766	48
13	1.1820	1.8757	47
14	1.1822	1.8749	46
15	1.1824	1.8740	45
16	1.1826	1.8731	44
17	1.1828	1.8723	43
18	1.1831	1.8714	42
19	1.1833	1.8706	41
20	1.1835	1.8697	40
21	1.1837	1.8688	39
22	1.1839	1.8680	38
23	1.1842	1.8671	37
24	1.1844	1.8663	36
25	1.1846	1.8654	35
26	1.1848	1.8646	34
27	1.1850	1.8637	33
28	1.1852	1.8629	32
29	1.1855	1.8620	31
30	1.1857	1.8612	30
31	1.1859	1.8603	29
32	1.1861	1.8595	28
33	1.1863	1.8586	27
34	1.1866	1.8578	26
35	1.1868	1.8569	25
36	1.1870	1.8561	24
37	1.1872	1.8552	23
38	1.1875	1.8544	22
39	1.1877	1.8535	21
40	1.1879	1.8527	20
41	1.1881	1.8519	19
42	1.1883	1.8510	18
43	1.1886	1.8502	17
44	1.1888	1.8494	16
45	1.1890	1.8485	15
46	1.1892	1.8477	14
47	1.1895	1.8468	13
48	1.1897	1.8460	12
49	1.1899	1.8452	11
50	1.1901	1.8443	10
51	1.1903	1.8435	9
52	1.1906	1.8427	8
53	1.1908	1.8419	7
54	1.1910	1.8410	6
55	1.1912	1.8402	5
56	1.1915	1.8394	4
57	1.1917	1.8385	3
58	1.1919	1.8377	2
59	1.1921	1.8369	1
60	1.1924	1.8361	0
′	Csc	Sec	′

122° (302°) (237°) **57°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

33° (213°) (326°) **146°**

′	Sec	Csc	′
0	1.1924	1.8361	60
1	1.1926	1.8353	59
2	1.1928	1.8344	58
3	1.1930	1.8336	57
4	1.1933	1.8328	56
5	1.1935	1.8320	55
6	1.1937	1.8312	54
7	1.1939	1.8303	53
8	1.1942	1.8295	52
9	1.1944	1.8287	51
10	1.1946	1.8279	50
11	1.1949	1.8271	49
12	1.1951	1.8263	48
13	1.1953	1.8255	47
14	1.1955	1.8247	46
15	1.1958	1.8238	45
16	1.1960	1.8230	44
17	1.1962	1.8222	43
18	1.1964	1.8214	42
19	1.1967	1.8206	41
20	1.1969	1.8198	40
21	1.1971	1.8190	39
22	1.1974	1.8182	38
23	1.1976	1.8174	37
24	1.1978	1.8166	36
25	1.1981	1.8158	35
26	1.1983	1.8150	34
27	1.1985	1.8142	33
28	1.1987	1.8134	32
29	1.1990	1.8126	31
30	1.1992	1.8118	30
31	1.1994	1.8110	29
32	1.1997	1.8102	28
33	1.1999	1.8094	27
34	1.2001	1.8086	26
35	1.2004	1.8078	25
36	1.2006	1.8070	24
37	1.2008	1.8062	23
38	1.2011	1.8055	22
39	1.2013	1.8047	21
40	1.2015	1.8039	20
41	1.2018	1.8031	19
42	1.2020	1.8023	18
43	1.2022	1.8015	17
44	1.2025	1.8007	16
45	1.2027	1.8000	15
46	1.2029	1.7992	14
47	1.2032	1.7984	13
48	1.2034	1.7976	12
49	1.2036	1.7968	11
50	1.2039	1.7960	10
51	1.2041	1.7953	9
52	1.2043	1.7945	8
53	1.2046	1.7937	7
54	1.2048	1.7929	6
55	1.2050	1.7922	5
56	1.2053	1.7914	4
57	1.2055	1.7906	3
58	1.2057	1.7898	2
59	1.2060	1.7891	1
60	1.2062	1.7883	0
′	Csc	Sec	′

123° (303°) (236°) **56°**

34° (214°) (325°) **145°**

′	Sec	Csc	′
0	1.2062	1.7883	60
1	1.2065	1.7875	59
2	1.2067	1.7868	58
3	1.2069	1.7860	57
4	1.2072	1.7852	56
5	1.2074	1.7844	55
6	1.2076	1.7837	54
7	1.2079	1.7829	53
8	1.2081	1.7821	52
9	1.2084	1.7814	51
10	1.2086	1.7806	50
11	1.2088	1.7799	49
12	1.2091	1.7791	48
13	1.2093	1.7783	47
14	1.2096	1.7776	46
15	1.2098	1.7768	45
16	1.2100	1.7761	44
17	1.2103	1.7753	43
18	1.2105	1.7745	42
19	1.2108	1.7738	41
20	1.2110	1.7730	40
21	1.2112	1.7723	39
22	1.2115	1.7715	38
23	1.2117	1.7708	37
24	1.2120	1.7700	36
25	1.2122	1.7693	35
26	1.2124	1.7685	34
27	1.2127	1.7678	33
28	1.2129	1.7670	32
29	1.2132	1.7663	31
30	1.2134	1.7655	30
31	1.2136	1.7648	29
32	1.2139	1.7640	28
33	1.2141	1.7633	27
34	1.2144	1.7625	26
35	1.2146	1.7618	25
36	1.2149	1.7610	24
37	1.2151	1.7603	23
38	1.2154	1.7596	22
39	1.2156	1.7588	21
40	1.2158	1.7581	20
41	1.2161	1.7573	19
42	1.2163	1.7566	18
43	1.2166	1.7559	17
44	1.2168	1.7551	16
45	1.2171	1.7544	15
46	1.2173	1.7537	14
47	1.2176	1.7529	13
48	1.2178	1.7522	12
49	1.2181	1.7515	11
50	1.2183	1.7507	10
51	1.2185	1.7500	9
52	1.2188	1.7493	8
53	1.2190	1.7485	7
54	1.2193	1.7478	6
55	1.2195	1.7471	5
56	1.2198	1.7463	4
57	1.2200	1.7456	3
58	1.2203	1.7449	2
59	1.2205	1.7442	1
60	1.2208	1.7434	0
′	Csc	Sec	′

124° (304°) (235°) **55°**

35° (215°) (324°) **144°**

′	Sec	Csc	′
0	1.2208	1.7434	60
1	1.2210	1.7427	59
2	1.2213	1.7420	58
3	1.2215	1.7413	57
4	1.2218	1.7406	56
5	1.2220	1.7398	55
6	1.2223	1.7391	54
7	1.2225	1.7384	53
8	1.2228	1.7377	52
9	1.2230	1.7370	51
10	1.2233	1.7362	50
11	1.2235	1.7355	49
12	1.2238	1.7348	48
13	1.2240	1.7341	47
14	1.2243	1.7334	46
15	1.2245	1.7327	45
16	1.2248	1.7320	44
17	1.2250	1.7312	43
18	1.2253	1.7305	42
19	1.2255	1.7298	41
20	1.2258	1.7291	40
21	1.2260	1.7284	39
22	1.2263	1.7277	38
23	1.2265	1.7270	37
24	1.2268	1.7263	36
25	1.2271	1.7256	35
26	1.2273	1.7249	34
27	1.2276	1.7242	33
28	1.2278	1.7235	32
29	1.2281	1.7228	31
30	1.2283	1.7221	30
31	1.2286	1.7213	29
32	1.2288	1.7206	28
33	1.2291	1.7199	27
34	1.2293	1.7192	26
35	1.2296	1.7185	25
36	1.2299	1.7179	24
37	1.2301	1.7172	23
38	1.2304	1.7165	22
39	1.2306	1.7158	21
40	1.2309	1.7151	20
41	1.2311	1.7144	19
42	1.2314	1.7137	18
43	1.2317	1.7130	17
44	1.2319	1.7123	16
45	1.2322	1.7116	15
46	1.2324	1.7109	14
47	1.2327	1.7102	13
48	1.2329	1.7095	12
49	1.2332	1.7088	11
50	1.2335	1.7081	10
51	1.2337	1.7075	9
52	1.2340	1.7068	8
53	1.2342	1.7061	7
54	1.2345	1.7054	6
55	1.2348	1.7047	5
56	1.2350	1.7040	4
57	1.2353	1.7033	3
58	1.2355	1.7027	2
59	1.2358	1.7020	1
60	1.2361	1.7013	0
′	Csc	Sec	′

125° (305°) (234°) **54°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

36° (216°) (323°) **143°**

′	Sec	Csc	′
0	1.2361	1.7013	60
1	1.2363	1.7006	59
2	1.2366	1.6999	58
3	1.2369	1.6993	57
4	1.2371	1.6986	56
5	1.2374	1.6979	55
6	1.2376	1.6972	54
7	1.2379	1.6966	53
8	1.2382	1.6959	52
9	1.2384	1.6952	51
10	1.2387	1.6945	50
11	1.2390	1.6939	49
12	1.2392	1.6932	48
13	1.2395	1.6925	47
14	1.2397	1.6918	46
15	1.2400	1.6912	45
16	1.2403	1.6905	44
17	1.2405	1.6898	43
18	1.2408	1.6892	42
19	1.2411	1.6885	41
20	1.2413	1.6878	40
21	1.2416	1.6871	39
22	1.2419	1.6865	38
23	1.2421	1.6858	37
24	1.2424	1.6852	36
25	1.2427	1.6845	35
26	1.2429	1.6838	34
27	1.2432	1.6832	33
28	1.2435	1.6825	32
29	1.2437	1.6818	31
30	1.2440	1.6812	30
31	1.2443	1.6805	29
32	1.2445	1.6799	28
33	1.2448	1.6792	27
34	1.2451	1.6785	26
35	1.2453	1.6779	25
36	1.2456	1.6772	24
37	1.2459	1.6766	23
38	1.2462	1.6759	22
39	1.2464	1.6753	21
40	1.2467	1.6746	20
41	1.2470	1.6739	19
42	1.2472	1.6733	18
43	1.2475	1.6726	17
44	1.2478	1.6720	16
45	1.2480	1.6713	15
46	1.2483	1.6707	14
47	1.2486	1.6700	13
48	1.2489	1.6694	12
49	1.2491	1.6687	11
50	1.2494	1.6681	10
51	1.2497	1.6674	9
52	1.2499	1.6668	8
53	1.2502	1.6661	7
54	1.2505	1.6655	6
55	1.2508	1.6649	5
56	1.2510	1.6642	4
57	1.2513	1.6636	3
58	1.2516	1.6629	2
59	1.2519	1.6623	1
60	1.2521	1.6616	0
′	Csc	Sec	′

126° (306°) (233°) **53°**

37° (217°) (322°) **142°**

′	Sec	Csc	′
0	1.2521	1.6616	60
1	1.2524	1.6610	59
2	1.2527	1.6604	58
3	1.2530	1.6597	57
4	1.2532	1.6591	56
5	1.2535	1.6584	55
6	1.2538	1.6578	54
7	1.2541	1.6572	53
8	1.2543	1.6565	52
9	1.2546	1.6559	51
10	1.2549	1.6553	50
11	1.2552	1.6546	49
12	1.2554	1.6540	48
13	1.2557	1.6534	47
14	1.2560	1.6527	46
15	1.2563	1.6521	45
16	1.2566	1.6515	44
17	1.2568	1.6508	43
18	1.2571	1.6502	42
19	1.2574	1.6496	41
20	1.2577	1.6489	40
21	1.2579	1.6483	39
22	1.2582	1.6477	38
23	1.2585	1.6471	37
24	1.2588	1.6464	36
25	1.2591	1.6458	35
26	1.2593	1.6452	34
27	1.2596	1.6446	33
28	1.2599	1.6439	32
29	1.2602	1.6433	31
30	1.2605	1.6427	30
31	1.2608	1.6421	29
32	1.2610	1.6414	28
33	1.2613	1.6408	27
34	1.2616	1.6402	26
35	1.2619	1.6396	25
36	1.2622	1.6390	24
37	1.2624	1.6383	23
38	1.2627	1.6377	22
39	1.2630	1.6371	21
40	1.2633	1.6365	20
41	1.2636	1.6359	19
42	1.2639	1.6353	18
43	1.2641	1.6346	17
44	1.2644	1.6340	16
45	1.2647	1.6334	15
46	1.2650	1.6328	14
47	1.2653	1.6322	13
48	1.2656	1.6316	12
49	1.2659	1.6310	11
50	1.2661	1.6303	10
51	1.2664	1.6297	9
52	1.2667	1.6291	8
53	1.2670	1.6285	7
54	1.2673	1.6279	6
55	1.2676	1.6273	5
56	1.2679	1.6267	4
57	1.2682	1.6261	3
58	1.2684	1.6255	2
59	1.2687	1.6249	1
60	1.2690	1.6243	0
′	Csc	Sec	′

127° (307°) (232°) **52°**

38° (218°) (321°) **141°**

′	Sec	Csc	′
0	1.2690	1.6243	60
1	1.2693	1.6237	59
2	1.2696	1.6231	58
3	1.2699	1.6225	57
4	1.2702	1.6219	56
5	1.2705	1.6213	55
6	1.2708	1.6207	54
7	1.2710	1.6201	53
8	1.2713	1.6195	52
9	1.2716	1.6189	51
10	1.2719	1.6183	50
11	1.2722	1.6177	49
12	1.2725	1.6171	48
13	1.2728	1.6165	47
14	1.2731	1.6159	46
15	1.2734	1.6153	45
16	1.2737	1.6147	44
17	1.2740	1.6141	43
18	1.2742	1.6135	42
19	1.2745	1.6129	41
20	1.2748	1.6123	40
21	1.2751	1.6117	39
22	1.2754	1.6111	38
23	1.2757	1.6105	37
24	1.2760	1.6099	36
25	1.2763	1.6093	35
26	1.2766	1.6087	34
27	1.2769	1.6082	33
28	1.2772	1.6076	32
29	1.2775	1.6070	31
30	1.2778	1.6064	30
31	1.2781	1.6058	29
32	1.2784	1.6052	28
33	1.2787	1.6046	27
34	1.2790	1.6040	26
35	1.2793	1.6035	25
36	1.2796	1.6029	24
37	1.2799	1.6023	23
38	1.2802	1.6017	22
39	1.2804	1.6011	21
40	1.2807	1.6005	20
41	1.2810	1.6000	19
42	1.2813	1.5994	18
43	1.2816	1.5988	17
44	1.2819	1.5982	16
45	1.2822	1.5976	15
46	1.2825	1.5971	14
47	1.2828	1.5965	13
48	1.2831	1.5959	12
49	1.2834	1.5953	11
50	1.2837	1.5948	10
51	1.2840	1.5942	9
52	1.2843	1.5936	8
53	1.2846	1.5930	7
54	1.2849	1.5925	6
55	1.2852	1.5919	5
56	1.2855	1.5913	4
57	1.2859	1.5907	3
58	1.2862	1.5902	2
59	1.2865	1.5896	1
60	1.2868	1.5890	0
′	Csc	Sec	′

128° (308°) (231°) **51°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

39° (219°) (320°) **140°**

′	Sec	Csc	′
0	1.2868	1.5890	60
1	1.2871	1.5884	59
2	1.2874	1.5879	58
3	1.2877	1.5873	57
4	1.2880	1.5867	56
5	1.2883	1.5862	55
6	1.2886	1.5856	54
7	1.2889	1.5850	53
8	1.2892	1.5845	52
9	1.2895	1.5839	51
10	1.2898	1.5833	50
11	1.2901	1.5828	49
12	1.2904	1.5822	48
13	1.2907	1.5816	47
14	1.2910	1.5811	46
15	1.2913	1.5805	45
16	1.2916	1.5800	44
17	1.2919	1.5794	43
18	1.2923	1.5788	42
19	1.2926	1.5783	41
20	1.2929	1.5777	40
21	1.2932	1.5771	39
22	1.2935	1.5766	38
23	1.2938	1.5760	37
24	1.2941	1.5755	36
25	1.2944	1.5749	35
26	1.2947	1.5744	34
27	1.2950	1.5738	33
28	1.2953	1.5732	32
29	1.2957	1.5727	31
30	1.2960	1.5721	30
31	1.2963	1.5716	29
32	1.2966	1.5710	28
33	1.2969	1.5705	27
34	1.2972	1.5699	26
35	1.2975	1.5694	25
36	1.2978	1.5688	24
37	1.2981	1.5683	23
38	1.2985	1.5677	22
39	1.2988	1.5672	21
40	1.2991	1.5666	20
41	1.2994	1.5661	19
42	1.2997	1.5655	18
43	1.3000	1.5650	17
44	1.3003	1.5644	16
45	1.3307	1.5639	15
46	1.3010	1.5633	14
47	1.3013	1.5628	13
48	1.3016	1.5622	12
49	1.3019	1.5617	11
50	1.3022	1.5611	10
51	1.3026	1.5606	9
52	1.3029	1.5601	8
53	1.3032	1.5595	7
54	1.3035	1.5590	6
55	1.3038	1.5584	5
56	1.3041	1.5579	4
57	1.3045	1.5573	3
58	1.3048	1.5568	2
59	1.3051	1.5563	1
60	1.3054	1.5557	0
′	Csc	Sec	′

129° (309°) (230°) **50°**

40° (220°) (319°) **139°**

′	Sec	Csc	′
0	1.3054	1.5557	60
1	1.3057	1.5552	59
2	1.3060	1.5546	58
3	1.3064	1.5541	57
4	1.3067	1.5536	56
5	1.3070	1.5530	55
6	1.3073	1.5525	54
7	1.3076	1.5520	53
8	1.3080	1.5514	52
9	1.3083	1.5509	51
10	1.3086	1.5504	50
11	1.3089	1.5498	49
12	1.3093	1.5493	48
13	1.3096	1.5488	47
14	1.3099	1.5482	46
15	1.3102	1.5477	45
16	1.3105	1.5472	44
17	1.3109	1.5466	43
18	1.3112	1.5461	42
19	1.3115	1.5456	41
20	1.3118	1.5450	40
21	1.3122	1.5445	39
22	1.3125	1.5440	38
23	1.3128	1.5435	37
24	1.3131	1.5429	36
25	1.3135	1.5424	35
26	1.3138	1.5419	34
27	1.3141	1.5413	33
28	1.3144	1.5408	32
29	1.3148	1.5403	31
30	1.3151	1.5398	30
31	1.3154	1.5392	29
32	1.3157	1.5387	28
33	1.3161	1.5382	27
34	1.3164	1.5377	26
35	1.3167	1.5372	25
36	1.3171	1.5366	24
37	1.3174	1.5361	23
38	1.3177	1.5356	22
39	1.3180	1.5351	21
40	1.3184	1.5345	20
41	1.3187	1.5340	19
42	1.3190	1.5335	18
43	1.3194	1.5330	17
44	1.3197	1.5325	16
45	1.3200	1.5320	15
46	1.3203	1.5314	14
47	1.3207	1.5309	13
48	1.3210	1.5304	12
49	1.3213	1.5299	11
50	1.3217	1.5294	10
51	1.3220	1.5289	9
52	1.3223	1.5283	8
53	1.3227	1.5278	7
54	1.3230	1.5273	6
55	1.3233	1.5268	5
56	1.3237	1.5263	4
57	1.3240	1.5258	3
58	1.3243	1.5253	2
59	1.3247	1.5248	1
60	1.3250	1.5243	0
′	Csc	Sec	′

130° (310°) (229°) **49°**

41° (221°) (318°) **138°**

′	Sec	Csc	′
0	1.3250	1.5243	60
1	1.3253	1.5237	59
2	1.3257	1.5232	58
3	1.3260	1.5227	57
4	1.3264	1.5222	56
5	1.3267	1.5217	55
6	1.3270	1.5212	54
7	1.3274	1.5207	53
8	1.3277	1.5202	52
9	1.3280	1.5197	51
10	1.3284	1.5192	50
11	1.3287	1.5187	49
12	1.3291	1.5182	48
13	1.3294	1.5177	47
14	1.3297	1.5172	46
15	1.3301	1.5167	45
16	1.3304	1.5162	44
17	1.3307	1.5156	43
18	1.3311	1.5151	42
19	1.3314	1.5146	41
20	1.3318	1.5141	40
21	1.3321	1.5136	39
22	1.3325	1.5131	38
23	1.3328	1.5126	37
24	1.3331	1.5121	36
25	1.3335	1.5116	35
26	1.3338	1.5111	34
27	1.3342	1.5107	33
28	1.3345	1.5102	32
29	1.3348	1.5097	31
30	1.3352	1.5092	30
31	1.3355	1.5087	29
32	1.3359	1.5082	28
33	1.3362	1.5077	27
34	1.3366	1.5072	26
35	1.3369	1.5067	25
36	1.3373	1.5062	24
37	1.3376	1.5057	23
38	1.3380	1.5052	22
39	1.3383	1.5047	21
40	1.3386	1.5042	20
41	1.3390	1.5037	19
42	1.3393	1.5032	18
43	1.3397	1.5027	17
44	1.3400	1.5023	16
45	1.3404	1.5018	15
46	1.3407	1.5013	14
47	1.3411	1.5008	13
48	1.3414	1.5003	12
49	1.3418	1.4998	11
50	1.3421	1.4993	10
51	1.3425	1.4988	9
52	1.3428	1.4984	8
53	1.3432	1.4979	7
54	1.3435	1.4974	6
55	1.3439	1.4969	5
56	1.3442	1.4964	4
57	1.3446	1.4959	3
58	1.3449	1.4954	2
59	1.3453	1.4950	1
60	1.3456	1.4945	0
′	Csc	Sec	′

131° (311°) (228°) **48°**

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

42° (222°) (317°) **137°**

′	Sec	Csc	′
0	1.3456	1.4945	60
1	1.3460	1.4940	59
2	1.3463	1.4935	58
3	1.3467	1.4930	57
4	1.3470	1.4925	56
5	1.3474	1.4921	55
6	1.3478	1.4916	54
7	1.3481	1.4911	53
8	1.3485	1.4906	52
9	1.3488	1.4901	51
10	1.3492	1.4897	50
11	1.3495	1.4892	49
12	1.3499	1.4887	48
13	1.3502	1.4882	47
14	1.3506	1.4878	46
15	1.3510	1.4873	45
16	1.3513	1.4868	44
17	1.3517	1.4863	43
18	1.3520	1.4859	42
19	1.3524	1.4854	41
20	1.3527	1.4849	40
21	1.3531	1.4844	39
22	1.3535	1.4840	38
23	1.3538	1.4835	37
24	1.3542	1.4830	36
25	1.3545	1.4825	35
26	1.3549	1.4821	34
27	1.3553	1.4816	33
28	1.3556	1.4811	32
29	1.3560	1.4807	31
30	1.3563	1.4802	30
31	1.3567	1.4797	29
32	1.3571	1.4792	28
33	1.3574	1.4788	27
34	1.3578	1.4783	26
35	1.3582	1.4778	25
36	1.3585	1.4774	24
37	1.3589	1.4769	23
38	1.3592	1.4764	22
39	1.3596	1.4760	21
40	1.3600	1.4755	20
41	1.3603	1.4750	19
42	1.3607	1.4746	18
43	1.3611	1.4741	17
44	1.3614	1.4737	16
45	1.3618	1.4732	15
46	1.3622	1.4727	14
47	1.3625	1.4723	13
48	1.3629	1.4718	12
49	1.3633	1.4713	11
50	1.3636	1.4709	10
51	1.3640	1.4704	9
52	1.3644	1.4700	8
53	1.3647	1.4695	7
54	1.3651	1.4690	6
55	1.3655	1.4686	5
56	1.3658	1.4681	4
57	1.3662	1.4677	3
58	1.3666	1.4672	2
59	1.3670	1.4667	1
60	1.3673	1.4663	0
′	Csc	Sec	′

132° (312°) (227°) **47°**

43° (223°) (316°) **136°**

′	Sec	Csc	′
0	1.3673	1.4663	60
1	1.3677	1.4658	59
2	1.3681	1.4654	58
3	1.3684	1.4649	57
4	1.3688	1.4645	56
5	1.3692	1.4640	55
6	1.3696	1.4635	54
7	1.3699	1.4631	53
8	1.3703	1.4626	52
9	1.3707	1.4622	51
10	1.3711	1.4617	50
11	1.3714	1.4613	49
12	1.3718	1.4608	48
13	1.3722	1.4604	47
14	1.3726	1.4599	46
15	1.3729	1.4595	45
16	1.3733	1.4590	44
17	1.3737	1.4586	43
18	1.3741	1.4581	42
19	1.3744	1.4577	41
20	1.3748	1.4572	40
21	1.3752	1.4568	39
22	1.3756	1.4563	38
23	1.3759	1.4559	37
24	1.3763	1.4554	36
25	1.3767	1.4550	35
26	1.3771	1.4545	34
27	1.3775	1.4541	33
28	1.3778	1.4536	32
29	1.3782	1.4532	31
30	1.3786	1.4527	30
31	1.3790	1.4523	29
32	1.3794	1.4518	28
33	1.3797	1.4514	27
34	1.3801	1.4510	26
35	1.3805	1.4505	25
36	1.3809	1.4501	24
37	1.3813	1.4496	23
38	1.3817	1.4492	22
39	1.3820	1.4487	21
40	1.3824	1.4483	20
41	1.3828	1.4479	19
42	1.3832	1.4474	18
43	1.3836	1.4470	17
44	1.3840	1.4465	16
45	1.3843	1.4461	15
46	1.3847	1.4457	14
47	1.3851	1.4452	13
48	1.3855	1.4448	12
49	1.3859	1.4443	11
50	1.3863	1.4439	10
51	1.3867	1.4435	9
52	1.3871	1.4430	8
53	1.3874	1.4426	7
54	1.3878	1.4422	6
55	1.3882	1.4417	5
56	1.3886	1.4413	4
57	1.3890	1.4409	3
58	1.3894	1.4404	2
59	1.3898	1.4400	1
60	1.3902	1.4396	0
′	Csc	Sec	′

133° (313°) (226°) **46°**

44° (224°) (315°) **135°**

′	Sec	Csc	′
0	1.3902	1.4396	60
1	1.3906	1.4391	59
2	1.3909	1.4387	58
3	1.3913	1.4383	57
4	1.3917	1.4378	56
5	1.3921	1.4374	55
6	1.3925	1.4370	54
7	1.3929	1.4365	53
8	1.3933	1.4361	52
9	1.3937	1.4357	51
10	1.3941	1.4352	50
11	1.3945	1.4348	49
12	1.3949	1.4344	48
13	1.3953	1.4340	47
14	1.3957	1.4335	46
15	1.3961	1.4331	45
16	1.3965	1.4327	44
17	1.3969	1.4322	43
18	1.3972	1.4318	42
19	1.3976	1.4314	41
20	1.3980	1.4310	40
21	1.3984	1.4305	39
22	1.3988	1.4301	38
23	1.3992	1.4297	37
24	1.3996	1.4293	36
25	1.4000	1.4288	35
26	1.4004	1.4284	34
27	1.4008	1.4280	33
28	1.4012	1.4276	32
29	1.4016	1.4271	31
30	1.4020	1.4267	30
31	1.4024	1.4263	29
32	1.4028	1.4259	28
33	1.4032	1.4255	27
34	1.4036	1.4250	26
35	1.4040	1.4246	25
36	1.4044	1.4242	24
37	1.4048	1.4238	23
38	1.4052	1.4234	22
39	1.4057	1.4229	21
40	1.4061	1.4225	20
41	1.4065	1.4221	19
42	1.4069	1.4217	18
43	1.4073	1.4213	17
44	1.4077	1.4208	16
45	1.4081	1.4204	15
46	1.4085	1.4200	14
47	1.4089	1.4196	13
48	1.4093	1.4192	12
49	1.4097	1.4188	11
50	1.4101	1.4183	10
51	1.4105	1.4179	9
52	1.4109	1.4175	8
53	1.4113	1.4171	7
54	1.4118	1.4167	6
55	1.4122	1.4163	5
56	1.4126	1.4159	4
57	1.4130	1.4154	3
58	1.4134	1.4150	2
59	1.4138	1.4146	1
60	1.4142	1.4142	0
′	Csc	Sec	′

134° (314°) (225°) **45°**

6.6 INVERSES OF THE TRIGONOMETRIC FUNCTIONS

Each of the trigonometric functions has the property that to each value of the function there corresponds more than one argument. Their inverses are thus not functions. However, if the domains of the trigonometric functions are restricted in such a way that to each value there is exactly one argument, then the functions defined on these restricted domains have inverses which are functions. In the definitions of the inverse trigonometric functions which follow, the restrictions of domain and range are noted.

6.7 DEFINITION

(1) $\arcsin x = \sin^{-1}(x)$. *Domain:* $[-1, 1]$. *Range:* $[-\pi/2, \pi/2]$.

(2) $\arccos x = \cos^{-1}(x)$. *Domain:* $[-1, 1]$. *Range:* $[0, \pi]$.

(3) $\arctan x = \tan^{-1}(x)$. *Domain:* $(-\infty, \infty)$. *Range:* $(-\pi/2, \pi/2)$.

(4) $\operatorname{arccot} x = \cot^{-1}(x)$. *Domain:* $(-\infty, \infty)$. *Range:* $(0, \pi)$.

(5) $\operatorname{arcsec} x = \sec^{-1}(x)$. *Domain:* $(-\infty, -1], [1, \infty)$. *Range:* $[0, \pi/2), [\pi, 3\pi/2)$.

(6) $\operatorname{arccsc} x = \csc^{-1}(x)$. *Domain:* $(-\infty, -1], [1, \infty)$. *Range:* $(0, \pi/2], (\pi, 3\pi/2]$.

6.8 THE LOGARITHMIC AND THE EXPONENTIAL FUNCTIONS

The natural logarithm function is defined by a differential equation with a boundary condition. If a is a positive number different from 1, then we may define a logarithmic function to the base a in terms of the natural logarithmic function. Since each logarithmic function has the property that for each value there is exactly one argument, each has an inverse which is a function. The exponential functions are the inverses of the logarithmic functions.

6.9 DEFINITION

The function **f** which satisfies the two equations

$$\mathbf{Df}(x) = \frac{1}{x} \quad \text{and} \quad \mathbf{f}(1) = 0$$

is called the **natural logarithm function** and is denoted by **ln.**

6.10 DEFINITION

The number x for which $\mathbf{ln}\ x = 1$ is called the **base of natural logarithms** and is denoted by e. The number $e = \lim_{n \to 0} (1 + n)^{1/n} =$ 2.7182818 to seven decimals.

6.11 DEFINITION

If $a > 0$ and $a \neq 1$, the **logarithm function to the base** a is defined by

$$\log_a x = \frac{\ln x}{\ln a}.$$

6.12 PROPERTIES OF LOGARITHMIC FUNCTIONS

(1) $\qquad \ln e = 1.$

(2) $\qquad \log_a a = 1.$

(3) $\qquad \ln x = \log_e x.$

$$\log_a x = \frac{\ln x}{\ln a}. \tag{4}$$

$$\log_a b \log_b a = 1. \tag{5}$$

$$\log_a 1 = 0. \tag{6}$$

$$\log_a x_1 x_2 = \log_a x_1 + \log_a x_2. \tag{7}$$

$$\log_a \frac{x_1}{x_2} = \log_a x_1 - \log_a x_2. \tag{8}$$

$$\log_a x_1^{x_2} = x_2 \log_a x_1. \tag{9}$$

6.13 TABLE OF NATURAL LOGARITHMS

Table 6.13

NATURAL (NAPIERIAN) LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0 0000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0672	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2.8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
N	0	1	2	3	4	5	6	7	8	9

Table 6.13 (Cont'd)

NATURAL (NAPIERIAN) LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
6.0	1.7 9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

6.14 TABLE OF COMMON LOGARITHMS ($\log_{10} x$)

Table 6.14

COMMON LOGARITHMS OF NUMBERS

100 — 150

N.		0	1	2	3	4	5	6	7	8	9
100	00	000	043	087	130	173	217	260	303	346	389
101		432	475	518	561	604	647	689	732	775	817
102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	407	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
109		743	782	822	862	902	941	981	*021	*060	*100
110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	156	189	222	254	287	320	352
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		435	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
150		609	638	667	696	725	754	782	811	840	869
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts			
	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4
	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7
	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Proportional parts

.00 000 — .17 869

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

150 — 200

N.		0	1	2	3	4	5	6	7	8	9
150	17	609	638	667	696	725	754	782	811	840	869
151		898	926	955	984	*013	*041	*070	*099	*127	*156
152	18	184	213	241	270	298	327	355	384	412	441
153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	167	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	779
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
177		797	822	846	871	895	920	944	969	993	*018
178	25	042	066	091	115	139	164	188	212	237	261
179		285	310	334	358	382	406	431	455	479	503
180		527	551	575	600	624	648	672	696	720	744
181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
183		245	269	293	316	340	364	387	411	435	458
184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	852
190		875	898	921	944	967	989	*012	*035	*058	*081
191	28	103	126	149	171	194	217	240	262	285	307
192		330	353	375	398	421	443	466	488	511	533
193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

Proportional parts

.17 609 — .30 298

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

200 — 250

N.		0	1	2	3	4	5	6	7	8	9
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	383	401	420	438	457
237		475	493	511	530	548	566	585	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777
250		794	811	829	846	863	881	898	915	933	950
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

Proportional parts

.30 103 — .39 950

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

250 — 300

N.		0	1	2	3	4	5	6	7	8	9
250	39	794	811	829	846	863	881	898	915	933	950
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40	140	157	175	192	209	226	243	261	278	295
253		312	329	346	364	381	398	415	432	449	466
254		483	500	518	535	552	569	586	603	620	637
255		654	671	688	705	722	739	756	773	790	807
256		824	841	858	875	892	909	926	943	960	976
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41	162	179	196	212	229	246	263	280	296	313
259		330	347	363	380	397	414	430	447	464	481
260		497	514	531	547	564	581	597	614	631	647
261		664	681	697	714	731	747	764	780	797	814
262		830	847	863	880	896	913	929	946	963	979
263		996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42	160	177	193	210	226	243	259	275	292	308
265		325	341	357	374	390	406	423	439	455	472
266		488	504	521	537	553	570	586	602	619	635
267		651	667	684	700	716	732	749	765	781	797
268		813	830	846	862	878	894	911	927	943	959
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43	136	152	169	185	201	217	233	249	265	281
271		297	313	329	345	361	377	393	409	425	441
272		457	473	489	505	521	537	553	569	584	600
273		616	632	648	664	680	696	712	727	743	759
274		775	791	807	823	838	854	870	886	902	917
275		933	949	965	981	996	*012	*028	*044	*059	*075
276	44	091	107	122	138	154	170	185	201	217	232
277		248	264	279	295	311	326	342	358	373	389
278		404	420	436	451	467	483	498	514	529	545
279		560	576	592	607	623	638	654	669	685	700
280		716	731	747	762	778	793	809	824	840	855
281		871	886	902	917	932	948	963	979	994	*010
282	45	025	040	056	071	086	102	117	133	148	163
283		179	194	209	225	240	255	271	286	301	317
284		332	347	362	378	393	408	423	439	454	469
285		484	500	515	530	545	561	576	591	606	621
286		637	652	667	682	697	712	728	743	758	773
287		788	803	818	834	849	864	879	894	909	924
288		939	954	969	984	*000	*015	*030	*045	*060	*075
289	46	090	105	120	135	150	165	180	195	210	225
290		240	255	270	285	300	315	330	345	359	374
291		389	404	419	434	449	464	479	494	509	523
292		538	553	568	583	598	613	627	642	657	672
293		687	702	716	731	746	761	776	790	805	820
294		835	850	864	879	894	909	923	938	953	967
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47	129	144	159	173	188	202	217	232	246	261
297		276	290	305	319	334	349	363	378	392	407
298		422	436	451	465	480	494	509	524	538	553
299		567	582	596	611	625	640	654	669	683	698
300		712	727	741	756	770	784	799	813	828	842
N		0	1	2	3	4	5	6	7	8	9

Proportional parts

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

$\log e = 0.43429$

Proportional parts

.39 794 — .47 842

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

300 — 350

N.		0	1	2	3	4	5	6	7	8	9
300	47	712	727	741	756	770	784	799	813	828	842
301		857	871	885	900	914	929	943	958	972	986
302	48	001	015	029	044	058	073	087	101	116	130
303		144	159	173	187	202	216	230	244	259	273
304		287	302	316	330	344	359	373	387	401	416
305		430	444	458	473	487	501	515	530	544	558
306		572	586	601	615	629	643	657	671	686	700
307		714	728	742	756	770	785	799	813	827	841
308		855	869	883	897	911	926	940	954	968	982
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49	136	150	164	178	192	206	220	234	248	262
311		276	290	304	318	332	346	360	374	388	402
312		415	429	443	457	471	485	499	513	527	541
313		554	568	582	596	610	624	638	651	665	679
314		693	707	721	734	748	762	776	790	803	817
315		831	845	859	872	886	900	914	927	941	955
316		969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50	106	120	133	147	161	174	188	202	215	229
318		243	256	270	284	297	311	325	338	352	365
319		379	393	406	420	433	447	461	474	488	501
320		515	529	542	556	569	583	596	610	623	637
321		651	664	678	691	705	718	732	745	759	772
322		786	799	813	826	840	853	866	880	893	907
323		920	934	947	961	974	987	*001	*014	*028	*041
324	51	055	068	081	095	108	121	135	148	162	175
325		188	202	215	228	242	255	268	282	295	308
326		322	335	348	362	375	388	402	415	428	441
327		455	468	481	495	508	521	534	548	561	574
328		587	601	614	627	640	654	667	680	693	706
329		720	733	746	759	772	786	799	812	825	838
330		851	865	878	891	904	917	930	943	957	970
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52	114	127	140	153	166	179	192	205	218	231
333		244	257	270	284	297	310	323	336	349	362
334		375	388	401	414	427	440	453	466	479	492
335		504	517	530	543	556	569	582	595	608	621
336		634	647	660	673	686	699	711	724	737	750
337		763	776	789	802	815	827	840	853	866	879
338		892	905	917	930	943	956	969	982	994	*007
339	53	020	033	046	058	071	084	097	110	122	135
340		148	161	173	186	199	212	224	237	250	263
341		275	288	301	314	326	339	352	364	377	390
342		403	415	428	441	453	466	479	491	504	517
343		529	542	555	567	580	593	605	618	631	643
344		656	668	681	694	706	719	732	744	757	769
345		782	794	807	820	832	845	857	870	882	895
346		908	920	933	945	958	970	983	995	*008	*020
347	54	033	045	058	070	083	095	108	120	133	145
348		158	170	183	195	208	220	233	245	258	270
349		283	295	307	320	332	345	357	370	382	394
350	—	407	419	432	444	456	469	481	494	506	518
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

$\log \pi = 0.49715$

Proportional parts

.47 712 — .54 518

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

350 — 400

N.		0	1	2	3	4	5	6	7	8	9
350	54	407	419	432	444	456	469	481	494	506	518
351		531	543	555	568	580	593	605	617	630	642
352		654	667	679	691	704	716	728	741	753	765
353		777	790	802	814	827	839	851	864	876	888
354		900	913	925	937	949	962	974	986	998	*011
355	55	023	035	047	060	072	084	096	108	121	133
356		145	157	169	182	194	206	218	230	242	255
357		267	279	291	303	315	328	340	352	364	376
358		388	400	413	425	437	449	461	473	485	497
359		509	522	534	546	558	570	582	594	606	618
360		630	642	654	666	678	691	703	715	727	739
361		751	763	775	787	799	811	823	835	847	859
362		871	883	895	907	919	931	943	955	967	979
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56	110	122	134	146	158	170	182	194	205	217
365		229	241	253	265	277	289	301	312	324	336
366		348	360	372	384	396	407	419	431	443	455
367		467	478	490	502	514	526	538	549	561	573
368		585	597	608	620	632	644	656	667	679	691
369		703	714	726	738	750	761	773	785	797	808
370		820	832	844	855	867	879	891	902	914	926
371		937	949	961	972	984	996	*008	*019	*031	*043
372	57	054	066	078	089	101	113	124	136	148	159
373		171	183	194	206	217	229	241	252	264	276
374		287	299	310	322	334	345	357	368	380	392
375		403	415	426	438	449	461	473	484	496	507
376		519	530	542	553	565	576	588	600	611	623
377		634	646	657	669	680	692	703	715	726	738
378		749	761	772	784	795	807	818	830	841	852
379		864	875	887	898	910	921	933	944	955	967
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58	092	104	115	127	138	149	161	172	184	195
382		206	218	229	240	252	263	274	286	297	309
383		320	331	343	354	365	377	388	399	410	422
384		433	444	456	467	478	490	501	512	524	535
385		546	557	569	580	591	602	614	625	636	647
386		659	670	681	692	704	715	726	737	749	760
387		771	782	794	805	816	827	838	850	861	872
388		883	894	906	917	928	939	950	961	973	984
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59	106	118	129	140	151	162	173	184	195	207
391		218	229	240	251	262	273	284	295	306	318
392		329	340	351	362	373	384	395	406	417	428
393		439	450	461	472	483	494	506	517	528	539
394		550	561	572	583	594	605	616	627	638	649
395		660	671	682	693	704	715	726	737	748	759
396		770	780	791	802	813	824	835	846	857	868
397		879	890	901	912	923	934	945	956	966	977
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60	097	108	119	130	141	152	163	173	184	195
400		206	217	228	239	249	260	271	282	293	304
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts										
	13									
1	1.3									
2	2.6									
3	3.9									
4	5.2									
5	6.5									
6	7.8									
7	9.1									
8	10.4									
9	11.7									
	12									
1	1.2									
2	2.4									
3	3.6									
4	4.8									
5	6.0									
6	7.2									
7	8.4									
8	9.6									
9	10.8									
	11									
1	1.1									
2	2.2									
3	3.3									
4	4.4									
5	5.5									
6	6.6									
7	7.7									
8	8.8									
9	9.9									
	10									
1	1.0									
2	2.0									
3	3.0									
4	4.0									
5	5.0									
6	6.0									
7	7.0									
8	8.0									
9	9.0									
Proportional parts										

.54 407 — .60 304

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

400 — 450

N.		0	1	2	3	4	5	6	7	8	9
400	60	206	217	228	239	249	260	271	282	293	304
401		314	325	336	347	358	369	379	390	401	412
402		423	433	444	455	466	477	487	498	509	520
403		531	541	552	563	574	584	595	606	617	627
404		638	649	660	670	681	692	703	713	724	735
405		746	756	767	778	788	799	810	821	831	842
406		853	863	874	885	895	906	917	927	938	949
407		959	970	981	991	*002	*013	*023	*034	*045	*055
408	61	066	077	087	098	109	119	130	140	151	162
409		172	183	194	204	215	225	236	247	257	268
410		278	289	300	310	321	331	342	352	363	374
411		384	395	405	416	426	437	448	458	469	479
412		490	500	511	521	532	542	553	563	574	584
413		595	606	616	627	637	648	658	669	679	690
414		700	711	721	731	742	752	763	773	784	794
415		805	815	826	836	847	857	868	878	888	899
416		909	920	930	941	951	962	972	982	993	*003
417	62	014	024	034	045	055	066	076	086	097	107
418		118	128	138	149	159	170	180	190	201	211
419		221	232	242	252	263	273	284	294	304	315
420		325	335	346	356	366	377	387	397	408	418
421		428	439	449	459	469	480	490	500	511	521
422		531	542	552	562	572	583	593	603	613	624
423		634	644	655	665	675	685	696	706	716	726
424		737	747	757	767	778	788	798	808	818	829
425		839	849	859	870	880	890	900	910	921	931
426		941	951	961	972	982	992	*002	*012	*022	*033
427	63	043	053	063	073	083	094	104	114	124	134
428		144	155	165	175	185	195	205	215	225	236
429		246	256	266	276	286	296	306	317	327	337
430		347	357	367	377	387	397	407	417	428	438
431		448	458	468	478	488	498	508	518	528	538
432		548	558	568	579	589	599	609	619	629	639
433		649	659	669	679	689	699	709	719	729	739
434		749	759	769	779	789	799	809	819	829	839
435		849	859	869	879	889	899	909	919	929	939
436		949	959	969	979	988	998	*008	*018	*028	*038
437	64	048	058	068	078	088	098	108	118	128	137
438		147	157	167	177	187	197	207	217	227	237
439		246	256	266	276	286	296	306	316	326	335
440		345	355	365	375	385	395	404	414	424	434
441		444	454	464	473	483	493	503	513	523	532
442		542	552	562	572	582	591	601	611	621	631
443		640	650	660	670	680	689	699	709	719	729
444		738	748	758	768	777	787	797	807	816	826
445		836	846	856	865	875	885	895	904	914	924
446		933	943	953	963	972	982	992	*002	*011	*021
447	65	031	040	050	060	070	079	089	099	108	118
448		128	137	147	157	167	176	186	196	205	215
449		225	234	244	254	263	273	283	292	302	312
450		321	331	341	350	360	369	379	389	398	408
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Proportional parts

.60 206 — .65 408

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

450 — 500

N.		0	1	2	3	4	5	6	7	8	9
450	65	321	331	341	350	360	369	379	389	398	408
451		418	427	437	447	456	466	475	485	495	504
452		514	523	533	543	552	562	571	581	591	600
453		610	619	629	639	648	658	667	677	686	696
454		706	715	725	734	744	753	763	772	782	792
455		801	811	820	830	839	849	858	868	877	887
456		896	906	916	925	935	944	954	963	973	982
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66	087	096	106	115	124	134	143	153	162	172
459		181	191	200	210	219	229	238	247	257	266
460		276	285	295	304	314	323	332	342	351	361
461		370	380	389	398	408	417	427	436	445	455
462		464	474	483	492	502	511	521	530	539	549
463		558	567	577	586	596	605	614	624	633	642
464		652	661	671	680	689	699	708	717	727	736
465		745	755	764	773	783	792	801	811	820	829
466		839	848	857	867	876	885	894	904	913	922
467		932	941	950	960	969	978	987	997	*006	*015
468	67	025	034	043	052	062	071	080	089	099	108
469		117	127	136	145	154	164	173	182	191	201
470		210	219	228	237	247	256	265	274	284	293
471		302	311	321	330	339	348	357	367	376	385
472		394	403	413	422	431	440	449	459	468	477
473		486	495	504	514	523	532	541	550	560	569
474		578	587	596	605	614	624	633	642	651	660
475		669	679	688	697	706	715	724	733	742	752
476		761	770	779	788	797	806	815	825	834	843
477		852	861	870	879	888	897	906	916	925	934
478		943	952	961	970	979	988	997	*006	*015	*024
479	68	034	043	052	061	070	079	088	097	106	115
480		124	133	142	151	160	169	178	187	196	205
481		215	224	233	242	251	260	269	278	287	296
482		305	314	323	332	341	350	359	368	377	386
483		395	404	413	422	431	440	449	458	467	476
484		485	494	502	511	520	529	538	547	556	565
485		574	583	592	601	610	619	728	637	646	655
486		664	673	681	690	699	708	717	726	735	744
487		753	762	771	780	789	797	806	815	824	833
488		842	851	860	869	878	886	895	904	913	922
489		931	940	949	958	966	975	984	993	*002	*011
490	69	020	028	037	046	055	064	073	082	090	099
491		108	117	126	135	144	152	161	170	179	188
492		197	205	214	223	232	241	249	258	267	276
493		285	294	302	311	320	329	338	346	355	364
494		373	381	390	399	408	417	425	434	443	452
495		461	469	478	487	496	504	513	522	531	539
496		548	557	566	574	583	592	601	609	618	627
497		636	644	653	662	671	679	688	697	705	714
498		723	732	740	749	758	767	775	784	793	801
499		810	819	827	836	845	854	862	871	880	888
500		897	906	914	923	932	940	949	958	966	975
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts	
	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

Proportional parts	
	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Proportional parts	
	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

.65 321 — .69 975

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

500 — 550

N.		0	1	2	3	4	5	6	7	8	9
500	69	897	906	914	923	932	940	949	958	966	975
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70	070	079	088	096	105	114	122	131	140	148
503		157	165	174	183	191	200	209	217	226	234
504		243	252	260	269	278	286	295	303	312	321
505		329	338	346	355	364	372	381	389	398	406
506		415	424	432	441	449	458	467	475	484	492
507		501	509	518	526	535	544	552	561	569	578
508		586	595	603	612	621	629	638	646	655	663
509		672	680	689	697	706	714	723	731	740	749
510		757	766	774	783	791	800	808	817	825	834
511		842	851	859	868	876	885	893	902	910	919
512		927	935	944	952	961	969	978	986	995	*003
513	71	012	020	029	037	046	054	063	071	079	088
514		096	105	113	122	130	139	147	155	164	172
515		181	189	198	206	214	223	231	240	248	257
516		265	273	282	290	299	307	315	324	332	341
517		349	357	366	374	383	391	399	408	416	425
518		433	441	450	458	466	475	483	492	500	508
519		517	525	533	542	550	559	567	575	584	592
520		600	609	617	625	634	642	650	659	667	675
521		684	692	700	709	717	725	734	742	750	759
522		767	775	784	792	800	809	817	825	834	842
523		850	858	867	875	883	892	900	908	917	925
524		933	941	950	958	966	975	983	991	999	*008
525	72	016	024	032	041	049	057	066	074	082	090
526		099	107	115	123	132	140	148	156	165	173
527		181	189	198	206	214	222	230	239	247	255
528		263	272	280	288	296	304	313	321	329	337
529		346	354	362	370	378	387	395	403	411	419
530		428	436	444	452	460	469	477	485	493	501
531		509	518	526	534	542	550	558	567	575	583
532		591	599	607	616	624	632	640	648	656	665
533		673	681	689	697	705	713	722	730	738	746
534		754	762	770	779	787	795	803	811	819	827
535		835	843	852	860	868	876	884	892	900	908
536		916	925	933	941	949	957	965	973	981	989
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73	078	086	094	102	111	119	127	135	143	151
539		159	167	175	183	191	199	207	215	223	231
540		239	247	255	263	272	280	288	296	304	312
541		320	328	336	344	352	360	368	376	384	392
542		400	408	416	424	432	440	448	456	464	472
543		480	488	496	504	512	520	528	536	544	552
544		560	568	576	584	592	600	608	616	624	632
545		640	648	656	664	672	679	687	695	703	711
546		719	727	735	743	751	759	767	775	783	791
547		799	807	815	823	830	838	846	854	862	870
548		878	886	894	902	910	918	926	933	941	949
549		957	965	973	981	989	997	*005	*013	*020	*028
550	74	036	044	052	060	068	076	084	092	099	107
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Proportional parts

.69 897 — .74 107

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

550 — 600

N.		0	1	2	3	4	5	6	7	8	9
550	74	036	044	052	060	068	076	084	092	099	107
551		115	123	131	139	147	155	162	170	178	186
552		194	202	210	218	225	233	241	249	257	265
553		273	280	288	296	304	312	320	327	335	343
554		351	359	367	374	382	390	398	406	414	421
555		429	437	445	453	461	468	476	484	492	500
556		507	515	523	531	539	547	554	562	570	578
557		586	593	601	609	617	624	632	640	648	656
558		663	671	679	687	695	702	710	718	726	733
559		741	749	757	764	772	780	788	796	803	811
560		819	827	834	842	850	858	865	873	881	889
561		896	904	912	920	927	935	943	950	958	966
562		974	981	989	997	*005	*012	*020	*028	*035	*043
563	75	051	059	066	074	082	089	097	105	113	120
564		128	136	143	151	159	166	174	182	189	197
565		205	213	220	228	236	243	251	259	266	274
566		282	289	297	305	312	320	328	335	343	351
567		358	366	374	381	389	397	404	412	420	427
568		435	442	450	458	465	473	481	488	496	504
569		511	519	526	534	542	549	557	565	572	580
570		587	595	603	610	618	626	633	641	648	656
571		664	671	679	686	694	702	709	717	724	732
572		740	747	755	762	770	778	785	793	800	808
573		815	823	831	838	846	853	861	868	876	884
574		891	899	906	914	921	929	937	944	952	959
575		967	974	982	989	997	*005	*012	*020	*027	*035
576	76	042	050	057	065	072	080	087	095	103	110
577		118	125	133	140	148	155	163	170	178	185
578		193	200	208	215	223	230	238	245	253	260
579		268	275	283	290	298	305	313	320	328	335
580		343	350	358	365	373	380	388	395	403	410
581		418	425	433	440	448	455	462	470	477	485
582		492	500	507	515	522	530	537	545	552	559
583		567	574	582	589	597	604	612	619	626	634
584		641	649	656	664	671	678	686	693	701	708
585		716	723	730	738	745	753	760	768	775	782
586		790	797	805	812	819	827	834	842	849	856
587		864	871	879	886	893	901	908	916	923	930
588		938	945	953	960	967	975	982	989	997	*004
589	77	012	019	026	034	041	048	056	063	070	078
590		085	093	100	107	115	122	129	137	144	151
591		159	166	173	181	188	195	203	210	217	225
592		232	240	247	254	262	269	276	283	291	298
593		305	313	320	327	335	342	349	357	364	371
594		379	386	393	401	408	415	422	430	437	444
595		452	459	466	474	481	488	495	503	510	517
596		525	532	539	546	554	561	568	576	583	590
597		597	605	612	619	627	634	641	648	656	663
598		670	677	685	692	699	706	714	721	728	735
599		743	750	757	764	772	779	786	793	801	808
600		815	822	830	837	844	851	859	866	873	880
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Proportional parts

.74 036 — .77 880

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

600 — 650

N.		0	1	2	3	4	5	6	7	8	9
600	77	815	822	830	837	844	851	859	866	873	880
601		887	895	902	909	916	924	931	938	945	952
602		960	967	974	981	988	996	*003	*010	*017	*025
603	78	032	039	046	053	061	068	075	082	089	097
604		104	111	118	125	132	140	147	154	161	168
605		176	183	190	197	204	211	219	226	233	240
606		247	254	262	269	276	283	290	297	305	312
607		319	326	333	340	347	355	362	369	376	383
608		390	398	405	412	419	426	433	440	447	455
609		462	469	476	483	490	497	504	512	519	526
610		533	540	547	554	561	569	576	583	590	597
611		604	611	618	625	633	640	647	654	661	668
612		675	682	689	696	704	711	718	725	732	739
613		746	753	760	767	774	781	789	796	803	810
614		817	824	831	838	845	852	859	866	873	880
615		888	895	902	909	916	923	930	937	944	951
616		958	965	972	979	986	993	*000	*007	*014	*021
617	79	029	036	043	050	057	064	071	078	085	092
618		099	106	113	120	127	134	141	148	155	162
619		169	176	183	190	197	204	211	218	225	232
620		239	246	253	260	267	274	281	288	295	302
621		309	316	323	330	337	344	351	358	365	372
622		379	386	393	400	407	414	421	428	435	442
623		449	456	463	470	477	484	491	498	505	511
624		518	525	532	539	546	553	560	567	574	581
625		588	595	602	609	616	623	630	637	644	650
626		657	664	671	678	685	692	699	706	713	720
627		727	734	741	748	754	761	768	775	782	789
628		796	803	810	817	824	831	837	844	851	858
629		865	872	879	886	893	900	906	913	920	927
630		934	941	948	955	962	969	975	982	989	996
631	80	003	010	017	024	030	037	044	051	058	065
632		072	079	085	092	099	106	113	120	127	134
633		140	147	154	161	168	175	182	188	195	202
634		209	216	223	229	236	243	250	257	264	271
635		277	284	291	298	305	312	318	325	332	339
636		346	353	359	366	373	380	387	393	400	407
637		414	421	428	434	441	448	455	462	468	475
638		482	489	496	502	509	516	523	530	536	543
639		550	557	564	570	577	584	591	598	604	611
640		618	625	632	638	645	652	659	665	672	679
641		686	693	699	706	713	720	726	733	740	747
642		754	760	767	774	781	787	794	801	808	814
643		821	828	835	841	848	855	862	868	875	882
644		889	895	902	909	916	922	929	936	943	949
645		956	963	969	976	983	990	996	*003	*010	*017
646	81	023	030	037	043	050	057	064	070	077	084
647		090	097	104	111	117	124	131	137	144	151
648		158	164	171	178	184	191	198	204	211	218
649		224	231	238	245	251	258	265	271	278	285
650		291	298	305	311	318	325	331	338	345	351
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Proportional parts

.77 815 — .81 351

Table 6.14 (Cont'd).

COMMON LOGARITHMS OF NUMBERS

650 — 700

N.		0	1	2	3	4	5	6	7	8	9
650	81	291	298	305	311	318	325	331	338	345	351
651		358	365	371	378	385	391	398	405	411	418
652		425	431	438	445	451	458	465	471	478	485
653		491	498	505	511	518	525	531	538	544	551
654		558	564	571	578	584	591	598	604	611	617
655		624	631	637	644	651	657	664	671	677	684
656		690	697	704	710	717	723	730	737	743	750
657		757	763	770	776	783	790	796	803	809	816
658		823	829	836	842	849	856	862	869	875	882
659		889	895	902	908	915	921	928	935	941	948
660		954	961	968	974	981	987	994	*000	*007	*014
661	82	020	027	033	040	046	053	060	066	073	079
662		086	092	099	105	112	119	125	132	138	145
663		151	158	164	171	178	184	191	197	204	210
664		217	223	230	236	243	249	256	263	269	276
665		282	289	295	302	308	315	321	328	334	341
666		347	354	360	367	373	380	387	393	400	406
667		413	419	426	432	439	445	452	458	465	471
668		478	484	491	497	504	510	517	523	530	536
669		543	549	556	562	569	575	582	588	595	601
670		607	614	620	627	633	640	646	653	659	666
671		672	679	685	692	698	705	711	718	724	730
672		737	743	750	756	763	769	776	782	789	795
673		802	808	814	821	827	834	840	847	853	860
674		866	872	879	885	892	898	905	911	918	924
675		930	937	943	950	956	963	969	975	982	988
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052
677	83	059	065	072	078	085	091	097	104	110	117
678		123	129	136	142	149	155	161	168	174	181
679		187	193	200	206	213	219	225	232	238	245
680		251	257	264	270	276	283	289	296	302	308
681		315	321	327	334	340	347	353	359	366	372
682		378	385	391	398	404	410	417	423	429	436
683		442	448	455	461	467	474	480	487	493	499
684		506	512	518	525	531	537	544	550	556	563
685		569	575	582	588	594	601	607	613	620	626
686		632	639	645	651	658	664	670	677	683	689
687		696	702	708	715	721	727	734	740	746	753
688		759	765	771	778	784	790	797	803	809	816
689		822	828	835	841	847	853	860	866	872	879
690		885	891	897	904	910	916	923	929	935	942
691		948	954	960	967	973	979	985	992	998	*004
692	84	011	017	023	029	036	042	048	055	061	067
693		073	080	086	092	098	105	111	117	123	130
694		136	142	148	155	161	167	173	180	186	192
695		198	205	211	217	223	230	236	242	248	255
696		261	267	273	280	286	292	298	305	311	317
697		323	330	336	342	348	354	361	367	373	379
698		386	392	398	404	410	417	423	429	435	442
699		448	454	460	466	473	479	485	491	497	504
700		510	516	522	528	535	541	547	553	559	566
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Proportional parts

.81 291 — .84 566

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

700 — 750

N.		0	1	2	3	4	5	6	7	8	9
700	84	510	516	522	528	535	541	547	553	559	566
701		572	578	584	590	597	603	609	615	621	628
702		634	640	646	652	658	665	671	677	683	689
703		696	702	708	714	720	726	733	739	745	751
704		757	763	770	776	782	788	794	800	807	813
705		819	825	831	837	844	850	856	862	868	874
706		880	887	893	899	905	911	917	924	930	936
707		942	948	954	960	967	973	979	985	991	997
708	85	003	009	016	022	028	034	040	046	052	058
709		065	071	077	083	089	095	101	107	114	120
710		126	132	138	144	150	156	163	169	175	181
711		187	193	199	205	211	217	224	230	236	242
712		248	254	260	266	272	278	285	291	297	303
713		309	315	321	327	333	339	345	352	358	364
714		370	376	382	388	394	400	406	412	418	425
715		431	437	443	449	455	461	467	473	479	485
716		491	497	503	509	516	522	528	534	540	546
717		552	558	564	570	576	582	588	594	600	606
718		612	618	625	631	637	643	649	655	661	667
719		673	679	685	691	697	703	709	715	721	727
720		733	739	745	751	757	763	769	775	781	788
721		794	800	806	812	818	824	830	836	842	848
722		854	860	866	872	878	884	890	896	902	908
723		914	920	926	932	938	944	950	956	962	968
724		974	980	986	992	998	*004	*010	*016	*022	*028
725	86	034	040	046	052	058	064	070	076	082	088
726		094	100	106	112	118	124	130	136	141	147
727		153	159	165	171	177	183	189	195	201	207
728		213	219	225	231	237	243	249	255	261	267
729		273	279	285	291	297	303	308	314	320	326
730		332	338	344	350	356	362	368	374	380	386
731		392	398	404	410	415	421	427	433	439	445
732		451	457	463	469	475	481	487	493	499	504
733		510	516	522	528	534	540	546	552	558	564
734		570	576	581	587	593	599	605	611	617	623
735		629	635	641	646	652	658	664	670	676	682
736		688	694	700	705	711	717	723	729	735	741
737		747	753	759	764	770	776	782	788	794	800
738		806	812	817	823	829	835	841	847	853	859
739		864	870	876	882	888	894	900	906	911	917
740		923	929	935	941	947	953	958	964	970	976
741		982	988	994	999	*005	*011	*017	*023	*029	*035
742	87	040	046	052	058	064	070	075	081	087	093
743		099	105	111	116	122	128	134	140	146	151
744		157	163	169	175	181	186	192	198	204	210
745		216	221	227	233	239	245	251	256	262	268
746		274	280	286	291	297	303	309	315	320	326
747		332	338	344	349	355	361	367	373	379	384
748		390	396	402	408	413	419	425	431	437	442
749		448	454	460	466	471	477	483	489	495	500
750		506	512	518	523	529	535	541	547	552	558
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

Proportional parts

.84 510 — .87 558

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

750 — 800

N.		0	1	2	3	4	5	6	7	8	9
750	87	506	512	518	523	529	535	541	547	552	558
751		564	570	576	581	587	593	599	604	610	616
752		622	628	633	639	645	651	656	662	668	674
753		679	685	691	697	703	708	714	720	726	731
754		737	743	749	754	760	766	772	777	783	789
755		795	800	806	812	818	823	829	835	841	846
756		852	858	864	869	875	881	887	892	898	904
757		910	915	921	927	933	938	944	950	955	961
758		967	973	978	984	990	996	*001	*007	*013	*018
759	88	024	030	036	041	047	053	058	064	070	076
760		081	087	093	098	104	110	116	121	127	133
761		138	144	150	156	161	167	173	178	184	190
762		195	201	207	213	218	224	230	235	241	247
763		252	258	264	270	275	281	287	292	298	304
764		309	315	321	326	332	338	343	349	355	360
765		366	372	377	383	389	395	400	406	412	417
766		423	429	434	440	446	451	457	463	468	474
767		480	485	491	497	502	508	513	519	525	530
768		536	542	547	553	559	564	570	576	581	587
769		593	598	604	610	615	621	627	632	638	643
770		649	655	660	666	672	677	683	689	694	700
771		705	711	717	722	728	734	739	745	750	756
772		762	767	773	779	784	790	795	801	807	812
773		818	824	829	835	840	846	852	857	863	868
774		874	880	885	891	897	902	908	913	919	925
775		930	936	941	947	953	958	964	969	975	981
776		986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89	042	048	053	059	064	070	076	081	087	092
778		098	104	109	115	120	126	131	137	143	148
779		154	159	165	170	176	182	187	193	198	204
780		209	215	221	226	232	237	243	248	254	260
781		265	271	276	282	287	293	298	304	310	315
782		321	326	332	337	343	348	354	360	365	371
783		376	382	387	393	398	404	409	415	421	426
784		432	437	443	448	454	459	465	470	476	481
785		487	492	498	504	509	515	520	526	531	537
786		542	548	553	559	564	570	575	581	586	592
787		597	603	609	614	620	625	631	636	642	647
788		653	658	664	669	675	680	686	691	697	702
789		708	713	719	724	730	735	741	746	752	757
790		763	768	774	779	785	790	796	801	807	812
791		818	823	829	834	840	845	851	856	862	867
792		873	878	883	889	894	900	905	911	916	922
793		927	933	938	944	949	955	960	966	971	977
794		982	988	993	998	*004	*009	*015	*020	*026	*031
795	90	037	042	048	053	059	064	069	075	080	086
796		091	097	102	108	113	119	124	129	135	140
797		146	151	157	162	168	173	179	184	189	195
798		200	206	211	217	222	227	233	238	244	249
799		255	260	266	271	276	282	287	293	298	304
800		309	314	320	325	331	336	342	347	352	358
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Proportional parts

.87 506 — .90 358

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

800 — 850

N.		0	1	2	3	4	5	6	7	8	9
800	90	309	314	320	325	331	336	342	347	352	358
801		363	369	374	380	385	390	396	401	407	412
802		417	423	428	434	439	445	450	455	461	466
803		472	477	482	488	493	499	504	509	515	520
804		526	531	536	542	547	553	558	563	569	574
805		580	585	590	596	601	607	612	617	623	628
806		634	639	644	650	655	660	666	671	677	682
807		687	693	698	703	709	714	720	725	730	736
808		741	747	752	757	763	768	773	779	784	789
809		795	800	806	811	816	822	827	832	838	843
810		849	854	859	865	870	875	881	886	891	897
811		902	907	913	918	924	929	934	940	945	950
812		956	961	966	972	977	982	988	993	998	*004
813	91	009	014	020	025	030	036	041	046	052	057
814		062	068	073	078	084	089	094	100	105	110
815		116	121	126	132	137	142	148	153	158	164
816		169	174	180	185	190	196	201	206	212	217
817		222	228	233	238	243	249	254	259	265	270
818		275	281	286	291	297	302	307	312	318	323
819		328	334	339	344	350	355	360	365	371	376
820		381	387	392	397	403	408	413	418	424	429
821		434	440	445	450	455	461	466	471	477	482
822		487	492	498	503	508	514	519	524	529	535
823		540	545	551	556	561	566	572	577	582	587
824		593	598	603	609	614	619	624	630	635	640
825		645	651	656	661	666	672	677	682	687	693
826		698	703	709	714	719	724	730	735	740	745
827		751	756	761	766	772	777	782	787	793	798
828		803	808	814	819	824	829	834	840	845	850
829		855	861	866	871	876	882	887	892	897	903
830		908	913	918	924	929	934	939	944	950	955
831		960	965	971	976	981	986	991	997	*002	*007
832	92	012	018	023	028	033	038	044	049	054	059
833		065	070	075	080	085	091	096	101	106	111
834		117	122	127	132	137	143	148	153	158	163
835		169	174	179	184	189	195	200	205	210	215
836		221	226	231	236	241	247	252	257	262	267
837		273	278	283	288	293	298	304	309	314	319
838		324	330	335	340	345	350	355	361	366	371
839		376	381	387	392	397	402	407	412	418	423
840		428	433	438	443	449	454	459	464	469	474
841		480	485	490	495	500	505	511	516	521	526
842		531	536	542	547	552	557	562	567	572	578
843		583	588	593	598	603	609	614	619	624	629
844		634	639	645	650	655	660	665	670	675	681
845		686	691	696	701	706	711	716	722	727	732
846		737	742	747	752	758	763	768	773	778	783
847		788	793	799	804	809	814	819	824	829	834
848		840	845	850	855	860	865	870	875	881	886
849		891	896	901	906	911	916	921	927	932	937
850		942	947	952	957	962	967	973	978	983	988
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Proportional parts	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Proportional parts

.90 309 — .92 988

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

850 — 900

N.		0	1	2	3	4	5	6	7	8	9
850	92	942	947	952	957	962	967	973	978	983	988
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93	044	049	054	059	064	069	075	080	085	090
853		095	100	105	110	115	120	125	131	136	141
854		146	151	156	161	166	171	176	181	186	192
855		197	202	207	212	217	222	227	232	237	242
856		247	252	258	263	268	273	278	283	288	293
857		298	303	308	313	318	323	328	334	339	344
858		349	354	359	364	369	374	379	384	389	394
859		399	404	409	414	420	425	430	435	440	445
860		450	455	460	465	470	475	480	485	490	495
861		500	505	510	515	520	526	531	536	541	546
862		551	556	561	566	571	576	581	586	591	596
863		601	606	611	616	621	626	631	636	641	646
864		651	656	661	666	671	676	682	687	692	697
865		702	707	712	717	722	727	732	737	742	747
866		752	757	762	767	772	777	782	787	792	797
867		802	807	812	817	822	827	832	837	842	847
868		852	857	862	867	872	877	882	887	892	897
869		902	907	912	917	922	927	932	937	942	947
870		952	957	962	967	972	977	982	987	992	997
871	94	002	007	012	017	022	027	032	037	042	047
872		052	057	062	067	072	077	082	086	091	096
873		101	106	111	116	121	126	131	136	141	146
874		151	156	161	166	171	176	181	186	191	196
875		201	206	211	216	221	226	231	236	240	245
876		250	255	260	265	270	275	280	285	290	295
877		300	305	310	315	320	325	330	335	340	345
878		349	354	359	364	369	374	379	384	389	394
879		399	404	409	414	419	424	429	433	438	443
880		448	453	458	463	468	473	478	483	488	493
881		498	503	507	512	517	522	527	532	537	542
882		547	552	557	562	567	571	576	581	586	591
883		596	601	606	611	616	621	626	630	635	640
884		645	650	655	660	665	670	675	680	685	689
885		694	699	704	709	714	719	724	729	734	738
886		743	748	753	758	763	768	773	778	783	787
887		792	797	802	807	812	817	822	827	832	836
888		841	846	851	856	861	866	871	876	880	885
889		890	895	900	905	910	915	919	924	929	934
890		939	944	949	954	959	963	968	973	978	983
891		988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95	036	041	046	051	056	061	066	071	075	080
893		085	090	095	100	105	109	114	119	124	129
894		134	139	143	148	153	158	163	168	173	177
895		182	187	192	197	202	207	211	216	221	226
896		231	236	240	245	250	255	260	265	270	274
897		279	284	289	294	299	303	308	313	318	323
898		328	332	337	342	347	352	357	361	366	371
899		376	381	386	390	395	400	405	410	415	419
900		424	429	434	439	444	448	453	458	463	468
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Proportional parts

.92 942 — .95 468

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

900 — 950

N.		0	1	2	3	4	5	6	7	8	9
900	95	424	429	434	439	444	448	453	458	463	468
901		472	477	482	487	492	497	501	506	511	516
902		521	525	530	535	540	545	550	554	559	564
903		569	574	578	583	588	593	598	602	607	612
904		617	622	626	631	636	641	646	650	655	660
905		665	670	674	679	684	689	694	698	703	708
906		713	718	722	727	732	737	742	746	751	756
907		761	766	770	775	780	785	789	794	799	804
908		809	813	818	823	828	832	837	842	847	852
909		856	861	866	871	875	880	885	890	895	899
910		904	909	914	918	923	928	933	938	942	947
911		952	957	961	966	971	976	980	985	990	995
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96	047	052	057	061	066	071	076	080	085	090
914		095	099	104	109	114	118	123	128	133	137
915		142	147	152	156	161	166	171	175	180	185
916		190	194	199	204	209	213	218	223	227	232
917		237	242	246	251	256	261	265	270	275	280
918		284	289	294	298	303	308	313	317	322	327
919		332	336	341	346	350	355	360	365	369	374
920		379	384	388	393	398	402	407	412	417	421
921		426	431	435	440	445	450	454	459	464	468
922		473	478	483	487	492	497	501	506	511	515
923		520	525	530	534	539	544	548	553	558	562
924		567	572	577	581	586	591	595	600	605	609
925		614	619	624	628	633	638	642	647	652	656
926		661	666	670	675	680	685	689	694	699	703
927		708	713	717	722	727	731	736	741	745	750
928		755	759	764	769	774	778	783	788	792	797
929		802	806	811	816	820	825	830	834	839	844
930		848	853	858	862	867	872	876	881	886	890
931		895	900	904	909	914	918	923	928	932	937
932		942	946	951	956	960	965	970	974	979	984
933		988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97	035	039	044	049	053	058	063	067	072	077
935		081	086	090	095	100	104	109	114	118	123
936		128	132	137	142	146	151	155	160	165	169
937		174	179	183	188	192	197	202	206	211	216
938		220	225	230	234	239	243	248	253	257	262
939		267	271	276	280	285	290	294	299	304	308
940		313	317	322	327	331	336	340	345	350	354
941		359	364	368	373	377	382	387	391	396	400
942		405	410	414	419	424	428	433	437	442	447
943		451	456	460	465	470	474	479	483	488	493
944		497	502	506	511	516	520	525	529	534	539
945		543	548	552	557	562	566	571	575	580	585
946		589	594	598	603	607	612	617	621	626	630
947		635	640	644	649	653	658	663	667	672	676
948		681	685	690	695	699	704	708	713	717	722
949		727	731	736	740	745	749	754	759	763	768
950		772	777	782	786	791	795	800	804	809	813
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts	
	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5
	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Proportional parts

.95 424 — .97 813

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

950 — 1000

N.		0	1	2	3	4	5	6	7	8	9
950	97	772	777	782	786	791	795	800	804	809	813
951		818	823	827	832	836	841	845	850	855	859
952		864	868	873	877	882	886	891	896	900	905
953		909	914	918	923	928	932	937	941	946	950
954		955	959	964	968	973	978	982	987	991	996
955	98	000	005	009	014	019	023	028	032	037	041
956		046	050	055	059	064	068	073	078	082	087
957		091	096	100	105	109	114	118	123	127	132
958		137	141	146	150	155	159	164	168	173	177
959		182	186	191	195	200	204	209	214	218	223
960		227	232	236	241	245	250	254	259	263	268
961		272	277	281	286	290	295	299	304	308	313
962		318	322	327	331	336	340	345	349	354	358
963		363	367	372	376	381	385	390	394	399	403
964		408	412	417	421	426	430	435	439	444	448
965		453	457	462	466	471	475	480	484	489	493
966		498	502	507	511	516	520	525	529	534	538
967		543	547	552	556	561	565	570	574	579	583
968		588	592	597	601	605	610	614	619	623	628
969		632	637	641	646	650	655	659	664	668	673
970		677	682	686	691	695	700	704	709	713	717
971		722	726	731	735	740	744	749	753	758	762
972		767	771	776	780	784	789	793	798	802	807
973		811	816	820	825	829	834	838	843	847	851
974		856	860	865	869	874	878	883	887	892	896
975		900	905	909	914	918	923	927	932	936	941
976		945	949	954	958	963	967	972	976	981	985
977		989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99	034	038	043	047	052	056	061	065	069	074
979		078	083	087	092	096	100	105	109	114	118
980		123	127	131	136	140	145	149	154	158	162
981		167	171	176	180	185	189	193	198	202	207
982		211	216	220	224	229	233	238	242	247	251
983		255	260	264	269	273	277	282	286	291	295
984		300	304	308	313	317	322	326	330	335	339
985		344	348	352	357	361	366	370	374	379	383
986		388	392	396	401	405	410	414	419	423	427
987		432	436	441	445	449	454	458	463	467	471
988		476	480	484	489	493	498	502	506	511	515
989		520	524	528	533	537	542	546	550	555	559
990		564	568	572	577	581	585	590	594	599	603
991		607	612	616	621	625	629	634	638	642	647
992		651	656	660	664	669	673	677	682	686	691
993		695	699	704	708	712	717	721	726	730	734
994		739	743	747	752	756	760	765	769	774	778
995		782	787	791	795	800	804	808	813	817	822
996		826	830	835	839	843	848	852	856	861	865
997		870	874	878	883	887	891	896	900	904	909
998		913	917	922	926	930	935	939	944	948	952
999		957	961	965	970	974	978	983	987	991	996
1000	00	000	004	009	013	017	022	026	030	035	039
N.		0	1	2	3	4	5	6	7	8	9

Proportional parts

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Proportional parts

.97 772 — .99 996

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1000 — 1050

N.		0	1	2	3	4	5	6	7	8	9	d.
1000	000	0000	0434	0869	1303	1737	2171	2605	3039	3473	3907	434
1001		4341	4775	5208	5642	6076	6510	6943	7377	7810	8244	434
1002		8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576	433
1003	001	3009	3442	3875	4308	4741	5174	5607	6039	6472	6905	433
1004		7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228	432
1005	002	1661	2093	2525	2957	3389	3821	4253	4685	5116	5548	432
1006		5980	6411	6843	7275	7706	8138	8569	9001	9432	9863	431
1007	003	0295	0726	1157	1588	2019	2451	2882	3313	3744	4174	431
1008		4605	5036	5467	5898	6328	6759	7190	7620	8051	8481	431
1009		8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784	430
1010	004	3214	3644	4074	4504	4933	5363	5793	6223	6652	7082	430
1011		7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376	429
1012	005	1805	2234	2663	3092	3521	3950	4379	4808	5237	5666	429
1013		6094	6523	6952	7380	7809	8238	8666	9094	9523	9951	429
1014	006	0380	0808	1236	1664	2092	2521	2949	3377	3805	4233	428
1015		4660	5088	5516	5944	6372	6799	7227	7655	8082	8510	428
1016		8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782	427
1017	007	3210	3637	4064	4490	4917	5344	5771	6198	6624	7051	427
1018		7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316	426
1019	008	1742	2168	2594	3020	3446	3872	4298	4724	5150	5576	426
1020		6002	6427	6853	7279	7704	8130	8556	8981	9407	9832	426
1021	009	0257	0683	1108	1533	1959	2384	2809	3234	3659	4084	425
1022		4509	4934	5359	5784	6208	6633	7058	7483	7907	8332	425
1023		8756	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575	424
1024	010	3000	3424	3848	4272	4696	5120	5544	5967	6391	6815	424
1025		7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050	424
1026	011	1474	1897	2320	2743	3166	3590	4013	4436	4859	5282	423
1027		5704	6127	6550	6973	7396	7818	8241	8664	9086	9509	423
1028		9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732	422
1029	012	4154	4576	4998	5420	5842	6264	6685	7107	7529	7951	422
1030		8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165	422
1031	013	2587	3008	3429	3850	4271	4692	5113	5534	5955	6376	421
1032		6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583	421
1033	014	1003	1424	1844	2264	2685	3105	3525	3945	4365	4785	420
1034		5205	5625	6045	6465	6885	7305	7725	8144	8564	8984	420
1035		9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178	420
1036	015	3598	4017	4436	4855	5274	5693	6112	6531	6950	7369	419
1037		7788	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555	419
1038	016	1974	2392	2810	3229	3647	4065	4483	4901	5319	5737	418
1039		6155	6573	6991	7409	7827	8245	8663	9080	9498	9916	418
1040	017	0333	0751	1168	1586	2003	2421	2838	3256	3673	4090	417
1041		4507	4924	5342	5759	6176	6593	7010	7427	7844	8260	417
1042		8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427	417
1043	018	2843	3259	3676	4092	4508	4925	5341	5757	6173	6589	416
1044		7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747	416
1045	019	1163	1578	1994	2410	2825	3240	3656	4071	4486	4902	415
1046		5317	5732	6147	6562	6977	7392	7807	8222	8637	9052	415
1047		9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198	415
1048	020	3613	4027	4442	4856	5270	5684	6099	6513	6927	7341	414
1049		7755	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479	414
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
N.		0	1	2	3	4	5	6	7	8	9	d.

.000 0000 — .021 5614

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1050 — 1100

N.		0	1	2	3	4	5	6	7	8	9	d.
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
1051		6027	6440	6854	7267	7680	8093	8506	8919	9332	9745	413
1052	022	0157	0570	0983	1396	1808	2221	2634	3046	3459	3871	413
1053		4284	4696	5109	5521	5933	6345	6758	7170	7582	7994	412
1054		8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113	412
1055	023	2525	2936	3348	3759	4171	4582	4994	5405	5817	6228	411
1056		6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339	411
1057	024	0750	1161	1572	1982	2393	2804	3214	3625	4036	4446	411
1058		4857	5267	5678	6088	6498	6909	7319	7729	8139	8549	410
1059		8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649	410
1060	025	3059	3468	3878	4288	4697	5107	5516	5926	6335	6744	410
1061		7154	7563	7972	8382	8791	9200	9609	*0018	*0427	*0836	409
1062	026	1245	1654	2063	2472	2881	3289	3698	4107	4515	4924	409
1063		5333	5741	6150	6558	6967	7375	7783	8192	8600	9008	408
1064		9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088	408
1065	027	3496	3904	4312	4719	5127	5535	5942	6350	6757	7165	408
1066		7572	7979	8387	8794	9201	9609	*0016	*0423	*0830	*1237	407
1067	028	1644	2051	2458	2865	3272	3679	4086	4492	4899	5306	407
1068		5713	6119	6526	6932	7339	7745	8152	8558	8964	9371	406
1069		9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432	406
1070	029	3838	4244	4649	5055	5461	5867	6272	6678	7084	7489	406
1071		7895	8300	8706	9111	9516	9922	*0327	*0732	*1138	*1543	405
1072	030	1948	2353	2758	3163	3568	3973	4378	4783	5188	5592	405
1073		5997	6402	6807	7211	7616	8020	8425	8830	9234	9638	405
1074	031	0043	0447	0851	1256	1660	2064	2468	2872	3277	3681	404
1075		4085	4489	4893	5296	5700	6104	6508	6912	7315	7719	404
1076		8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754	403
1077	032	2157	2560	2963	3367	3770	4173	4576	4979	5382	5785	403
1078		6188	6590	6993	7396	7799	8201	8604	9007	9409	9812	403
1079	033	0214	0617	1019	1422	1824	2226	2629	3031	3433	3835	402
1080		4238	4640	5042	5444	5846	6248	6650	7052	7453	7855	402
1081		8257	8659	9060	9462	9864	*0265	*0667	*1068	*1470	*1871	402
1082	034	2273	2674	3075	3477	3878	4279	4680	5081	5482	5884	401
1083		6285	6686	7087	7487	7888	8289	8690	9091	9491	9892	401
1084	035	0293	0693	1094	1495	1895	2296	2696	3096	3497	3897	400
1085		4297	4698	5098	5498	5898	6298	6698	7098	7498	7898	400
1086		8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896	400
1087	036	2295	2695	3094	3494	3893	4293	4692	5091	5491	5890	399
1088		6289	6688	7087	7486	7885	8284	8683	9082	9481	9880	399
1089	037	0279	0678	1076	1475	1874	2272	2671	3070	3468	3867	399
1090		4265	4663	5062	5460	5858	6257	6655	7053	7451	7849	398
1091		8248	8646	9044	9442	9839	*0237	*0635	*1033	*1431	*1829	398
1092	038	2226	2624	3022	3419	3817	4214	4612	5009	5407	5804	398
1093		6202	6599	6996	7393	7791	8188	8585	8982	9379	9776	397
1094	039	0173	0570	0967	1364	1761	2158	2554	2951	3348	3745	397
1095		4141	4538	4934	5331	5727	6124	6520	6917	7313	7709	397
1096		8106	8502	8898	9294	9690	*0086	*0482	*0878	*1274	*1670	396
1097	040	2066	2462	2858	3254	3650	4045	4441	4837	5232	5628	396
1098		6023	6419	6814	7210	7605	8001	8396	8791	9187	9582	395
1099		9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532	395
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
N.		0	1	2	3	4	5	6	7	8	9	d.

.021 1893 — .041 7479

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1100 — 1150

N.		0	1	2	3	4	5	6	7	8	9	d.
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
1101		7873	8268	8662	9056	9451	9845	*0239	*0633	*1028	*1422	394
1102	042	1816	2210	2604	2998	3392	3786	4180	4574	4968	5361	394
1103		5755	6149	6543	6936	7330	7723	8117	8510	8904	9297	394
1104		9691	*0084	*0477	*0871	*1264	*1657	*2050	*2444	*2837	*3230	393
1105	043	3623	4016	4409	4802	5195	5587	5980	6373	6766	7159	393
1106		7551	7944	8337	8729	9122	9514	9907	*0299	*0692	*1084	393
1107	044	1476	1869	2261	2653	3045	3437	3829	4222	4614	5006	392
1108		5398	5790	6181	6573	6965	7357	7749	8140	8532	8924	392
1109		9315	9707	*0099	*0490	*0882	*1273	*1664	*2056	*2447	*2839	392
1110	045	3230	3621	4012	4403	4795	5186	5577	5968	6359	6750	391
1111		7141	7531	7922	8313	8704	9095	9485	9876	*0267	*0657	391
1112	046	1048	1438	1829	2219	2610	3000	3391	3781	4171	4561	390
1113		4952	5342	5732	6122	6512	6902	7292	7682	8072	8462	390
1114		8852	9242	9632	*0021	*0411	*0801	*1190	*1580	*1970	*2359	390
1115	047	2749	3138	3528	3917	4306	4696	5085	5474	5864	6253	389
1116		6642	7031	7420	7809	8198	8587	8976	9365	9754	*0143	389
1117	048	0532	0921	1309	1698	2087	2475	2864	3253	3641	4030	389
1118		4418	4806	5195	5583	5972	6360	6748	7136	7525	7913	388
1119		8301	8689	9077	9465	9853	*0241	*0629	*1017	*1405	*1792	388
1120	049	2180	2568	2956	3343	3731	4119	4506	4894	5281	5669	388
1121		6056	6444	6831	7218	7606	7993	8380	8767	9154	9541	387
1122		9929	*0316	*0703	*1090	*1477	*1863	*2250	*2637	*3024	*3411	387
1123	050	3798	4184	4571	4958	5344	5731	6117	6504	6890	7277	387
1124		7663	8049	8436	8822	9208	9595	9981	*0367	*0753	*1139	386
1125	051	1525	1911	2297	2683	3069	3455	3841	4227	4612	4998	386
1126		5384	5770	6155	6541	6926	7312	7697	8083	8468	8854	386
1127		9239	9624	*0010	*0395	*0780	*1166	*1551	*1936	*2321	*2706	385
1128	052	3091	3476	3861	4246	4631	5016	5400	5785	6170	6555	385
1129		6939	7324	7709	8093	8478	8862	9247	9631	*0016	*0400	385
1130	053	0784	1169	1553	1937	2321	2706	3090	3474	3858	4242	384
1131		4626	5010	5394	5778	6162	6546	6929	7313	7697	8081	384
1132		8464	8848	9232	9615	9999	*0382	*0766	*1149	*1532	*1916	384
1133	054	2299	2682	3066	3449	3832	4215	4598	4981	5365	5748	383
1134		6131	6514	6896	7279	7662	8045	8428	8811	9193	9576	383
1135		9959	*0341	*0724	*1106	*1489	*1871	*2254	*2636	*3019	*3401	382
1136	055	3783	4166	4548	4930	5312	5694	6077	6459	6841	7223	382
1137		7605	7987	8369	8750	9132	9514	9896	*0278	*0659	*1041	382
1138	056	1423	1804	2186	2567	2949	3330	3712	4093	4475	4856	381
1139		5237	5619	6000	6381	6762	7143	7524	7905	8287	8668	381
1140		9049	9429	9810	*0191	*0572	*0953	*1334	*1714	*2095	*2476	381
1141	057	2856	3237	3618	3998	4379	4759	5140	5520	5900	6281	381
1142		6661	7041	7422	7802	8182	8562	8942	9322	9702	*0082	380
1143	058	0462	0842	1222	1602	1982	2362	2741	3121	3501	3881	380
1144		4260	4640	5019	5399	5778	6158	6537	6917	7296	7676	380
1145		8055	8434	8813	9193	9572	9951	*0330	*0709	*1088	*1467	379
1146	059	1846	2225	2604	2983	3362	3741	4119	4498	4877	5256	379
1147		5634	6013	6391	6770	7148	7527	7905	8284	8662	9041	379
1148		9419	9797	*0175	*0554	*0932	*1310	*1688	*2066	*2444	*2822	378
1149	060	3200	3578	3956	4334	4712	5090	5468	5845	6223	6601	378
1150		6978	7356	7734	8111	8489	8866	9244	9621	9999	*0376	378
N.		0	1	2	3	4	5	6	7	8	9	d.

.041 3927 — .061 0376

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1150 — 1200

N.		0	1	2	3	4	5	6	7	8	9	d.
1150	060	6978	7356	7734	8111	8489	8866	9244	9621	9999	*0376	378
1151	061	0753	1131	1508	1885	2262	2639	3017	3394	3771	4148	377
1152		4525	4902	5279	5656	6032	6409	6786	7163	7540	7916	377
1153		8293	8670	9046	9423	9799	*0176	*0552	*0929	*1305	*1682	377
1154	062	2058	2434	2811	3187	3563	3939	4316	4692	5068	5444	376
1155		5820	6196	6572	6948	7324	7699	8075	8451	8827	9203	376
1156		9578	9954	*0330	*0705	*1081	*1456	*1832	*2207	*2583	*2958	376
1157	063	3334	3709	4084	4460	4835	5210	5585	5960	6335	6711	375
1158		7086	7461	7836	8211	8585	8960	9335	9710	*0085	*0460	375
1159	064	0834	1209	1584	1958	2333	2708	3082	3457	3831	4205	375
1160		4580	4954	5329	5703	6077	6451	6826	7200	7574	7948	374
1161		8322	8696	9070	9444	9818	*0192	*0566	*0940	*1314	*1688	374
1162	065	2061	2435	2809	3182	3556	3930	4303	4677	5050	5424	374
1163		5797	6171	6544	6917	7291	7664	8037	8410	8784	9157	373
1164		9530	9903	*0276	*0649	*1022	*1395	*1768	*2141	*2514	*2886	373
1165	066	3259	3632	4005	4377	4750	5123	5495	5868	6241	6613	373
1166		6986	7358	7730	8103	8475	8847	9220	9592	9964	*0336	372
1167	067	0709	1081	1453	1825	2197	2569	2941	3313	3685	4057	372
1168		4428	4800	5172	5544	5915	6287	6659	7030	7402	7774	372
1169		8145	8517	8888	9259	9631	*0002	*0374	*0745	*1116	*1487	371
1170	068	1859	2230	2601	2972	3343	3714	4085	4456	4827	5198	371
1171		5569	5940	6311	6681	7052	7423	7794	8164	8535	8906	371
1172		9276	9647	*0017	*0388	*0758	*1129	*1499	*1869	*2240	*2610	370
1173	069	2980	3350	3721	4091	4461	4831	5201	5571	5941	6311	370
1174		6681	7051	7421	7791	8160	8530	8900	9270	9639	*0009	370
1175	070	0379	0748	1118	1487	1857	2226	2596	2965	3335	3704	369
1176		4073	4442	4812	5181	5550	5919	6288	6658	7027	7396	369
1177		7765	8134	8503	8871	9240	9609	9978	*0347	*0715	*1084	369
1178	071	1453	1822	2190	2559	2927	3296	3664	4033	4401	4770	369
1179		5138	5506	5875	6243	6611	6979	7348	7716	8084	8452	368
1180		8820	9188	9556	9924	*0292	*0660	*1028	*1396	*1763	*2131	368
1181	072	2499	2867	3234	3602	3970	4337	4705	5072	5440	5807	368
1182		6175	6542	6910	7277	7644	8011	8379	8746	9113	9480	367
1183		9847	*0215	*0582	*0949	*1316	*1683	*2050	*2416	*2783	*3150	367
1184	073	3517	3884	4251	4617	4984	5351	5717	6084	6450	6817	367
1185		7184	7550	7916	8283	8649	9016	9382	9748	*0114	*0481	366
1186	074	0847	1213	1579	1945	2311	2677	3043	3409	3775	4141	366
1187		4507	4873	5239	5605	5970	6336	6702	7068	7433	7799	366
1188		8164	8530	8895	9261	9626	9992	*0357	*0723	*1088	*1453	365
1189	075	1819	2184	2549	2914	3279	3644	4010	4375	4740	5105	365
1190		5470	5835	6199	6564	6929	7294	7659	8024	8388	8753	365
1191		9118	9482	9847	*0211	*0576	*0940	*1305	*1669	*2034	*2398	364
1192	076	2763	3127	3491	3855	4220	4584	4948	5312	5676	6040	364
1193		6404	6768	7132	7496	7860	8224	8588	8952	9316	9680	364
1194	077	0043	0407	0771	1134	1498	1862	2225	2589	2952	3316	364
1195		3679	4042	4406	4769	5133	5496	5859	6222	6585	6949	363
1196		7312	7675	8038	8401	8764	9127	9490	9853	*0216	*0579	363
1197	078	0942	1304	1667	2030	2393	2755	3118	3480	3843	4206	363
1198		4568	4931	5293	5656	6018	6380	6743	7105	7467	7830	362
1199		8192	8554	8916	9278	9640	*0003	*0365	*0727	*1089	*1451	362
1200	079	1812	2174	2536	2898	3260	3622	3983	4345	4707	5068	362
N.		0	1	2	3	4	5	6	7	8	9	d.

.060 6978 — .079 5068

6.15 TABLE OF COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Table 6.15

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

0° (180°) (359°) **179°**

″	′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	0						10.00 000	60
60	1	6.46 373	30103	6.46 373	30103	13.53 627	10.00 000	59
120	2	6.76 476	17609	6.76 476	17609	13.23 524	10.00 000	58
180	3	6.94 085	12494	6.94 085	12494	13.05 915	10.00 000	57
240	4	7.06 579	9691	7.06 579	9691	12.93 421	10.00 000	56
300	5	7.16 270	7918	7.16 270	7918	12.83 730	10.00 000	55
360	6	7.24 188	6694	7.24 188	6694	12.75 812	10.00 000	54
420	7	7.30 882	5800	7.30 882	5800	12.69 118	10.00 000	53
480	8	7.36 682	5115	7.36 682	5115	12.63 318	10.00 000	52
540	9	7.41 797	4576	7.41 797	4576	12.58 203	10.00 000	51
600	10	7.46 373	4139	7.46 373	4139	12.53 627	10.00 000	50
660	11	7.50 512	3779	7.50 512	3779	12.49 488	10.00 000	49
720	12	7.54 291	3476	7.54 291	3476	12.45 709	10.00 000	48
780	13	7.57 767	3218	7.57 767	3219	12.42 233	10.00 000	47
840	14	7.60 985	2997	7.60 986	2996	12.39 014	10.00 000	46
900	15	7.63 982	2802	7.63 982	2803	12.36 018	10.00 000	45
960	16	7.66 784	2633	7.66 785	2633	12.33 215	10.00 000	44
1020	17	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
1080	18	7.71 900	2348	7.71 900	2348	12.28 100	9.99 999	42
1140	19	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
1200	20	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	40
1260	21	7.78 594	2021	7.78 595	2020	12.21 405	9.99 999	39
1320	22	7.80 615	1930	7.80 615	1931	12.19 385	9.99 999	38
1380	23	7.82 545	1848	7.82 546	1848	12.17 454	9.99 999	37
1440	24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
1500	25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
1560	26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
1620	27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
1680	28	7.91 088	1524	7.91 089	1524	12.08 911	9.99 999	32
1740	29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
1800	30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	30
1860	31	7.95 508	1379	7.95 510	1379	12.04 490	9.99 998	29
1920	32	7.96 887	1336	7.96 889	1336	12.03 111	9.99 998	28
1980	33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
2040	34	7.99 520	1259	7.99 522	1259	12.00 478	9.99 998	26
2100	35	8.00 779	1223	8.00 781	1223	11.99 219	9.99 998	25
2160	36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
2220	37	8.03 192	1158	8.03 194	1159	11.96 806	9.99 997	23
2280	38	8.04 350	1128	8.04 353	1128	11.95 647	9.99 997	22
2340	39	8.05 478	1100	8.05 481	1100	11.94 519	9.99 997	21
2400	40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	20
2460	41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
2520	42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
2580	43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
2640	44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
2700	45	8.11 693	954	8.11 696	955	11.88 304	9.99 996	15
2760	46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
2820	47	8.13 581	914	8.13 585	915	11.86 415	9.99 996	13
2880	48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
2940	49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
3000	50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	10
3060	51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
3120	52	8.17 971	827	8.17 976	828	11.82 024	9.99 995	8
3180	53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
3240	54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
3300	55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
3360	56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
3420	57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
3480	58	8.22 713	743	8.22 720	742	11.77 280	9.99 994	2
3540	59	8.23 456	730	8.23 462	730	11.76 538	9.99 994	1
3600	60	8.24 186		8.24 192		11.75 808	9.99 993	0
″	′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

90° (270°) (269°) **89°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

1° (181°) (358°) **178°**

″	′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
3600	0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	60
3660	1	8.24 903	706	8.24 910	706	11.75 090	9.99 993	59
3720	2	8.25 609	695	8.25 616	696	11.74 384	9.99 993	58
3780	3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
3840	4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
3900	5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	55
3960	6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
4020	7	8.28 997	644	8.28 986	643	11.71 014	9.99 992	53
4080	8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
4140	9	8.30 255	624	8.30 263	625	11.69 737	9.99 991	51
4200	10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	50
4260	11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
4320	12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
4380	13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
4440	14	8.33 292	583	8.33 302	584	11.66 698	9.99 990	46
4500	15	8.33 875	575	8.33 886	575	11.66 114	9.99 990	45
4560	16	8.34 450	568	8.34 461	568	11.65 539	9.99 989	44
4620	17	8.35 018	560	8.35 029	561	11.64 971	9.99 989	43
4680	18	8.35 578	553	8.35 590	553	11.64 410	9.99 989	42
4740	19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
4800	20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	40
4860	21	8.37 217	533	8.37 229	533	11.62 771	9.99 988	39
4920	22	8.37 750	526	8.37 762	527	11.62 238	9.99 988	38
4980	23	8.38 276	520	8.38 289	520	11.61 711	9.99 987	37
5040	24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
5100	25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	35
5160	26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
5220	27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
5280	28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
5340	29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
5400	30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	30
5460	31	8.42 272	474	8.42 287	475	11.57 713	9.99 985	29
5520	32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
5580	33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
5640	34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
5700	35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	25
5760	36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
5820	37	8.45 044	445	8.45 061	446	11.54 939	9.99 983	23
5880	38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
5940	39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
6000	40	8.46 366	433	8.46 385	432	11.53 615	9.99 982	20
6060	41	8.46 799	427	8.46 817	428	11.53 183	9.99 981	19
6120	42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
6180	43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
6240	44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
6300	45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	15
6360	46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
6420	47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
6480	48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
6540	49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
6600	50	8.50 504	393	8.50 527	393	11.49 473	9.99 978	10
6660	51	8.50 897	390	8.50 920	390	11.49 080	9.99 977	9
6720	52	8.51 287	386	8.51 310	386	11.48 690	9.99 977	8
6780	53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
6840	54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
6900	55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	5
6960	56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
7020	57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
7080	58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
7140	59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
7200	60	8.54 282		8.54 308		11.45 692	9.99 974	0
″	′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

91° (271°) (268°) **88°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

2° (182°) (357°) **177°**

″	′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
7200	0	8.54 282	360	8.54 308	361	11.45 692	9.99 974	60
7260	1	8.54 642	357	8.54 669	358	11.45 331	9.99 973	59
7320	2	8.54 999	355	8.55 027	355	11.44 973	9.99 973	58
7380	3	8.55 354	351	8.55 382	352	11.44 618	9.99 972	57
7440	4	8.55 705	349	8.55 734	349	11.44 266	9.99 972	56
7500	5	8.56 054	346	8.56 083	346	11.43 917	9.99 971	55
7560	6	8.56 400	343	8.56 429	344	11.43 571	9.99 971	54
7620	7	8.56 743	341	8.56 773	341	11.43 227	9.99 970	53
7680	8	8.57 084	337	8.57 114	338	11.42 886	9.99 970	52
7740	9	8.57 421	336	8.57 452	336	11.42 548	9.99 969	51
7800	10	8.57 757	332	8.57 788	333	11.42 212	9.99 969	50
7860	11	8.58 089	330	8.58 121	330	11.41 879	9.99 968	49
7920	12	8.58 419	328	8.58 451	328	11.41 549	9.99 968	48
7980	13	8.58 747	325	8.58 779	326	11.41 221	9.99 967	47
8040	14	8.59 072	323	8.59 105	323	11.40 895	9.99 967	46
8100	15	8.59 395	320	8.59 428	321	11.40 572	9.99 967	45
8160	16	8.59 715	318	8.59 749	319	11.40 251	9.99 966	44
8220	17	8.60 033	316	8.60 068	316	11.39 932	9.99 966	43
8280	18	8.60 349	313	8.60 384	314	11.39 616	9.99 965	42
8340	19	8.60 662	311	8.60 698	311	11.39 302	9.99 964	41
8400	20	8.60 973	309	8.61 009	310	11.38 991	9.99 964	40
8460	21	8.61 282	307	8.61 319	307	11.38 681	9.99 963	39
8520	22	8.61 589	305	8.61 626	305	11.38 374	9.99 963	38
8580	23	8.61 894	302	8.61 931	303	11.38 069	9.99 962	37
8640	24	8.62 196	301	8.62 234	301	11.37 766	9.99 962	36
8700	25	8.62 497	298	8.62 535	299	11.37 465	9.99 961	35
8760	26	8.62 795	296	8.62 834	297	11.37 166	9.99 961	34
8820	27	8.63 091	294	8.63 131	295	11.36 869	9.99 960	33
8880	28	8.63 385	293	8.63 426	292	11.36 574	9.99 960	32
8940	29	8.63 678	290	8.63 718	291	11.36 282	9.99 959	31
9000	30	8.63 968	288	8.64 009	289	11.35 991	9.99 959	30
9060	31	8.64 256	287	8.64 298	287	11.35 702	9.99 958	29
9120	32	8.64 543	284	8.64 585	285	11.35 415	9.99 958	28
9180	33	8.64 827	283	8.64 870	284	11.35 130	9.99 957	27
9240	34	8.65 110	281	8.65 154	281	11.34 846	9.99 956	26
9300	35	8.65 391	279	8.65 435	280	11.34 565	9.99 956	25
9360	36	8.65 670	277	8.65 715	278	11.34 285	9.99 955	24
9420	37	8.65 947	276	8.65 993	276	11.34 007	9.99 955	23
9480	38	8.66 223	274	8.66 269	274	11.33 731	9.99 954	22
9540	39	8.66 497	272	8.66 543	273	11.33 457	9.99 954	21
9600	40	8.66 769	270	8.66 816	271	11.33 184	9.99 953	20
9660	41	8.67 039	269	8.67 087	269	11.32 913	9.99 952	19
9720	42	8.67 308	267	8.67 356	268	11.32 644	9.99 952	18
9780	43	8.67 575	266	8.67 624	266	11.32 376	9.99 951	17
9840	44	8.67 841	263	8.67 890	264	11.32 110	9.99 951	16
9900	45	8.68 104	263	8.68 154	263	11.31 846	9.99 950	15
9960	46	8.68 367	260	8.68 417	261	11.31 583	9.99 949	14
10020	47	8.68 627	259	8.68 678	260	11.31 322	9.99 949	13
10080	48	8.68 886	258	8.68 938	258	11.31 062	9.99 948	12
10140	49	8.69 144	256	8.69 196	257	11.30 804	9.99 948	11
10200	50	8.69 400	254	8.69 453	255	11.30 547	9.99 947	10
10260	51	8.69 654	253	8.69 708	254	11.30 292	9.99 946	9
10320	52	8.69 907	252	8.69 962	252	11.30 038	9.99 946	8
10380	53	8.70 159	250	8.70 214	251	11.29 786	9.99 945	7
10440	54	8.70 409	249	8.70 465	249	11.29 535	9.99 944	6
10500	55	8.70 658	247	8.70 714	248	11.29 286	9.99 944	5
10560	56	8.70 905	246	8.70 962	246	11.29 038	9.99 943	4
10620	57	8.71 151	244	8.71 208	245	11.28 792	9.99 942	3
10680	58	8.71 395	243	8.71 453	244	11.28 547	9.99 942	2
10740	59	8.71 638	242	8.71 697	243	11.28 303	9.99 941	1
10800	60	8.71 880		8.71 940		11.28 060	9.99 940	0
″	′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

92° (272°) (267°) **87°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

3° (183°) (356°) 176°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	8.71 880	240	8.71 940	241	11.28 060	9.99 940	60
1	8.72 120	239	8.72 181	239	11.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	11.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	11.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	11.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	11.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	11.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	11.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	11.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	11.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	11.25 708	9.99 934	50
11	8.74 454	226	8.74 521	227	11.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	11.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	11.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	11.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	11.24 577	9.99 930	45
16	8.75 575	220	8.75 645	222	11.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	11.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	11.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	11.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	11.23 475	9.99 926	40
21	8.76 667	216	8.76 742	216	11.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	11.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	11.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	11.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	11.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	11.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	11.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	11.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	11.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	11.21 351	9.99 919	30
31	8.78 774	205	8.78 855	206	11.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	11.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	11.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	11.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	11.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	11.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	11.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	11.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	11.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	11.19 326	9.99 911	20
41	8.80 782	196	8.80 872	196	11.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	11.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	11.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	11.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	11.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	11.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	11.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	11.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	11.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	11.17 390	9.99 903	10
51	8.82 701	187	8.82 799	188	11.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	11.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	11.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	11.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	11.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	11.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	11.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	11.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	11.15 718	9.99 895	1
60	8.84 358		8.84 464		11.15 536	9.99 894	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts				
″	241	239	237	235	234
1	4.0	4.0	4.0	3.9	3.9
2	8.0	8.0	7.9	7.8	7.8
3	12.0	12.0	11.8	11.8	11.7
4	16.1	15.9	15.8	15.7	15.6
5	20.1	19.9	19.8	19.6	19.5
6	24.1	23.9	23.7	23.5	23.4
7	28.1	27.9	27.6	27.4	27.3
8	32.1	31.9	31.6	31.3	31.2
9	36.2	35.8	35.6	35.2	35.1
″	232	229	227	225	223
1	3.9	3.8	3.8	3.8	3.7
2	7.7	7.6	7.6	7.5	7.4
3	11.6	11.4	11.4	11.2	11.2
4	15.5	15.3	15.1	15.0	14.9
5	19.3	19.1	18.9	18.8	18.6
6	23.2	22.9	22.7	22.5	22.3
7	27.1	26.7	26.5	26.2	26.0
8	30.9	30.5	30.3	30.0	29.7
9	34.8	34.4	34.0	33.8	33.4
″	222	220	217	215	213
1	3.7	3.7	3.6	3.6	3.6
2	7.4	7.3	7.2	7.2	7.1
3	11.1	11.0	10.8	10.8	10.6
4	14.8	14.7	14.5	14.3	14.2
5	18.5	18.3	18.1	17.9	17.8
6	22.2	22.0	21.7	21.5	21.3
7	25.9	25.7	25.3	25.1	24.8
8	29.6	29.3	28.9	28.7	28.4
9	33.3	33.0	32.6	32.2	32.0
″	211	208	206	203	201
1	3.5	3.5	3.4	3.4	3.4
2	7.0	6.9	6.9	6.8	6.7
3	10.6	10.4	10.3	10.2	10.0
4	14.1	13.9	13.7	13.5	13.4
5	17.6	17.3	17.2	16.9	16.8
6	21.1	20.8	20.6	20.3	20.1
7	24.6	24.3	24.0	23.7	23.4
8	28.1	27.7	27.5	27.1	26.8
9	31.6	31.2	30.9	30.4	30.2
″	199	197	195	193	192
1	3.3	3.3	3.2	3.2	3.2
2	6.6	6.6	6.5	6.4	6.4
3	10.0	9.8	9.8	9.6	9.6
4	13.3	13.1	13.0	12.9	12.8
5	16.6	16.4	16.2	16.1	16.0
6	19.9	19.7	19.5	19.3	19.2
7	23.2	23.0	22.8	22.5	22.4
8	26.5	26.3	26.0	25.7	25.6
9	29.8	29.6	29.2	29.0	28.8
″	189	187	185	183	181
1	3.2	3.1	3.1	3.0	3.0
2	6.3	6.2	6.2	6.1	6.0
3	9.4	9.4	9.2	9.2	9.0
4	12.6	12.5	12.3	12.2	12.1
5	15.8	15.6	15.4	15.2	15.1
6	18.9	18.7	18.5	18.3	18.1
7	22.0	21.8	21.6	21.4	21.1
8	25.2	24.9	24.7	24.4	24.1
9	28.4	28.0	27.8	27.4	27.2

Proportional parts

93° (273°) (266°) 86°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

4° (184°) (355°) **175°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	60
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	11.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	50
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	11.12 894	9.99 830	45
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	11.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	35
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	30
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	11.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	11.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	11.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	11.08 043	9.99 851	15
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	11.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	10
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	11.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1
60	8.94 030		8.94 195		11.05 805	9.99 834	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts

″	182	181	179	178	177
1	3.0	3.0	3.0	3.0	3.0
2	6.1	6.0	6.0	5.9	5.9
3	9.1	9.0	9.0	8.9	8.8
4	12.1	12.1	11.9	11.9	11.8
5	15.2	15.1	14.9	14.8	14.8
6	18.2	18.1	17.9	17.8	17.7
7	21.2	21.1	20.9	20.8	20.6
8	24.3	24.1	23.9	23.7	23.6
9	27.3	27.2	26.8	26.7	26.6

″	176	175	174	173	172
1	2.9	2.9	2.9	2.9	2.9
2	5.9	5.8	5.8	5.8	5.7
3	8.8	8.8	8.7	8.6	8.6
4	11.7	11.7	11.6	11.5	11.5
5	14.7	14.6	14.5	14.4	14.3
6	17.6	17.5	17.4	17.3	17.2
7	20.5	20.4	20.3	20.2	20.1
8	23.5	23.3	23.2	23.1	22.9
9	26.4	26.2	26.1	26.0	25.8

″	171	170	169	168	167
1	2.8	2.8	2.8	2.8	2.8
2	5.7	5.7	5.6	5.6	5.6
3	8.6	8.5	8.4	8.4	8.4
4	11.4	11.3	11.3	11.2	11.1
5	14.2	14.2	14.1	14.0	13.9
6	17.1	17.0	16.9	16.8	16.7
7	20.0	19.8	19.7	19.6	19.5
8	22.8	22.7	22.5	22.4	22.3
9	25.6	25.5	25.4	25.2	25.0

″	166	165	164	163	162
1	2.8	2.8	2.7	2.7	2.7
2	5.5	5.5	5.5	5.4	5.4
3	8.3	8.2	8.2	8.2	8.1
4	11.1	11.0	10.9	10.9	10.8
5	13.8	13.8	13.7	13.6	13.5
6	16.6	16.5	16.4	16.3	16.2
7	19.4	19.2	19.1	19.0	18.9
8	22.1	22.0	21.9	21.7	21.6
9	24.9	24.8	24.6	24.4	24.3

″	161	160	159	158	157
1	2.7	2.7	2.6	2.6	2.6
2	5.4	5.3	5.3	5.3	5.2
3	8.0	8.0	8.0	7.9	7.8
4	10.7	10.7	10.6	10.5	10.5
5	13.4	13.3	13.2	13.2	13.1
6	16.1	16.0	15.9	15.8	15.7
7	18.8	18.7	18.6	18.4	18.3
8	21.5	21.3	21.2	21.1	20.9
9	24.2	24.0	23.8	23.7	23.6

″	156	155	154	153	152
1	2.6	2.6	2.6	2.6	2.5
2	5.2	5.2	5.1	5.1	5.1
3	7.8	7.8	7.7	7.6	7.6
4	10.4	10.3	10.3	10.2	10.1
5	13.0	12.9	12.8	12.8	12.7
6	15.6	15.5	15.4	15.3	15.2
7	18.2	18.1	18.0	17.8	17.7
8	20.8	20.7	20.5	20.4	20.3
9	23.4	23.2	23.1	23.0	22.8

Proportional parts

94° (274°) (265°) **85°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

5° (185°) (354°) **174°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	8.94 030	144	8.94 195	145	11.05 805	9.99 834	60
1	8.94 174	143	8.94 340	145	11.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	11.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	11.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	11.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	11.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	11.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	11.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	11.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	11.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	11.04 373	9.99 823	50
11	8.95 589	139	8.95 767	141	11.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	11.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	11.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	11.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	11.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	11.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	11.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	11.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	11.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	11.02 987	9.99 812	40
21	8.96 960	135	8.97 150	135	11.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	11.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	11.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	11.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	11.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	11.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	11.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	11.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	11.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	11.01 642	9.99 800	30
31	8.98 288	131	8.98 490	132	11.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	11.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	11.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	11.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	11.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	11.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	11.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	11.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	11.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	11.00 338	9.99 787	20
41	8.99 577	127	8.99 791	128	11.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	11.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	10.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	10.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	10.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	10.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	10.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	10.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	10.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	10.99 070	9.99 775	10
51	9.00 828	123	9.01 055	124	10.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	10.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	10.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	10.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	10.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	10.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	10.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	10.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	10.97 960	9.99 763	1
60	9.01 923		9.02 162		10.97 838	9.99 761	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts					
″	151	149	148	147	146
1	2.5	2.5	2.5	2.4	2.4
2	5.0	5.0	4.9	4.9	4.9
3	7.6	7.4	7.4	7.4	7.3
4	10.1	9.9	9.9	9.8	9.7
5	12.6	12.4	12.3	12.2	12.2
6	15.1	14.9	14.8	14.7	14.6
7	17.6	17.4	17.3	17.2	17.0
8	20.1	19.9	19.7	19.6	19.5
9	22.6	22.4	22.2	22.0	21.9
″	145	144	143	142	141
1	2.4	2.4	2.4	2.4	2.4
2	4.8	4.8	4.8	4.7	4.7
3	7.2	7.2	7.2	7.1	7.0
4	9.7	9.6	9.5	9.5	9.4
5	12.1	12.0	11.9	11.8	11.8
6	14.5	14.4	14.3	14.2	14.1
7	16.9	16.8	16.7	16.6	16.4
8	19.3	19.2	19.1	18.9	18.8
9	21.8	21.6	21.4	21.3	21.2
″	140	139	138	137	136
1	2.3	2.3	2.3	2.3	2.3
2	4.7	4.6	4.6	4.6	4.5
3	7.0	7.0	6.9	6.8	6.8
4	9.3	9.3	9.2	9.1	9.1
5	11.7	11.6	11.5	11.4	11.3
6	14.0	13.9	13.8	13.7	13.6
7	16.3	16.2	16.1	16.0	15.9
8	18.7	18.5	18.4	18.3	18.1
9	21.0	20.8	20.7	20.6	20.4
″	135	134	133	132	131
1	2.2	2.2	2.2	2.2	2.2
2	4.5	4.5	4.4	4.4	4.4
3	6.8	6.7	6.6	6.6	6.6
4	9.0	8.9	8.9	8.8	8.7
5	11.2	11.2	11.1	11.0	10.9
6	13.5	13.4	13.3	13.2	13.1
7	15.8	15.6	15.5	15.4	15.3
8	18.0	17.9	17.7	17.6	17.5
9	20.2	20.1	20.0	19.8	19.6
″	130	129	128	127	126
1	2.2	2.2	2.1	2.1	2.1
2	4.3	4.3	4.3	4.2	4.2
3	6.5	6.4	6.4	6.4	6.3
4	8.7	8.6	8.5	8.5	8.4
5	10.8	10.8	10.7	10.6	10.5
6	13.0	12.9	12.8	12.7	12.6
7	15.2	15.0	14.9	14.8	14.7
8	17.3	17.2	17.1	16.9	16.8
9	19.5	19.4	19.2	19.0	18.9
″	125	124	123	122	121
1	2.1	2.1	2.0	2.0	2.0
2	4.2	4.1	4.1	4.1	4.0
3	6.2	6.2	6.2	6.1	6.0
4	8.3	8.3	8.2	8.1	8.1
5	10.4	10.3	10.2	10.2	10.1
6	12.5	12.4	12.3	12.2	12.1
7	14.6	14.5	14.4	14.2	14.1
8	16.7	16.5	16.4	16.3	16.1
9	18.8	18.6	18.4	18.3	18.2

Proportional parts

95° (275°) (264°) **84°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

6° (186°) (353°) **173°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	9.01 923	120	9.02 162	121	10.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	10.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	10.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	10.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	10.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	10.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	10.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	10.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	10.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	10.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	10.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	10.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	10.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	10.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	10.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	10.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	10.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	10.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	10.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	10.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	10.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	10.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	10.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	10.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	10.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	10.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	10.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	10.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	10.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	10.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	10.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	10.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	10.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	10.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	10.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	10.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	10.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	10.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	10.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	10.93 334	9.99 707	21
40	9.06 481	103	9.06 775	110	10.93 225	9.99 705	20
41	9.06 589	107	9.06 885	109	10.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	10.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	10.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	10.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	10.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	10.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	10.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	10.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	10.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	10.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	10.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	10.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	10.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	10.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	10.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	10.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	10.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	10.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	10.91 190	9.99 677	1
60	9.08 589		9.08 914		10.91 086	9.99 675	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts				
″	121	120	119	118
1	2.0	2.0	2.0	2.0
2	4.0	4.0	4.0	3.9
3	6.0	6.0	6.0	5.9
4	8.1	8.0	7.9	7.9
5	10.1	10.0	9.9	9.8
6	12.1	12.0	11.9	11.8
7	14.1	14.0	13.9	13.8
8	16.1	16.0	15.9	15.7
9	18.2	18.0	17.8	17.7
10	20.2	20.0	19.8	19.7
20	40.3	40.0	39.7	39.3
30	60.5	60.0	59.5	59.0
40	80.7	80.0	79.3	78.7
50	100.8	100.0	99.2	98.3

″	117	116	115	114
1	2.0	1.9	1.9	1.9
2	3.9	3.9	3.8	3.8
3	5.8	5.8	5.8	5.7
4	7.8	7.7	7.7	7.6
5	9.8	9.7	9.6	9.5
6	11.7	11.6	11.5	11.4
7	13.6	13.5	13.4	13.3
8	15.6	15.5	15.3	15.2
9	17.6	17.4	17.2	17.1
10	19.5	19.3	19.2	19.0
20	39.0	38.7	38.3	38.0
30	58.5	58.0	57.5	57.0
40	78.0	77.3	76.7	76.0
50	97.5	96.7	95.8	95.0

″	113	112	111	110
1	1.9	1.9	1.8	1.8
2	3.8	3.7	3.7	3.7
3	5.6	5.6	5.6	5.5
4	7.5	7.5	7.4	7.3
5	9.4	9.3	9.2	9.2
6	11.3	11.2	11.1	11.0
7	13.2	13.1	13.0	12.8
8	15.1	14.9	14.8	14.7
9	17.0	16.8	16.6	16.5
10	18.8	18.7	18.5	18.3
20	37.7	37.3	37.0	36.7
30	56.5	56.0	55.5	55.0
40	75.3	74.7	74.0	73.3
50	94.2	93.3	92.5	91.7

″	109	108	107	106
1	1.8	1.8	1.8	1.8
2	3.6	3.6	3.6	3.5
3	5.4	5.4	5.4	5.3
4	7.3	7.2	7.1	7.1
5	9.1	9.0	8.9	8.8
6	10.9	10.8	10.7	10.6
7	12.7	12.6	12.5	12.4
8	14.5	14.4	14.3	14.1
9	16.4	16.2	16.0	15.9
10	18.2	18.0	17.8	17.7
20	36.3	36.0	35.7	35.3
30	54.5	54.0	53.5	53.0
40	72.7	72.0	71.3	70.7
50	90.8	90.0	89.2	88.3

Proportional parts

96° (276°) (263°) **83°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

7° (187°) (352°) 172°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	9.08 589	103	9.08 914	105	10.91 086	9.99 675	60
1	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	10.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	10.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	50
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	10.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	10.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	10.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	10.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	10.89 044	9.99 643	40
21	9.10 697	98	9.11 056	99	10.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	10.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	10.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	10.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	10.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	10.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	10.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	10.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	10.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	10.88 057	9.99 627	30
31	9.11 666	95	9.12 040	98	10.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	10.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	10.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	10.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	10.87 572	9.99 618	25
36	9.12 142	94	9.12 525	96	10.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	10.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	10.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	10.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	10.87 091	9.99 610	20
41	9.12 612	94	9.13 004	95	10.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	10.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	10.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	10.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	10.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	10.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	10.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	10.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	10.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	10.86 146	9.99 593	10
51	9.13 539	91	9.13 948	93	10.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	10.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	10.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	10.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	10.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	10.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	10.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	10.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	10.85 312	9.99 577	1
60	9.14 356		9.14 780		10.85 220	9.99 575	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts				
′′	105	104	103	102
1	1.8	1.7	1.7	1.7
2	3.5	3.5	3.4	3.4
3	5.2	5.2	5.2	5.1
4	7.0	6.9	6.9	6.8
5	8.8	8.7	8.6	8.5
6	10.5	10.4	10.3	10.2
7	12.2	12.1	12.0	11.9
8	14.0	13.9	13.7	13.6
9	15.8	15.6	15.4	15.3
10	17.5	17.3	17.2	17.0
20	35.0	34.7	34.3	34.0
30	52.5	52.0	51.5	51.0
40	70.0	69.3	68.7	68.0
50	87.5	86.7	85.8	85.0

′′	101	100	99	98
1	1.7	1.7	1.6	1.6
2	3.4	3.3	3.3	3.3
3	5.0	5.0	5.0	4.9
4	6.7	6.7	6.6	6.5
5	8.4	8.3	8.2	8.2
6	10.1	10.0	9.9	9.8
7	11.8	11.7	11.6	11.4
8	13.5	13.3	13.2	13.1
9	15.2	15.0	14.8	14.7
10	16.8	16.7	16.5	16.3
20	33.7	33.3	33.0	32.7
30	50.5	50.0	49.5	49.0
40	67.3	66.7	66.0	65.3
50	84.2	83.3	82.5	81.7

′′	97	96	95	94
1	1.6	1.6	1.6	1.6
2	3.2	3.2	3.2	3.1
3	4.8	4.8	4.8	4.7
4	6.5	6.4	6.3	6.3
5	8.1	8.0	7.9	7.8
6	9.7	9.6	9.5	9.4
7	11.3	11.2	11.1	11.0
8	12.9	12.8	12.7	12.5
9	14.6	14.4	14.2	14.1
10	16.2	16.0	15.8	15.7
20	32.3	32.0	31.7	31.3
30	48.5	48.0	47.5	47.0
40	64.7	64.0	63.3	62.7
50	80.8	80.0	79.2	78.3

′′	93	92	91	90
1	1.6	1.5	1.5	1.5
2	3.1	3.1	3.0	3.0
3	4.6	4.6	4.6	4.5
4	6.2	6.1	6.1	6.0
5	7.8	7.7	7.6	7.5
6	9.3	9.2	9.1	9.0
7	10.8	10.7	10.6	10.5
8	12.4	12.3	12.1	12.0
9	14.0	13.8	13.6	13.5
10	15.5	15.3	15.2	15.0
20	31.0	30.7	30.3	30.0
30	46.5	36.0	45.5	45.0
40	62.0	61.3	60.7	60.0
50	77.5	76.7	75.8	75.0

Proportional parts

97° (277°) (262°) 82°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

8° (188°) (351°) 171°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	9.14 356	89	9.14 780	92	10.85 220	9.99 575	60
1	9.14 455	90	9.14 872	91	10.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	10.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	10.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	10.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	10.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	10.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	10.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	10.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	10.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	10.84 312	9.99 557	50
11	9.15 333	88	9.15 777	90	10.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	10.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	10.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	10.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	10.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	10.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	10.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	10.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	30
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	10.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	10.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	10.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	10.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	20
41	9.17 890	83	9.18 391	84	10.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	10.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	10.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	10.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	10.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	10.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	10.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	10.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	10.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	10.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	10.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	10.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	10.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	10.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	10.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	10.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	10.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	10.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	10.80 111	9.99 464	1
60	9.19 433		9.19 971		10.80 029	9.99 462	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Proportional parts			
′′	92	91	90
1	1.5	1.5	1.5
2	3.1	3.0	3.0
3	4.6	4.6	4.5
4	6.1	6.1	6.0
5	7.7	7.6	7.5
6	9.2	9.1	9.0
7	10.7	10.6	10.5
8	12.3	12.1	12.0
9	13.8	13.6	13.5
10	15.3	15.2	15.0
20	30.7	30.3	30.0
30	46.0	45.5	45.0
40	61.3	60.7	60.0
50	76.7	75.8	75.0
′′	89	88	87
1	1.5	1.5	1.4
2	3.0	2.9	2.9
3	4.4	4.4	4.4
4	5.9	5.9	5.8
5	7.4	7.3	7.2
6	8.9	8.8	8.7
7	10.4	10.3	10.2
8	11.9	11.7	11.6
9	13.4	13.2	13.0
10	14.8	14.7	14.5
20	29.7	29.3	29.0
30	44.5	44.0	43.5
40	59.3	58.7	58.0
50	74.2	73.3	72.5
′′	86	85	84
1	1.4	1.4	1.4
2	2.9	2.8	2.8
3	4.3	4.2	4.2
4	5.7	5.7	5.6
5	7.2	7.1	7.0
6	8.6	8.5	8.4
7	10.0	9.9	9.8
8	11.5	11.3	11.2
9	12.9	12.8	12.6
10	14.3*	14.2	14.0
20	28.7	28.3	28.0
30	43.0	42.5	42.0
40	57.3	56.7	56.0
50	71.7	70.8	70.0
′′	83	82	81
1	1.4	1.4	1.4
2	2.8	2.7	2.7
3	4.2	4.1	4.0
4	5.5	5.5	5.4
5	6.9	6.8	6.8
6	8.3	8.2	8.1
7	9.7	9.6	9.4
8	11.1	10.9	10.8
9	12.4	12.3	12.2
10	13.8	13.7	13.5
20	27.7	27.3	27.0
30	41.5	41.0	40.5
40	55.3	54.7	54.0
50	69.2	68.3	67.5

Proportional parts

98° (278°) (261°) 81°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

9° (189°) (350°) **170°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	9.19 433	80	9.19 971	82	10.80 029	9.99 462	60
1	9.19 513	79	9.20 053	81	10.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	10.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	10.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	10.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	10.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	10.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	10.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	10.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	10.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	10.79 218	9.99 442	50
11	9.20 302	78	9.20 862	80	10.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	10.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	10.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	10.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	10.78 818	9.99 432	45
16	9.20 691	77	9.21 261	80	10.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	10.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	10.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	10.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	10.78 422	9.99 421	40
21	9.21 076	77	9.21 657	79	10.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	10.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	10.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	10.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	10.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	10.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	10.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	10.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	10.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	10.77 639	9.99 400	30
31	9.21 836	76	9.22 438	78	10.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	10.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	10.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	10.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	10.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	10.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	10.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	10.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	10.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	10.76 870	9.99 379	20
41	9.22 583	74	9.23 206	77	10.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	10.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	10.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	10.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	10.76 490	9.99 368	15
46	9.22 952	73	9.23 586	75	10.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	10.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	10.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	10.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	10.76 113	9.99 357	10
51	9.23 317	73	9.23 962	75	10.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	10.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	10.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	10.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	10.75 739	9.99 346	5
56	9.23 679	73	9.24 335	75	10.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	10.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	10.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	10.75 442	9.99 337	1
60	9.23 967		9.24 632		10.75 368	9.99 335	0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

99° (279°) (260°) **80°**

Proportional parts

″	80	79	78	77
1	1.3	1.3	1.3	1.3
2	2.7	2.6	2.6	2.6
3	4.0	4.0	3.9	3.8
4	5.3	5.3	5.2	5.1
5	6.7	6.6	6.5	6.4
6	8.0	7.9	7.8	7.7
7	9.3	9.2	9.1	9.0
8	10.7	10.5	10.4	10.3
9	12.0	11.8	11.7	11.6
10	13.3	13.2	13.0	12.8
20	26.7	26.3	26.0	25.7
30	40.0	39.5	39.0	38.5
40	53.3	52.7	52.0	51.3
50	66.7	65.8	65.0	64.2

″	76	75	74	73
1	1.3	1.2	1.2	1.2
2	2.5	2.5	2.5	2.4
3	3.8	3.8	3.7	3.6
4	5.1	5.0	4.9	4.9
5	6.3	6.2	6.2	6.1
6	7.6	7.5	7.4	7.3
7	8.9	8.8	8.6	8.5
8	10.1	10.0	9.9	9.7
9	11.4	11.2	11.1	11.0
10	12.7	12.5	12.3	12.2
20	25.3	25.0	24.7	24.3
30	38.0	37.5	37.0	36.5
40	50.7	50.0	49.3	48.7
50	63.3	62.5	61.7	60.8

″	72	71	3	2
1	1.2	1.2	0.0	0.0
2	2.4	2.4	0.1	0.1
3	3.6	3.6	0.2	0.1
4	4.8	4.7	0.2	0.1
5	6.0	5.9	0.2	0.2
6	7.2	7.1	0.3	0.2
7	8.4	8.3	0.4	0.2
8	9.6	9.5	0.4	0.3
9	10.8	10.6	0.4	0.3
10	12.0	11.8	0.5	0.3
20	24.0	23.7	1.0	0.7
30	36.0	35.5	1.5	1.0
40	48.0	47.3	2.0	1.3
50	60.0	59.2	2.5	1.7

Proportional parts

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

10° (190°) (349°) **169°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.23 967	72	9.24 632	74	10.75 368	9.99 335	2	60
1	9.24 039	71	9.24 706	73	10.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	10.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	10.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	10.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	10.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	10.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	10.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	10.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	10.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	10.74 635	9.99 313	3	50
11	9.24 748	70	9.25 437	73	10.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	10.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	10.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	10.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	10.74 273	9.99 301	2	45
16	9.25 098	70	9.25 799	72	10.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	10.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	10.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	10.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	10.73 914	9.99 290	2	40
21	9.25 445	69	9.26 158	71	10.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	10.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	10.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	10.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	10.73 557	9.99 278	2	35
26	9.25 790	68	9.26 514	71	10.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	10.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	10.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	10.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	10.73 203	9.99 267	3	30
31	9.26 131	68	9.26 867	70	10.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	10.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	10.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	10.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	10.72 852	9.99 255	3	25
36	9.26 470	68	9.27 218	70	10.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	10.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	10.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	10.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	10.72 504	9.99 243	2	20
41	9.26 806	67	9.27 566	69	10.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	10.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	10.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	10.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	10.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	10.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	10.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	10.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	10.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	10.71 814	9.99 219	2	10
51	9.27 471	66	9.28 254	69	10.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	10.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	10.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	10.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	10.71 473	9.99 207	3	5
56	9.27 799	65	9.28 595	67	10.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	10.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	10.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	10.71 202	9.99 197	2	1
60	9.28 060		9.28 865		10.71 135	9.99 195		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″			
	74	73	72
1	1 2	1.2	1.2
2	2.5	2.4	2.4
3	3.7	3.6	3.6
4	4.9	4.9	4.8
5	6.2	6.1	6.0
6	7.4	7.3	7.2
7	8.6	8.5	8.4
8	9.9	9.7	9.6
9	11.1	11.0	10.8
10	12.3	12.2	12.0
20	24.7	24.3	24.0
30	37.0	36.5	36.0
40	49.3	48.7	48.0
50	61.7	60.8	60.0

″			
	71	70	69
1	1.2	1.2	1.2
2	2.4	2.3	2.3
3	3.6	3.5	3.4
4	4.7	4.7	4.6
5	5.9	5.8	5.8
6	7.1	7.0	6.9
7	8.3	8.2	8.0
8	9.5	9.3	9.2
9	10.6	10.5	10.4
10	11.8	11.7	11.5
20	23.7	23.3	23.0
30	35.5	35.0	34.5
40	47.3	46.7	46.0
50	59.2	58.3	57.5

″			
	68	67	66
1	1.1	1.1	1.1
2	2.3	2.2	2.2
3	3.4	3.4	3.3
4	4.5	4.5	4.4
5	5.7	5.6	5.5
6	6.8	6.7	6.6
7	7.9	7.8	7.7
8	9.1	8.9	8.8
9	10.2	10.0	9.9
10	11.3	11.2	11.0
20	22.7	22.3	22.0
30	34.0	33.5	33.0
40	45.3	44.7	44.0
50	56.7	55.8	55.0

Proportional parts

100° (280°) (259°) **79°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

11° (191°) (348°) **168°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.28 060	65	9.28 865	68	10.71 135	9.99 195	3	60
1	9.28 125	65	9.28 933	67	10.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	10.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	10.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	10.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	10.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	10.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	10.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	10.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	10.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	10.70 465	9.99 170	3	50
11	9.28 769	64	9.29 601	67	10.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	10.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	10.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	10.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	10.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	10.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	10.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	10.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	10.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	10.69 805	9.99 145	3	40
21	9.29 403	63	9.30 261	65	10.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	10.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	10.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	10.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	10.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	10.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	10.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	10.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	10.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	10.69 154	9.99 119	2	30
31	9.30 028	62	9.30 911	64	10.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	10.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	10.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	10.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	10.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	10.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	10.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	10.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	10.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	10.68 511	9.99 093	2	20
41	9.30 643	61	9.31 552	64	10.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	10.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	10.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	10.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	10.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	10.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	10.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	10.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	10.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	10.67 878	9.99 067	3	10
51	9.31 250	60	9.32 185	63	10.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	10.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	10.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	10.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	10.67 564	9.99 054	3	5
56	9.31 549	60	9.32 498	63	10.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	10.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	10.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	10.67 315	9.99 043	3	1
60	9.31 788		9.32 747		10.67 253	9.99 040		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts			
″	65	64	63
1	1.1	1.1	1.0
2	2.2	2.1	2.1
3	3.2	3.2	3.2
4	4.3	4.3	4.2
5	5.4	5.3	5.2
6	6.5	6.4	6.3
7	7.6	7.5	7.4
8	8.7	8.5	8.4
9	9.8	9.6	9.4
10	10.8	10.7	10.5
20	21.7	21.3	21.0
30	32.5	32.0	31.5
40	43.3	42.7	42.0
50	54.2	53.3	52.5

″	62	61	60
1	1.0	1.0	1.0
2	2.1	2.0	2.0
3	3.1	3.0	3.0
4	4.1	4.1	4.0
5	5.2	5.1	5.0
6	6.2	6.1	6.0
7	7.2	7.1	7.0
8	8.3	8.1	8.0
9	9.3	9.2	9.0
10	10.3	10.2	10.0
20	20.7	20.3	20.0
30	31.0	30.5	30.0
40	41.3	40.7	40.0
50	51.7	50.8	50.0

″	59	3	2
1	1.0	0.0	0.0
2	2.0	0.1	0.1
3	3.0	0.2	0.1
4	3.9	0.2	0.1
5	4.9	0.2	0.2
6	5.9	0.3	0.2
7	6.9	0.4	0.2
8	7.9	0.4	0.3
9	8.8	0.4	0.3
10	9.8	0.5	0.3
20	19.7	1.0	0.7
30	29.5	1.5	1.0
40	39.3	2.0	1.3
50	49.2	2.5	1.7

Proportional parts

101° (281°) (258°) **78°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

12° (192°) (347°) 167°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.31 788	59	9.32 747	63	10.67 253	9.99 040	2	60
1	9.31 847	60	9.32 810	62	10.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	10.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	10.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	10.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	10.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	61	10.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	10.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	10.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	10.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	10.66 635	9.99 013	2	50
11	9.32 437	58	9.33 426	61	10.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	10.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	10.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	10.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	10.66 330	9.99 000	3	45
16	9.32 728	58	9.33 731	61	10.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	10.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	10.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	10.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	10.66 026	9.98 986	3	40
21	9.33 018	57	9.34 034	61	10.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	10.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	10.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	10.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	10.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	10.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	10.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	10.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	10.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	10.65 424	9.98 958	3	30
31	9.33 591	56	9.34 635	60	10.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	10.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	10.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	10.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	10.65 126	9.98 944	3	25
36	9.33 874	57	9.34 933	59	10.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	10.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	10.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	10.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	10.64 830	9.98 930	3	20
41	9.34 156	56	9.35 229	59	10.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	10.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	10.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	10.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	10.64 536	9.98 916	3	15
46	9.34 436	55	9.35 523	58	10.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	10.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	10.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	10.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	10.64 243	9.98 901	3	10
51	9.34 713	56	9.35 815	58	10.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	10.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	10.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	10.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	10.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	10.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	10.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	10.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	10.63 721	9.98 875	3	1
60	9.35 209		9.36 336		10.63 664	9.98 872		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

//	63	62	61
1	1.0	1.0	1.0
2	2.1	2.1	2.0
3	3.2	3.1	3.0
4	4.2	4.1	4.1
5	5.2	5.2	5.1
6	6.3	6.2	6.1
7	7.4	7.2	7.1
8	8.4	8.3	8.1
9	9.4	9.3	9.2
10	10.5	10.3	10.2
20	21.0	20.7	20.3
30	31.5	31.0	30.5
40	42.0	41.3	40.7
50	52.5	51.7	50.8

//	60	59	58
1	1.0	1.0	1.0
2	2.0	2.0	1.9
3	3.0	3.0	2.9
4	4.0	3.9	3.9
5	5.0	4.9	4.8
6	6.0	5.9	5.8
7	7.0	6.9	6.8
8	8.0	7.9	7.7
9	9.0	8.8	8.7
10	10.0	9.8	9.7
20	20.0	19.7	19.3
30	30.0	29.5	29.0
40	40.0	39.3	38.7
50	50.0	49.2	48.3

//	57	56	55
1	1.0	0.9	0.9
2	1.9	1.9	1.8
3	2.8	2.8	2.8
4	3.8	3.7	3.7
5	4.8	4.7	4.6
6	5.7	5.6	5.5
7	6.6	6.5	6.4
8	7.6	7.5	7.3
9	8.6	8.4	8.2
10	9.5	9.3	9.2
20	19.0	18.7	18.3
30	28.5	28.0	27.5
40	38.0	37.3	36.7
50	47.5	46.7	45.8

Proportional parts

102° (282°) (257°) 77°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

13° (193°) (346°) **166°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.35 209	54	9.36 336	58	10.63 664	9.98 872	3	60
1	9.35 263	55	9.36 394	58	10.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	10.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	10.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	10.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	10.63 376	9.98 858	3	55
6	9.35 536	54	9.36 681	57	10.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	10.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	10.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	10.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	10.63 091	9.98 843	3	50
11	9.35 806	54	9.36 966	57	10.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	10.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	10.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	10.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	10.62 807	9.98 828	3	45
16	9.36 075	54	9.37 250	56	10.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	10.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	10.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	10.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	10.62 524	9.98 813	3	40
21	9.36 342	53	9.37 532	56	10.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	10.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	10.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	10.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	10.62 244	9.98 798	3	35
26	9.36 608	52	9.37 812	56	10.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	10.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	10.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	10.62 020	9.98 786	3	31
30	9.36 819	52	9.38 035	56	10.61 965	9.98 783	3	30
31	9.36 871	53	9.38 091	56	10.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	10.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	10.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	10.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	10.61 687	9.98 768	3	25
36	9.37 133	52	9.38 368	55	10.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	10.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	10.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	10.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	10.61 411	9.98 753	3	20
41	9.37 393	52	9.38 644	55	10.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	10.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	10.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	10.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	10.61 137	9.98 737	3	15
46	9.37 652	51	9.38 918	54	10.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	10.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	10.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	10.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	10.60 864	9.98 722	3	10
51	9.37 909	51	9.39 190	55	10.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	10.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	10.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	10.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	10.60 593	9.98 706	3	5
56	9.38 164	51	9.39 461	54	10.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	10.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	10.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	10.60 377	9.98 694	4	1
60	9.38 368		9.39 677		10.60 323	9.98 690		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	57	56	55
1	1.0	0.9	0.9
2	1.9	1.9	1.8
3	2.8	2.8	2.8
4	3.8	3.7	3.7
5	4.8	4.7	4.6
6	5.7	5.6	5.5
7	6.6	6.5	6.4
8	7.6	7.5	7.3
9	8.6	8.4	8.2
10	9.5	9.3	9.2
20	19.0	18.7	18.3
30	28.5	28.0	27.5
40	38.0	37.3	36.7
50	47.5	46.7	45.8

″	54	53	52
1	0.9	0.9	0.9
2	1.8	1.8	1.7
3	2.7	2.6	2.6
4	3.6	3.5	3.5
5	4.5	4.4	4.3
6	5.4	5.3	5.2
7	6.3	6.2	6.1
8	7.2	7.1	6.9
9	8.1	8.0	7.8
10	9.0	8.8	8.7
20	18.0	17.7	17.3
30	27.0	26.5	26.0
40	36.0	35.3	34.7
50	45.0	44.2	43.3

″	51	4	3	2
1	0.8	0.1	0.0	0.0
2	1.7	0.1	0.1	0.1
3	2.6	0.2	0.2	0.1
4	3.4	0.3	0.2	0.1
5	4.2	0.3	0.2	0.2
6	5.1	0.4	0.3	0.2
7	6.0	0.5	0.4	0.2
8	6.8	0.5	0.4	0.3
9	7.6	0.6	0.4	0.3
10	8.5	0.7	0.5	0.3
20	17.0	1.3	1.0	0.7
30	25.5	2.0	1.5	1.0
40	34.0	2.7	2.0	1.3
50	42.5	3.3	2.5	1.7

103° (283°) (256°) **76°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

14° (194°) (345°) 165°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.38 368	50	9.39 677	54	10.60 323	9.98 690	3	60
1	9.38 418	51	9.39 731	54	10.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	53	10.60 215	9.98 684	3	58
3	9.38 519	51	9.39 838	54	10.60 162	9.98 681	3	57
4	9.38 570	50	9.39 892	53	10.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	54	10.60 055	9.98 675	4	55
6	9.38 670	51	9.39 999	53	10.60 001	9.98 671	3	54
7	9.38 721	50	9.40 052	54	10.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	10.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	10.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	54	10.59 788	9.98 659	3	50
11	9.38 921	50	9.40 266	53	10.59 734	9.98 656	4	49
12	9.38 971	50	9.40 319	53	10.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	10.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	10.59 575	9.98 646	3	46
15	9.39 121	49	9.40 478	53	10.59 522	9.98 643	3	45
16	9.39 170	50	9.40 531	53	10.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	52	10.59 416	9.98 636	3	43
18	9.39 270	49	9.40 636	53	10.59 364	9.98 633	3	42
19	9.39 319	50	9.40 689	53	10.59 311	9.98 630	3	41
20	9.39 369	49	9.40 742	53	10.59 258	9.98 627	4	40
21	9.39 418	49	9.40 795	52	10.59 205	9.98 623	3	39
22	9.39 467	50	9.40 847	53	10.59 153	9.98 620	3	38
23	9.39 517	49	9.40 900	52	10.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	53	10.59 048	9.98 614	4	36
25	9.39 615	49	9.41 005	52	10.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	53	10.58 839	9.98 601	4	32
29	9.39 811	49	9.41 214	52	10.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	3	30
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	10.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	10.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	10.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	10.58 474	9.98 578	4	25
36	9.40 152	48	9.41 578	51	10.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	10.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	10.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	10.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	10.58 216	9.98 561	3	20
41	9.40 394	48	9.41 836	51	10.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	10.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	10.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	10.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	10.57 959	9.98 545	4	15
46	9.40 634	48	9.42 093	51	10.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	10.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	10.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	10.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	10.57 703	9.98 528	3	10
51	9.40 873	48	9.42 348	51	10.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	10.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	10.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	10.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	10.57 448	9.98 511	3	5
56	9.41 111	47	9.42 603	50	10.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	10.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	10.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	10.57 245	9.98 498	4	1
60	9.41 300		9.42 805		10.57 195	9.98 494		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

104° (284°) (255°) 75°

Proportional parts

″	54	53	52
1	0.9	0.9	0.9
2	1.8	1.8	1.7
3	2.7	2.6	2.6
4	3.6	3.5	3.5
5	4.5	4.4	4.3
6	5.4	5.3	5.2
7	6.3	6.2	6.1
8	7.2	7.1	6.9
9	8.1	8.0	7.8
10	9.0	8.8	8.7
20	18.0	17.7	17.3
30	27.0	26.5	26.0
40	36.0	35.3	34.7
50	45.0	44.2	43.3

″	51	50	49
1	0.8	0.8	0.8
2	1.7	1.7	1.6
3	2.6	2.5	2.4
4	3.4	3.3	3.3
5	4.2	4.2	4.1
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.6	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

″	48	47	4	3
1	0.8	0.8	0.1	0.0
2	1.6	1.6	0.1	0.1
3	2.4	2.4	0.2	0.2
4	3.2	3.1	0.3	0.2
5	4.0	3.9	0.3	0.2
6	4.8	4.7	0.4	0.3
7	5.6	5.5	0.5	0.4
8	6.4	6.3	0.5	0.4
9	7.2	7.0	0.6	0.4
10	8.0	7.8	0.7	0.5
20	16.0	15.7	1.3	1.0
30	24.0	23.5	2.0	1.5
40	32.0	31.3	2.7	2.0
50	40.0	39.2	3.3	2.5

Proportional parts

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

15° (195°) (344°) 164°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	3	60
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	3	55
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	3	50
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	4	46
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	50	10.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	10.56 194	9.98 426	4	40
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	4	35
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	3	30
31	9.42 735	46	9.44 348	49	10.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	10.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	3	25
36	9.42 962	46	9.44 592	49	10.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	4	20
41	9.43 188	45	9.44 836	48	10.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	10.55 067	9.99 345	3	17
44	9.43 323	44	9.44 981	48	10.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	10.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	3	10
51	9.43 635	45	9.45 319	48	10.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	10.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	10.54 489	9.98 302	3	5
56	9.43 857	44	9.45 559	47	10.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	10.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	10.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	10.54 298	9.98 288	4	1
60	9.44 034		9.45 750		10.54 250	9.98 284		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	51	50	49
1	0.8	0.8	0.8
2	1.7	1.7	1.6
3	2.6	2.5	2.4
4	3.4	3.3	3.3
5	4.2	4.2	4.1
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.6	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

″	48	47	46
1	0.8	0.8	0.8
2	1.6	1.6	1.5
3	2.4	2.4	2.3
4	3.2	3.1	3.1
5	4.0	3.9	3.8
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

″	45	44	4	3
1	0.8	0.7	0.1	0.0
2	1.5	1.5	0.1	0.1
3	2.2	2.2	0.2	0.2
4	3.0	2.9	0.3	0.2
5	3.8	3.7	0.3	0.2
6	4.5	4.4	0.4	0.3
7	5.2	5.1	0.5	0.4
8	6.0	5.9	0.5	0.4
9	6.8	6.6	0.6	0.4
10	7.5	7.3	0.7	0.5
20	15.0	14.7	1.3	1.0
30	22.5	22.0	2.0	1.5
40	30.0	29.3	2.7	2.0
50	37.5	36.7	3.3	2.5

Proportional parts

105° (285°) (254°) 74°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

16° (196°) (343°) **163°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.44 034	44	9.45 750	47	10.54 250	9.98 284	3	60
1	9.44 078	44	9.45 797	48	10.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	10.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	10.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	10.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	10.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	47	10.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	10.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	10.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	10.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	10.53 776	9.98 248	4	50
11	9.44 516	43	9.46 271	48	10.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	10.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	10.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	10.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	10.53 540	9.98 229	3	45
16	9.44 733	43	9.46 507	47	10.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	10.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	10.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	10.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	10.53 306	9.98 211	4	40
21	9.44 948	44	9.46 741	47	10.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	10.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	10.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	10.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	10.53 072	9.98 192	3	35
26	9.45 163	43	9.46 975	46	10.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	10.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	4	30
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	10.52 608	9.98 155	4	25
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	10.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	4	20
41	9.45 801	42	9.47 668	46	10.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	10.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	10.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	10.51 920	9.98 098	4	10
51	9.46 220	42	9.48 126	45	10.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	10.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	10.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	10.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	10.51 693	9.98 079	4	5
56	9.46 428	41	9.48 353	45	10.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	10.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	10.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	10.51 511	9.98 063	3	1
60	9 46 594		9.48 534		10.51 466	9.98 060		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	48	47	46
1	0.8	0.8	0.8
2	1.6	1.6	1.5
3	2.4	2.4	2.3
4	3.2	3.1	3.1
5	4.0	3.9	3.8
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

″	45	44	43
1	0.8	0.7	0.7
2	1.5	1.5	1.4
3	2.2	2.2	2.2
4	3.0	2.9	2.9
5	3.8	3.7	3.6
6	4.5	4.4	4.3
7	5.2	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.3	28.7
50	37.5	36.7	35.8

″	42	41	4	3
1	0.7	0.7	0.1	0.0
2	1.4	1.4	0.1	0.1
3	2.1	2.0	0.2	0.2
4	2.8	2.7	0.3	0.2
5	3.5	3.4	0.3	0.2
6	4.2	4.1	0.4	0.3
7	4.9	4.8	0.5	0.4
8	5.6	5.5	0.5	0.4
9	6.3	6.2	0.6	0.4
10	7.0	6.8	0.7	0.5
20	14.0	13.7	1.3	1.0
30	21.0	20.5	2.0	1.5
40	28.0	27.3	2.7	2.0
50	35.0	34.2	3.3	2.5

Proportional parts

106° (286°) (253°) **73°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

17° (197°) (342°) **162°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.46 594	41	9.48 534	45	10.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	10.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	10.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	10.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	10.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	10.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	10.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	10.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	10.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	10.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	10.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	10.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	10.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	10.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	10.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	10.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	10.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	10.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	10.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	10.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	10.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	10.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	10.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	10.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	10.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	10.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	10.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	10.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	10.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	10.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	10.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	10.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	10.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	10.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	10.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	10.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	10.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	10.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	10.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	10.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	10.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	10.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	10.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	10.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	10.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	10.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	10.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	10.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	10.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	10.49 297	9.97 866	6	11
50	9.48 607	40	9.50 746	43	10.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	10.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	10.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	10.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	10.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	10.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	10.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	10.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	10.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	10.48 865	9.97 825	4	1
60	9.48 998		9.51 178		10.48 822	9.97 821		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	45	44	43
1	0.8	0.7	0.7
2	1.5	1.5	1.4
3	2.2	2.2	2.2
4	3.0	2.9	2.9
5	3.8	3.7	3.6
6	4.5	4.4	4.3
7	5.2	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.2	28.7
50	37.5	36.7	35.8

″	42	41	40
1	0.7	0.7	0.7
2	1.4	1.4	1.3
3	2.1	2.0	2.0
4	2.8	2.7	2.7
5	3.5	3.4	3.3
6	4.2	4.1	4.0
7	4.9	4.8	4.7
8	5.6	5.5	5.3
9	6.3	6.2	6.0
10	7.0	6.8	6.7
20	14.0	13.7	13.3
30	21.0	20.5	20.0
40	28.0	27.3	26.7
50	35.0	34.2	33.3

″	39	5	4	3
1	0.6	0.1	0.1	0.0
2	1.3	0.2	0.1	0.1
3	2.0	0.2	0.2	0.2
4	2.6	0.3	0.3	0.2
5	3.2	0.4	0.3	0.2
6	3.9	0.5	0.4	0.3
7	4.6	0.6	0.5	0.4
8	5.2	0.7	0.5	0.5
9	5.8	0.8	0.6	0.4
10	6.5	0.8	0.7	0.5
20	13.0	1.7	1.3	1.0
30	19.5	2.5	2.0	1.5
40	26.0	3.3	2.7	2.0
20	32.5	4.2	3.3	2.5

Proportional parts

107° (287°) (252°) **72°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

18° (198°) (341°) **161°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.48 998	39	9.51 178	43	10.48 822	9.97 821	4	60
1	9.49 037	39	9.51 221	43	10.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	10.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	10.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	10.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	10.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	43	10.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	10.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	10.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	10.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	10.48 394	9.97 779	4	50
11	9.49 424	38	9.51 648	43	10.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	10.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	10.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	10.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	10.48 181	9.97 759	5	45
16	9.49 615	39	9.51 861	42	10.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	10.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	10.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	10.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	10.47 969	9.97 738	4	40
21	9.49 806	38	9.52 073	42	10.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	10.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	10.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	10.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	10.47 758	9.97 717	4	35
26	9.49 996	38	9.52 284	42	10.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	10.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	10.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	10.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	10.47 548	9.97 696	5	30
31	9.50 185	38	9.52 494	42	10.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	10.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	10.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	10.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	10.47 339	9.97 674	4	25
36	9.50 374	37	9.52 703	42	10.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	10.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	10.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	10.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	10.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	41	10.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	10.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	10.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	10.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	10.46 922	9.97 632	4	15
46	9.50 747	37	9.53 120	41	10.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	10.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	10.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	10.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	10.46 715	9.97 610	4	10
51	9.50 933	37	9.53 327	41	10.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	10.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	10.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	10.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	10.46 508	9.97 589	5	5
56	9.51 117	37	9.53 533	41	10.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	10.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	10.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	10.46 344	9.97 571	4	1
60	9.51 264		9.53 697		10.46 303	9.97 567		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	43	42	41
1	0.7	0.7	0.7
2	1.4	1.4	1.4
3	2.2	2.1	2.0
4	2.9	2.8	2.7
5	3.6	3.5	3.4
6	4.3	4.2	4.1
7	5.0	4.9	4.8
8	5.7	5.6	5.5
9	6.4	6.3	6.2
10	7.2	7.0	6.8
20	14.3	14.0	13.7
30	21.5	21.0	20.5
40	28.7	28.0	27.3
50	35.8	35.0	34.2

″	39	38	37
1	0.6	0.6	0.6
2	1.3	1.3	1.2
3	2.0	1.9	1.8
4	2.6	2.5	2.5
5	3.2	3.2	3.1
6	3.9	3.8	3.7
7	4.6	4.4	4.3
8	5.2	5.1	4.9
9	5.8	5.7	5.6
10	6.5	6.3	6.2
20	13.0	12.7	12.3
30	19.5	19.0	18.5
40	26.0	25.3	24.7
50	32.5	31.7	30.8

″	36	5	4
1	0.6	0.1	0.1
2	1.2	0.2	0.1
3	1.8	0.2	0.2
4	2.4	0.3	0.3
5	3.0	0.4	0.3
6	3.6	0.5	0.4
7	4.2	0.6	0.5
8	4.8	0.7	0.5
9	5.4	0.8	0.6
10	6.0	0.8	0.7
20	12.0	1.7	1.3
30	18.0	2.5	2.0
40	24.0	3.3	2.7
50	30.0	4.2	3.3

Proportional parts

108° (288°) (251°) **71°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

19° (199°) (340°) **160°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.51 264	37	9.53 697	41	10.46 303	9.97 567	4	60
1	9.51 301	37	9.53 738	41	10.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	10.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	10.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	10.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	10.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	10.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	10.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	10.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	10.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	10.45 894	9.97 523	4	50
11	9.51 666	36	9.54 147	40	10.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	10.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	10.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	10.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	10.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	10.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	10.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	10.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	10.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	10.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	41	10.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	10.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	10.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	10.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	10.45 286	9.97 457	4	35
26	9.52 207	35	9.54 754	40	10.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	10.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	10.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	10.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	10.45 085	9.97 435	5	30
31	9.52 385	36	9.54 955	40	10.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	10.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	10.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	10.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	10.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	10.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	10.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	10.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	10.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	10.44 685	9.97 390	5	20
41	9.52 740	35	9.55 355	40	10.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	10.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	10.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	10.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	10.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	10.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	10.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	10.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	10.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	10.44 288	9.97 344	4	10
51	9.53 092	34	9.55 752	39	10.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	10.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	10.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	10.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	10.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	10.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	10.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	10.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	10.43 933	9.97 303	4	1
60	9.53 405		9.56 107		10.43 893	9.97 299		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	41	40	39
1	0.7	0.7	0.6
2	1.4	1.3	1.3
3	2.0	2.0	2.0
4	2.7	2.7	2.6
5	3.4	3.3	3.2
6	4.1	4.0	3.9
7	4.8	4.7	4.6
8	5.5	5.3	5.2
9	6.2	6.0	5.8
10	6.8	6.7	6.5
20	13.7	13.3	13.0
30	20.5	20.0	19.5
40	27.3	26.7	26.0
50	34.2	33.3	32.5

″	37	36	35
1	0.6	0.6	0.6
2	1.2	1.2	1.2
3	1.8	1.8	1.8
4	2.5	2.4	2.3
5	3.1	3.0	2.9
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.2
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

″	34	5	4
1	0.6	0.1	0.1
2	1.1	0.2	0.1
3	1.7	0.2	0.2
4	2.3	0.3	0.3
5	2.8	0.4	0.3
6	3.4	0.5	0.4
7	4.0	0.6	0.5
8	4.5	0.7	0.5
9	5.1	0.8	0.6
10	5.7	0.8	0.7
20	11.3	1.7	1.3
30	17.0	2.5	2.0
40	22.7	3.3	2.7
50	28.3	4.2	3.3

109° (289°) (250°) **70°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

20° (200°) (339°) **159°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.53 405	35	9.56 107	39	10.43 893	9.97 299	6	60
1	9.53 440	35	9.56 146	39	10.43 854	9.97 294	6	59
2	9.53 475	34	9.56 185	39	10.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	10.43 776	9.97 285	6	57
4	9.53 544	34	9.56 264	39	10.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	10.43 697	9.97 276	6	55
6	9.53 613	34	9.56 342	39	10.43 658	9.97 271	6	54
7	9.53 647	35	9.56 381	39	10.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	10.43 580	9.97 262	6	52
9	9.53 716	35	9.56 459	39	10.43 541	9.97 257	6	51
10	9.53 751	34	9.56 498	39	10.43 502	9.97 252	4	50
11	9.53 785	34	9.56 537	39	10.43 463	9.97 248	6	49
12	9.53 819	35	9.56 576	39	10.43 424	9.97 243	6	48
13	9.53 854	34	9.56 615	39	10.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	10.43 346	9.97 234	6	46
15	9.53 922	35	9.56 693	39	10.43 307	9.97 229	6	45
16	9.53 957	34	9.56 732	39	10.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	10.43 229	9.97 220	6	43
18	9.54 025	34	9.56 810	39	10.43 190	9.97 215	6	42
19	9.54 059	34	9.56 849	38	10.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	10.43 113	9.97 206	6	40
21	9.54 127	34	9.56 926	39	10.43 074	9.97 201	6	39
22	9.54 161	34	9.56 965	39	10.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	10.42 996	9.97 192	6	37
24	9.54 229	34	9.57 042	39	10.42 958	9.97 187	6	36
25	9.54 263	34	9.57 081	39	10.42 919	9.97 182	4	35
26	9.54 297	34	9.57 120	38	10.42 880	9.97 178	6	34
27	9.54 331	34	9.57 158	39	10.42 842	9.97 173	6	33
28	9.54 365	34	9.57 197	38	10.42 803	9.97 168	6	32
29	9.54 399	34	9.57 235	39	10.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	10.42 726	9.97 159	6	30
31	9.54 466	34	9.57 312	39	10.42 688	9.97 154	6	29
32	9.54 500	34	9.57 351	38	10.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	10.42 611	9.97 145	6	27
34	9.54 567	34	9.57 428	38	10.42 572	9.97 140	6	26
35	9.54 601	34	9.57 466	38	10.42 534	9.97 135	6	25
36	9.54 635	33	9.57 504	39	10.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	10.42 457	9.97 126	6	23
38	9.54 702	33	9.57 581	38	10.42 419	9.97 121	6	22
39	9.54 735	34	9.57 619	39	10.42 381	9.97 116	6	21
40	9.54 769	33	9.57 658	38	10.42 342	9.97 111	4	20
41	9.54 802	34	9.57 696	38	10.42 304	9.97 107	6	19
42	9.54 836	33	9.57 734	38	10.42 266	9.97 102	6	18
43	9.54 869	34	9.57 772	38	10.42 228	9.97 097	6	17
44	9.54 903	33	9.57 810	39	10.42 190	9.97 092	6	16
45	9.54 936	33	9.57 849	38	10.42 151	9.97 087	4	15
46	9.54 969	34	9.57 887	38	10.42 113	9.97 083	6	14
47	9.55 003	33	9.57 925	38	10.42 075	9.97 078	6	13
48	9.55 036	33	9.57 963	38	10.42 037	9.97 073	6	12
49	9.55 069	33	9.58 001	38	10.41 999	9.97 068	6	11
50	9.55 102	34	9.58 039	38	10.41 961	9.97 063	4	10
51	9.55 136	33	9.58 077	38	10.41 923	9.97 059	6	9
52	9.55 169	33	9.58 115	38	10.41 885	9.97 054	6	8
53	9.55 202	33	9.58 153	38	10.41 847	9.97 049	6	7
54	9.55 235	33	9.58 191	38	10.41 809	9.97 044	6	6
55	9.55 268	33	9.58 229	38	10.41 771	9.97 039	4	5
56	9.55 301	33	9.58 267	37	10.41 733	9.97 035	6	4
57	9.55 334	33	9.58 304	38	10.41 696	9.97 030	6	3
58	9.55 367	33	9.58 342	38	10.41 658	9.97 025	6	2
59	9.55 400	33	9.58 380	38	10.41 620	9.97 020	6	1
60	9.55 433		9.58 418		10.41 582	9.97 015		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	40	39	38
1	0.7	0.6	0.6
2	1.3	1.3	1.3
3	2.0	2.0	1.9
4	2.7	2.6	2.5
5	3.3	3.2	3.2
6	4.0	3.9	3.8
7	4.7	4.6	4.4
8	5.3	5.2	5.1
9	6.0	5.8	5.7
10	6.7	6.5	6.3
20	13.3	13.0	12.7
30	20.0	19.5	19.0
40	26.7	26.0	25.3
50	33.3	32.5	31.7

″	37	35	34
1	0.6	0.6	0.6
2	1.2	1.2	1.1
3	1.8	1.8	1.7
4	2.5	2.3	2.3
5	3.1	2.9	2.8
6	3.7	3.5	3.4
7	4.3	4.1	4.0
8	4.9	4.7	4.5
9	5.6	5.2	5.1
10	6.2	5.8	5.7
20	12.3	11.7	11.3
30	18.5	17.5	17.0
40	24.7	23.3	22.7
50	30.8	29.2	28.3

″	33	5	4
1	0.6	0.1	0.1
2	1.1	0.2	0.1
3	1.6	0.2	0.2
4	2.2	0.3	0.3
5	2.8	0.4	0.3
6	3.3	0.5	0.4
7	3.8	0.6	0.5
8	4.4	0.7	0.5
9	5.0	0.8	0.6
10	5.5	0.8	0.7
20	11.0	1.7	1.3
30	16.5	2.5	2.0
40	22.0	3.3	2.7
50	27.5	4.2	3.3

Proportional parts

110° (290°) (249°) **69°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

21° (201°) (338°) **158°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.55 433	33	9.58 418	37	10.41 582	9.97 015	5	60
1	9.55 466	33	9.58 455	38	10.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	10.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	10.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	10.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	10.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	37	10.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	10.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	10.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	10.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	10.41 206	9.96 966	4	50
11	9.55 793	33	9.58 832	37	10.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	10.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	10.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	10.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	10.41 019	9.96 942	5	45
16	9.55 956	32	9.59 019	37	10.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	10.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	10.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	10.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	10.40 832	9.96 917	5	40
21	9.56 118	32	9.59 205	38	10.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	10.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	10.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	10.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	10.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	38	10.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	10.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	10.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	10.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	10.40 460	9.96 868	5	30
31	9.56 440	32	9.59 577	37	10.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	10.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	10.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	10.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	10.40 275	9.96 843	5	25
36	9.56 599	32	9.59 762	37	10.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	10.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	10.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	10.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	10.40 091	9.96 818	5	20
41	9.56 759	31	9.59 946	37	10.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	10.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	10.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	10.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	10.39 907	9.96 793	5	15
46	9.56 917	32	9.60 130	36	10.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	10.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	10.39 797	9.96 778	5	12
49	9.57 012	32	9.60 240	36	10.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	10.39 724	9.96 767	5	10
51	9.57 075	32	9.60 313	36	10.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	10.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	10.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	10.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	10.39 541	9.96 742	5	5
56	9.57 232	32	9.60 495	37	10.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	10.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	10.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	10.39 395	9.96 722	5	1
60	9.57 358		9.60 641		10.39 359	9.96 717		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	38	37	36
1	0.6	0.6	0.6
2	1.3	1.2	1.2
3	1.9	1.8	1.8
4	2.5	2.5	2.4
5	3.2	3.1	3.0
6	3.8	3.7	3.6
7	4.4	4.3	4.2
8	5.1	4.9	4.8
9	5.7	5.6	5.4
10	6.3	6.2	6.0
20	12.7	12.3	12.0
30	19.0	18.5	18.0
40	25.3	24.7	24.0
50	31.7	30.8	30.0

″	33	32	31
1	0.6	0.5	0.5
2	1.1	1.1	1.0
3	1.6	1.6	1.6
4	2.2	2.1	2.1
5	2.8	2.7	2.6
6	3.3	3.2	3.1
7	3.8	3.7	3.6
8	4.4	4.3	4.1
9	5.0	4.8	4.6
10	5.5	5.3	5.2
20	11.0	10.7	10.3
30	16.5	16.0	15.5
40	22.0	21.3	20.7
50	27.5	26.7	25.8

″	6	5	4
1	0.1	0.1	0.1
2	0.2	0.2	0.1
3	0.3	0.2	0.2
4	0.4	0.3	0.3
5	0.5	0.4	0.3
6	0.6	0.5	0.4
7	0.7	0.6	0.5
8	0.8	0.7	0.5
9	0.9	0.8	0.6
10	1.0	0.8	0.7
20	2.0	1.7	1.3
30	3.0	2.5	2.0
40	4.0	3.3	2.7
50	5.0	4.2	3.3

Proportional parts

111° (291°) (248°) **68°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

22° (202°) (337°) **157°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.57 358	31	9.60 641	36	10.39 359	9.96 717	6	60
1	9.57 389	31	9.60 677	37	10.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	10.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	10.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	10.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	10.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	10.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	10.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	10.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	10.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	10.38 996	9.96 665	5	50
11	9.57 700	31	9.61 040	36	10.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	10.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	10.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	10.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	10.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	10.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	10.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	10.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	10.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	10.38 636	9.96 614	6	40
21	9.58 008	31	9.61 400	36	10.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	10.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	10.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	10.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	10.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	10.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	10.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	10.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	10.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	10.38 278	9.96 562	6	30
31	9.58 314	31	9.61 758	36	10.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	10.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	10.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	10.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	10.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	10.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	10.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	10.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	10.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	10.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	36	10.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	10.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	10.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	10.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	10.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	10.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	10.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	10.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	10.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	10.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	36	10.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	10.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	10.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	10.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	10.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	10.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	10.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	10.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	10.37 250	9.96 408	5	1
60	9.59 188		9.62 785		10.37 215	9.96 403		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

//	37	36	35
1	0.6	0.6	0.6
2	1.2	1.2	1.2
3	1.8	1.8	1.8
4	2.5	2.4	2.3
5	3.1	3.0	2.9
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.2
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

//	32	31	30
1	0.5	0.5	0.5
2	1.1	1.0	1.0
3	1.6	1.6	1.5
4	2.1	2.1	2.0
5	2.7	2.6	2.5
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

//	29	6	5
1	0.5	0.1	0.1
2	1.0	0.2	0.2
3	1.4	0.3	0.2
4	1.9	0.4	0.3
5	2.4	0.5	0.4
6	2.9	0.6	0.5
7	3.4	0.7	0.6
8	3.9	0.8	0.7
9	4.4	0.9	0.8
10	4.8	1.0	0.8
20	9.7	2.0	1.7
30	14.5	3.0	2.5
40	19.3	4.0	3.3
50	24.2	5.0	4.2

Proportional parts

112° (292°) (247°) **67°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

23° (203°) (336°) **156°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.59 188	30	9.62 785	35	10.37 215	9.96 403	6	60
1	9.59 218	29	9.62 820	35	10.37 180	9.96 397	6	59
2	9.59 247	30	9.62 855	35	10.37 145	9.96 392	6	58
3	9.59 277	30	9.62 890	36	10.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	10.37 074	9.96 381	6	56
5	9.59 336	30	9.62 961	35	10.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	10.37 004	9.96 370	6	54
7	9.59 396	29	9.63 031	35	10.36 969	9.96 365	6	53
8	9.59 425	30	9.63 066	35	10.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	10.36 899	9.96 354	6	51
10	9.59 484	30	9.63 135	35	10.36 865	9.96 349	6	50
11	9.59 514	29	9.63 170	35	10.36 830	9.96 343	6	49
12	9.59 543	30	9.63 205	35	10.36 795	9.96 338	6	48
13	9.59 573	29	9.63 240	35	10.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	10.36 725	9.96 327	6	46
15	9.59 632	29	9.63 310	35	10.36 690	9.96 322	6	45
16	9.59 661	29	9.63 345	34	10.36 655	9.96 316	6	44
17	9.59 690	30	9.63 379	35	10.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	10.36 586	9.96 305	6	42
19	9.59 749	29	9.63 449	35	10.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	10.36 516	9.96 294	6	40
21	9.59 808	29	9.63 519	34	10.36 481	9.96 289	6	39
22	9.59 837	29	9.63 553	35	10.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	10.36 412	9.96 278	6	37
24	9.59 895	29	9.63 623	34	10.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	10.36 343	9.96 267	6	35
26	9.59 954	29	9.63 692	34	10.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	10.36 274	9.96 256	6	33
28	9.60 012	29	9.63 761	35	10.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	10.36 204	9.96 245	6	31
30	9.60 070	29	9.63 830	35	10.36 170	9.96 240	6	30
31	9.60 099	29	9.63 865	34	10.36 135	9.96 234	6	29
32	9.60 128	29	9.63 899	35	10.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	10.36 066	9.96 223	6	27
34	9.60 186	29	9.63 968	35	10.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	10.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	35	10.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	10.35 928	9.96 201	6	23
38	9.60 302	29	9.64 106	34	10.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	10.35 860	9.96 190	6	21
40	9.60 359	29	9.64 175	34	10.35 825	9.96 185	6	20
41	9.60 388	29	9.64 209	34	10.35 791	9.96 179	6	19
42	9.60 417	29	9.64 243	35	10.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	10.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	10.35 688	9.96 162	6	16
45	9.60 503	29	9.64 346	35	10.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	10.35 619	9.96 151	6	14
47	9.60 561	28	9.64 415	34	10.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	10.35 551	9.96 140	6	12
49	9.60 618	28	9.64 483	34	10.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	10.35 483	9.96 129	6	10
51	9.60 675	29	9.64 552	34	10.35 448	9.96 123	6	9
52	9.60 704	28	9.64 586	34	10.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	10.35 380	9.96 112	6	7
54	9.60 761	28	9.64 654	34	10.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	10.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	10.35 278	9.96 095	6	4
57	9.60 846	29	9.64 756	34	10.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	10.35 210	9.96 084	6	2
59	9.60 903	28	9.64 824	34	10.35 176	9.96 079	6	1
60	9.60 931		9.64 858		10.35 142	9.96 073		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	36	35	34
1	0.6	0.6	0.6
2	1.2	1.2	1.1
3	1.8	1.8	1.7
4	2.4	2.3	2.3
5	3.0	2.9	2.8
6	3.6	3.5	3.4
7	4.2	4.1	4.0
8	4.8	4.7	4.5
9	5.4	5.2	5.1
10	6.0	5.8	5.7
20	12.0	11.7	11.3
30	18.0	17.5	17.0
40	24.0	23.3	22.7
50	30.0	29.2	28.3

″	30	29	28
1	0.5	0.5	0.5
2	1.0	1.0	0.9
3	1.5	1.4	1.4
4	2.0	1.9	1.9
5	2.5	2.4	2.3
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

″	6	5
1	0.1	0.1
2	0.2	0.2
3	0.3	0.2
4	0.4	0.3
5	0.5	0.4
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

Proportional parts

113° (293°) (246°) **66°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

24° (204°) (335°) **155**°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.60 931	29	9.64 858	34	10.35 142	9.96 073	6	60
1	9.60 960	28	9.64 892	34	10.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	10.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	10.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	10.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	10.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	10.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	10.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	10.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	10.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	10.34 803	9.96 017	6	50
11	9.61 242	28	9.65 231	34	10.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	10.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	10.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	10.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	10.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	10.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	10.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	10.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	10.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	10.34 465	9.95 960	6	40
21	9.61 522	28	9.65 568	34	10.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	10.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	10.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	10.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	10.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	10.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	10.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	10.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	10.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	10.34 130	9.95 902	5	30
31	9.61 800	28	9.65 904	33	10.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	10.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	10.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	10.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	10.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	10.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	10.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	10.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	10.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	10.33 796	9.95 844	5	20
41	9.62 076	28	9.66 238	33	10.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	10.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	10.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	10.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	10.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	10.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	10.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	10.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	10.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	10.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	10.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	10.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	10.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	10.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	10.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	10.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	10.33 232	9.98 745	6	3
58	9.62 541	27	9.66 801	33	10.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	10.33 166	9.95 733	5	1
60	9.62 595		9.66 867		10.33 133	9.95 728		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	34	33
1	0.6	0.6
2	1.1	1.1
3	1.7	1.6
4	2.3	2.2
5	2.8	2.8
6	3.4	3.3
7	4.0	3.8
8	4.5	4.4
9	5.1	5.0
10	5.7	5.5
20	11.3	11.0
30	17.0	16.5
40	22.7	22.0
50	28.3	27.5

″	29	28	27
1	0.5	0.5	0.4
2	1.0	0.9	0.9
3	1.4	1.4	1.4
4	1.9	1.9	1.8
5	2.4	2.3	2.2
6	2.9	2.8	2.7
7	3.4	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.0
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

″	6	5
1	0.1	0.1
2	0.2	0.2
3	0.3	0.2
4	0.4	0.3
5	0.5	0.4
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

Proportional parts

114° (294°) (245°) **65**°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

25° (205°) (334°) **154°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.62 595	27	9.66 867	33	10.33 133	9.95 728	6	60
1	9.62 622	27	9.66 900	33	10.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	10.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	10.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	10.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	10.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	10.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	10.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	10.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	10.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	10.32 804	9.95 668	6	50
11	9.62 892	26	9.67 229	33	10.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	10.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	10.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	10.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	10.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	10.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	10.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	10.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	10.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	10.32 476	9.95 609	6	40
21	9.63 159	27	9.67 556	33	10.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	10.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	10.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	10.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	10.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	10.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	10.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	10.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	10.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	10.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	33	10.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	10.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	10.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	10.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	10.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	10.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	10.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	10.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	10.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	10.31 826	9.95 488	6	20
41	9.63 689	26	9.68 206	33	10.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	10.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	10.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	10.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	10.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	10.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	10.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	10.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	10.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	10.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	10.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	10.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	10.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	10.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	10.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	10.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	10.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	10.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	10.31 214	9.95 372	6	1
60	9.64 184		9.68 818		10.31 182	9.95 366		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	33	32
1	0.6	0.5
2	1.1	1.1
3	1.6	1.6
4	2.2	2.1
5	2.8	2.7
6	3.3	3.2
7	3.8	3.7
8	4.4	4.3
9	5.0	4.8
10	5.5	5.3
20	11.0	10.7
30	16.5	16.0
40	22.0	21.3
50	27.5	26.7

″	27	26
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

″	7	6	5
1	0.1	0.1	0.1
2	0.2	0.2	0.2
3	0.4	0.3	0.2
4	0.5	0.4	0.3
5	0.6	0.5	0.4
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.7
9	1.0	0.9	0.8
10	1.2	1.0	0.8
20	2.3	2.0	1.7
30	3.5	3.0	2.5
40	4.7	4.0	3.3
50	5.8	5.0	4.2

115° (295°) (244°) **64°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

26° (206°) (333°) **153**°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.64 184	26	9.68 818	32	10.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	10.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	10.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	10.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	10.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	10.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	10.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	10.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	10.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	10.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	10.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	10.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	10.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	10.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	10.30 734	9.95 279	6	46
15	9.64 471	25	9.69 298	31	10.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	10.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	10.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	10.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	10.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	10.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	10.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	10.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	10.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	10.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	10.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	10.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	10.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	10.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	10.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	10.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	10.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	10.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	10.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	10.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	10.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	10.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	10.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	10.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	10.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	10.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	10.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	10.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	10.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	10.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	10.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	10.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	10.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	10.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	10.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	10.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	10.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	10.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	10.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	10.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	10.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	10.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	10.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	10.29 346	9.95 001	6	2
59	0.65 680	25	9.70 685	32	10.29 315	9.94 995	7	1
60	9.65 705		9.70 717		10.29 283	9.94 988		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	32	31
1	0.5	0.5
2	1.1	1.0
3	1.6	1.6
4	2.1	2.1
5	2.7	2.6
6	3.2	3.1
7	3.7	3.6
8	4.3	4.1
9	4.8	4.6
10	5.3	5.2
20	10.7	10.3
30	16.0	15.5
40	21.3	20.7
50	26.7	25.8

″	26	25	24
1	0.4	0.4	0.4
2	0.9	0.8	0.8
3	1.3	1.2	1.2
4	1.7	1.7	1.6
5	2.2	2.1	2.0
6	2.6	2.5	2.4
7	3.0	2.9	2.8
8	3.5	3.3	3.2
9	3.9	3.8	3.6
10	4.3	4.2	4.0
20	8.7	8.3	8.0
30	13.0	12.5	12.0
40	17.3	16.7	16.0
50	21.7	20.8	20.0

″	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

Proportional parts

116° (296°) (243°) **63**°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

27° (207°) (332°) 152°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.65 705	24	9.70 717	31	10.29 283	9.94 988	6	60
1	9.65 729	25	9.70 748	31	10.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	10.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	10.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	10.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	10.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	10.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	10.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	10.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	10.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	10.28 972	9.94 923	6	50
11	9.65 976	25	9.71 059	31	10.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	10.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	10.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	10.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	10.28 816	9.94 891	6	45
16	9.66 099	25	9.71 215	31	10.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	10.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	10.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	10.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	10.28 661	9.94 858	6	40
21	9.66 221	25	9.71 370	31	10.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	10.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	10.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	10.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	10.28 507	9.94 826	7	35
26	9.66 343	25	9.71 524	31	10.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	10.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	10.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	10.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	10.28 352	9.94 793	7	30
31	9.66 465	24	9.71 679	30	10.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	10.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	10.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	10.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	10.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	30	10.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	10.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	10.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	10.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	10.28 045	9.94 727	7	20
41	9.66 706	25	9.71 986	31	10.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	10.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	10.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	10.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	10.27 891	9.94 694	7	15
46	9.66 827	24	9.72 140	30	10.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	10.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	10.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	10.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	10.27 738	9.94 660	6	10
51	9.66 946	24	9.72 293	30	10.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	10.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	10.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	10.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	10.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	31	10.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	10.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	10.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	10.27 463	9.94 600	7	1
60	9.67 161		9.72 567		10.27 433	9.94 593		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	32	31	30
1	0.5	0.5	0.5
2	1.1	1.0	1.0
3	1.6	1.6	1.5
4	2.1	2.1	2.0
5	2.7	2.6	2.5
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

″	25	24	23
1	0.4	0.4	0.4
2	0.8	0.8	0.8
3	1.2	1.2	1.2
4	1.7	1.6	1.5
5	2.1	2.0	1.9
6	2.5	2.4	2.3
7	2.9	2.8	2.7
8	3.3	3.2	3.1
9	3.8	3.6	3.4
10	4.2	4.0	3.8
20	8.3	8.0	7.7
30	12.5	12.0	11.5
40	16.7	16.0	15.3
50	20.8	20.0	19.2

″	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

Proportional parts

117° (297°) (242°) 62°

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

28° (208°) (331°) **151°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.67 161	24	9.72 567	31	10.27 433	9.94 593	6	60
1	9.67 185	23	9.72 598	30	10.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	10.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	10.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	10.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	10.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	10.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	10.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	10.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	10.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	10.27 128	9.94 526	7	50
11	9.67 421	24	9.72 902	30	10.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	10.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	10.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	10.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	10.26 977	9.94 492	7	45
16	9.67 539	23	9.73 054	30	10.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	10.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	10.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	10.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	10.26 825	9.94 458	7	40
21	9.67 656	24	9.73 205	30	10.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	10.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	10.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	10.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	10.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	10.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	10.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	10.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	10.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	10.26 524	9.94 390	7	30
31	9.67 890	23	9.73 507	30	10.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	10.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	10.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	10.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	10.26 373	9.94 355	6	25
36	9.68 006	23	9.73 657	30	10.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	10.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	10.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	10.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	10.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	10.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	10.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	10.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	10.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	10.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	10.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	10.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	10.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	10.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	10.25 923	9.94 252	7	10
51	9.68 351	23	9.74 107	30	10.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	10.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	10.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	10.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	10.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	10.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	10.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	10.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	10.25 655	9.94 189	7	1
60	9.68 557		9.74 375		10.25 625	9.94 182		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	31	30	29
1	0.5	0.5	0.5
2	1.0	1.0	1.0
3	1.6	1.5	1.4
4	2.1	2.0	1.9
5	2.6	2.5	2.4
6	3.1	3.0	2.9
7	3.6	3.5	3.4
8	4.1	4.0	3.9
9	4.6	4.5	4.4
10	5.2	5.0	4.8
20	10.3	10.0	9.7
30	15.5	15.0	14.5
40	20.7	20.0	19.3
50	25.8	25.0	24.2

″	24	23	22
1	0.4	0.4	0.4
2	0.8	0.8	0.7
3	1.2	1.2	1.1
4	1.6	1.5	1.5
5	2.0	1.9	1.8
6	2.4	2.3	2.2
7	2.8	2.7	2.6
8	3.2	3.1	2.9
9	3.6	3.4	3.3
10	4.0	3.8	3.7
20	8.0	7.7	7.3
30	12.0	11.5	11.0
40	16.0	15.3	14.7
50	20.0	19.2	18.3

″	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

Proportional parts

118° (298°) (241°) **61°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

29° (209°) (330°) 150°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.68 557	23	9.74 375	30	10.25 625	9.94 182	7	60
1	9.68 580	23	9.74 405	30	10.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	10.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	10.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	10.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	10.25 476	9.94 147	7	55
6	9.68 694	22	9.74 554	29	10.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	10.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	10.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	10.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	10.25 327	9.94 112	7	50
11	9.68 807	22	9.74 702	30	10.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	10.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	10.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	10.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	10.25 179	9.94 076	7	45
16	9.68 920	22	9.74 851	29	10.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	10.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	10.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	10.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	10.25 031	9.94 041	7	40
21	9.69 032	23	9.74 998	30	10.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	10.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	10.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	10.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	10.24 883	9.94 005	7	35
26	9.69 144	23	9.75 146	30	10.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	10.24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	10.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	10.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	10.24 736	9.93 970	7	30
31	9.69 256	23	9.75 294	29	10.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	10.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	10.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	10.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	10.24 589	9.93 934	7	25
36	9.69 368	22	9.75 441	29	10.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	10.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	10.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	10.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	10.24 442	9.93 898	7	20
41	9.69 479	22	9.75 588	29	10.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	10.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	10.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	10.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	10.24 295	9.93 862	7	15
46	9.69 589	22	9.75 735	29	10.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	10.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	10.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	10.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	10.24 148	9.93 826	7	10
51	9.69 699	22	9.75 881	29	10.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	10.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	10.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	10.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	10.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	10.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	10.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	10.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	10.23 885	9.93 760	7	1
60	9.69 897		9.76 144		10.23 856	9.93 753		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	30	29	23
1	0.5	0.5	0.4
2	1.0	1.0	0.8
3	1.5	1.4	1.2
4	2.0	1.9	1.5
5	2.5	2.4	1.9
6	3.0	2.9	2.3
7	3.5	3.4	2.7
8	4.0	3.9	3.1
9	4.5	4.4	3.4
10	5.0	4.8	3.8
20	10.0	9.7	7.7
30	15.0	14.5	11.5
40	20.0	19.3	15.3
50	25.0	24.2	19.2

″	22	8	7
1	0.4	0.1	0.1
2	0.7	0.3	0.2
3	1.1	0.4	0.4
4	1.5	0.5	0.5
5	1.8	0.7	0.6
6	2.2	0.8	0.7
7	2.6	0.9	0.8
8	2.9	1.1	0.9
9	3.3	1.2	1.0
10	3.7	1.3	1.2
20	7.3	2.7	2.3
30	11.0	4.0	3.5
40	14.7	5.3	4.7
50	18.3	6.7	5.8

Proportional parts

119° (299) (240°) 60°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

30° (210) (329°) **149°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.69 897	22	9.76 144	29	10.23 856	9.93 753	7	60
1	9.69 919	22	9.76 173	29	10.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	10.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	10.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	10.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	10.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	10.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	10.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	10.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	10.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	10.23 565	9.93 680	7	50
11	9.70 137	22	9.76 464	29	10.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	10.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	10.23 478	9.93 758	8	47
14	9.70 202	22	9.76 551	29	10.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	10.23 420	9.93 643	7	45
16	9.70 245	22	9.76 609	30	10.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	10.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	10.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	10.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	10.23 275	9.93 606	7	40
21	9.70 353	22	9.76 754	29	10.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	10.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	10.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	10.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	10.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	10.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	10.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	10.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	10.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	10.22 985	9.93 532	7	30
31	9.70 568	22	9.77 044	29	10.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	10.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	10.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	10.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	10.22 841	9.93 495	8	25
36	9.70 675	22	9.77 188	29	10.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	10.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	10.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	10.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	10.22 697	9.93 457	7	20
41	9.70 782	21	9.77 332	29	10.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	10.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	10.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	10.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	10.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	10.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	10.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	10.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	10.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	10.22 409	9.93 382	7	10
51	9.70 994	21	9.77 619	29	10.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	10.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	10.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	10.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	10.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	28	10.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	10.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	10.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	10.22 151	9.93 314	7	1
60	9.71 184		9.77 877		10.22 123	9.93 307		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

120° (300°) (239°) **59°**

Proportional parts

′′	30	29	28
1	0.5	0.5	0.5
2	1.0	1.0	0.9
3	1.5	1.4	1.4
4	2.0	1.9	1.9
5	2.5	2.4	2.3
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

′′	22	21
1	0.4	0.4
2	0.7	0.7
3	1.1	1.0
4	1.5	1.4
5	1.8	1.8
6	2.2	2.1
7	2.6	2.4
8	2.9	2.8
9	3.3	3.2
10	3.7	3.5
20	7.3	7.0
30	11.0	10.5
40	14.7	14.0
50	18.3	17.5

′′	8	7
1	0.1	0.1
2	0.3	0.2
3	0.4	0.4
4	0.5	0.5
5	0.7	0.6
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

Proportional parts

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

31° (211°) (328°) **148°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.71 184	21	9.77 877	29	10.22 123	9.93 307	8	60
1	9.71 205	21	9.77 906	29	10.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	10.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	10.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	10.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	10.21 980	9.93 269	8	55
6	9.71 310	21	9.78 049	28	10.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	10.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	10.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	10.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	10.21 837	9.93 230	7	50
11	9.71 414	21	9.78 192	28	10.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	10.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	10.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	10.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	10.21 694	9.93 192	8	45
16	9.71 519	20	9.78 334	29	10.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	10.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	10.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	10.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	10.21 552	9.93 154	8	40
21	9.71 622	21	9.78 476	29	10.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	10.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	10.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	10.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	10.21 410	9.93 115	7	35
26	9.71 726	21	9.78 618	29	10.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	10.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	10.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	10.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	10.21 268	9.93 077	8	30
31	9.71 829	21	9.78 760	29	10.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	10.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	10.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	10.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	10.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	10.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	10.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	10.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	10.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	10.20 985	9.92 999	8	20
41	9.72 034	21	9.79 043	29	10.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	10.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	10.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	10.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	10.20 844	9.92 960	8	15
46	9.72 137	20	9.79 185	28	10.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	10.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	10.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	10.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	10.20 703	9.92 921	8	10
51	9.72 238	21	9.79 326	28	10.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	10.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	10.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	10.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	10.20 562	9.92 881	7	5
56	9.72 340	20	9.79 466	29	10.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	10.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	10.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	10.20 449	9.92 850	8	1
60	9.72 421		9.79 579		10.20 421	9.92 842		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	29	28
1	0.5	0.5
2	1.0	0.9
3	1.4	1.4
4	1.9	1.9
5	2.4	2.3
6	2.9	2.8
7	3.4	3.3
8	3.9	3.7
9	4.4	4.2
10	4.8	4.7
20	9.7	9.3
30	14.5	14.0
40	19.3	18.7
50	24.2	23.3

″	21	20
1	0.4	0.3
2	0.7	0.7
3	1.0	1.0
4	1.4	1.3
5	1.8	1.7
6	2.1	2.0
7	2.4	2.3
8	2.8	2.7
9	3.2	3.0
10	3.5	3.3
20	7.0	6.7
30	10.5	10.0
40	14.0	13.3
50	17.5	16.7

″	8	7
1	0.1	0.1
2	0.3	0.2
3	0.4	0.4
4	0.5	0.5
5	0.7	0.6
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

Proportional parts

121° (301°) (238°) **58°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

32° (212°) (327°) 147°

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.72 421	20	9.79 579	28	10.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	10.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	10.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	10.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	10.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	10.20 281	9.92 803	8	55
6	9.72 542	20	9.79 747	29	10.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	10.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	10.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	10.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	10.20 140	9.92 763	8	50
11	9.72 643	20	9.79 888	28	10.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	10.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	10.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	10.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	10.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	10.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	10.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	10.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	10.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	10.19 860	9.92 683	8	40
21	9.72 843	20	9.80 168	27	10.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	10.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	10.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	10.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	10.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	10.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	10.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	10.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	10.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	10.19 581	9.92 603	8	30
31	9.73 041	20	9.80 447	27	10.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	10.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	10.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	10.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	10.19 442	9.92 563	8	25
36	9.73 140	20	9.80 586	28	10.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	10.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	10.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	10.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	10.19 303	9.92 522	8	20
41	9.73 239	20	9.80 725	28	10.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	10.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	10.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	10.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	10.19 164	9.92 482	9	15
46	9.73 337	20	9.80 864	28	10.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	10.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	10.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	10.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	10.19 025	9.92 441	8	10
51	9.73 435	20	9.81 003	27	10.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	10.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	10.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	10.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	10.18 887	9.92 400	8	5
56	9.73 533	19	9.81 141	28	10.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	10.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	10.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	10.18 776	9.92 367	8	1
60	9.73 611		9.81 252		10.18 748	9.92 359		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

122° (302°) (237°) 57°

Proportional parts

″	29	28	27
1	0.5	0.5	0.4
2	1.0	0.9	0.9
3	1.4	1.4	1.4
4	1.9	1.9	1.8
5	2.4	2.3	2.2
6	2.9	2.8	2.7
7	3.4	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.0
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

″	21	20	19
1	0.4	0.3	0.3
2	0.7	0.7	0.6
3	1.0	1.0	1.0
4	1.4	1.3	1.3
5	1.8	1.7	1.6
6	2.1	2.0	1.9
7	2.4	2.3	2.2
8	2.8	2.7	2.5
9	3.2	3.0	2.8
10	3.5	3.3	3.2
20	7.0	6.7	6.3
30	10.5	10.0	9.5
40	14.0	13.3	12.7
50	17.5	16.7	15.8

″	9	8	7
1	0.2	0.1	0.1
2	0.3	0.3	0.2
3	0.4	0.4	0.4
4	0.6	0.5	0.5
5	0.8	0.7	0.6
6	0.9	0.8	0.7
7	1.0	0.9	0.8
8	1.2	1.1	0.9
9	1.4	1.2	1.0
10	1.5	1.3	1.2
20	3.0	2.7	2.3
30	4.5	4.0	3.5
40	6.0	5.3	4.7
50	7.5	6.7	5.8

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

33° (213°) (326°) **146°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.73 611	19	9.81 252	27	10.18 748	9.92 359	8	60
1	9.73 630	20	9.81 279	28	10.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	10.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	10.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	10.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	10.18 610	9.92 318	8	55
6	9.73 727	20	9.81 418	27	10.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	10.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	10.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	10.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	10.18 472	9.92 277	8	50
11	9.73 824	19	9.81 556	27	10.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	10.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	10.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	10.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	10.18 334	9.92 235	8	45
16	9.73 921	19	9.81 693	28	10.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	10.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	10.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	10.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	10.18 197	9.92 194	8	40
21	9.74 017	19	9.81 831	27	10.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	10.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	10.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	10.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	10.18 059	9.92 152	8	35
26	9.74 113	19	9.81 968	28	10.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	10.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	10.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	10.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	10.17 922	9.92 111	9	30
31	9.74 208	19	9.82 106	27	10.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	10.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	10.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	10.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	10.17 785	9.92 069	9	25
36	9.74 303	19	9.82 243	27	10.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	10.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	10.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	10.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	10.17 648	9.92 027	9	20
41	9.74 398	19	9.82 380	27	10.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	10.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	10.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	10.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	10.17 511	9.91 985	9	15
46	9.74 493	19	9.82 517	27	10.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	10.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	10.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	10.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	10.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	28	10.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	10.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	10.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	10.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	10.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	27	10.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	10.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	10.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	10.17 129	9.91 866	9	1
60	9.74 756		9.82 899		10.17 101	9.91 857		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	28	27
1	0.5	0.4
2	0.9	0.9
3	1.4	1.4
4	1.9	1.8
5	2.3	2.2
6	2.8	2.7
7	3.3	3.2
8	3.7	3.6
9	4.2	4.0
10	4.7	4.5
20	9.3	9.0
30	14.0	13.5
40	18.7	18.0
50	23.3	22.5

″	20	19	18
1	0.3	0.3	0.3
2	0.7	0.6	0.6
3	1.0	1.0	0.9
4	1.3	1.3	1.2
5	1.7	1.6	1.5
6	2.0	1.9	1.8
7	2.3	2.2	2.1
8	2.7	2.5	2.4
9	3.0	2.8	2.7
10	3.3	3.2	3.0
20	6.7	6.3	6.0
30	10.0	9.5	9.0
40	13.3	12.7	12.0
50	16.7	15.8	15.0

″	9	8
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

Proportional parts

123° (303°) (236°) **56°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

34° (214°) (325°) **145°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.74 756	19	9.82 899	27	10.17 101	9.91 857	8	60
1	9.74 775	19	9.82 926	27	10.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	10.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	10.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	10.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	10.16 965	9.91 815	9	55
6	9.74 868	19	9.83 062	27	10.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	10.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	10.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	10.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	10.16 829	9.91 772	9	50
11	9.74 961	19	9.83 198	27	10.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	10.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	10.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	10.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	10.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	10.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	10.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	10.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	10.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	10.16 558	9.91 686	9	40
21	9.75 147	18	9.83 470	27	10.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	10.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	10.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	10.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	10.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	10.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	10.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	10.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	10.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	10.16 287	9.91 599	8	30
31	9.75 331	19	9.83 740	28	10.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	10.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	10.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	10.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	10.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	10.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	10.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	10.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	10.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	10.16 016	9.91 512	8	20
41	9.75 514	19	9.84 011	27	10.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	10.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	10.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	10.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	10.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	10.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	10.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	10.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	10.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	10.15 746	9.91 425	9	10
51	9.75 696	18	9.84 280	27	10.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	10.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	10.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	10.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	10.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	10.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	10.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	10.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	10.15 504	9.91 345	9	1
60	9.75 859		9.84 523		10.15 477	9.91 336		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

124° (304°) (235°) **55°**

Proportional parts

″	28	27	26
1	0.5	0.4	0.4
2	0.9	0.9	0.9
3	1.4	1.4	1.3
4	1.9	1.8	1.7
5	2.3	2.2	2.2
6	2.8	2.7	2.6
7	3.3	3.2	3.0
8	3.7	3.6	3.5
9	4.2	4.0	3.9
10	4.7	4.5	4.3
20	9.3	9.0	8.7
30	14.0	13.5	13.0
40	18.7	18.0	17.3
50	23.3	22.5	21.7

″	19	18
1	0.3	0.3
2	0.6	0.6
3	1.0	0.9
4	1.3	1.2
5	1.6	1.5
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.7
10	3.2	3.0
20	6.3	6.0
30	9.5	9.0
40	12.7	12.0
50	15.8	15.0

″	9	8
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

Proportional parts

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

35° (215°) (324°) **144°**

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	′
0	9.75 859	18	9.84 523	27	10.15 477	9.91 336	8	60
1	9.75 877	18	9.84 550	26	10.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	10.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	10.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	10.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	10.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	10.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	10.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	10.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	10.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	10.15 209	9.91 248	9	50
11	9.76 057	18	9.84 818	27	10.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	10.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	10.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	10.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	10.15 075	9.91 203	9	45
16	9.76 146	18	9.84 952	27	10.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	10.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	10.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	10.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	10.14 941	9.91 158	9	40
21	9.76 236	17	9.85 086	27	10.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	10.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	10.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	10.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	10.14 807	9.91 114	9	35
26	9.76 324	18	9.85 220	27	10.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	10.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	10.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	10.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	10.14 673	9.91 069	9	30
31	9.76 413	18	9.85 354	26	10.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	10.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	10.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	10.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	10.14 540	9.91 023	9	25
36	9.76 501	18	9.85 487	27	10.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	10.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	10.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	10.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	10.14 406	9.90 978	9	20
41	9.76 590	17	9.85 620	27	10.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	10.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	10.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	10.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	10.14 273	9.90 933	9	15
46	9.76 677	18	9.85 754	26	10.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	10.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	10.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	10.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	10.14 140	9.90 887	9	10
51	9.76 765	17	9.85 887	26	10.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	10.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	10.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	10.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	10.14 007	9.90 842	10	5
56	9.76 852	18	9.86 020	26	10.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	10.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	10.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	10.13 900	9.90 805	9	1
60	9.76 922		9.86 126		10.13 874	9.90 796		0
′	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Proportional parts

″	27	26	18
1	0.4	0.4	0.3
2	0.9	0.9	0.6
3	1.4	1.3	0.9
4	1.8	1.7	1.2
5	2.2	2.2	1.5
6	2.7	2.6	1.8
7	3.2	3.0	2.1
8	3.6	3.5	2.4
9	4.0	3.9	2.7
10	4.5	4.3	3.0
20	9.0	8.7	6.0
30	13.5	13.0	9.0
40	18.0	17.3	12.0
50	22.5	21.7	15.0

″	17	10
1	0.3	0.2
2	0.6	0.3
3	0.8	0.5
4	1.1	0.7
5	1.4	0.8
6	1.7	1.0
7	2.0	1.2
8	2.3	1.3
9	2.6	1.5
10	2.8	1.7
20	5.7	3.3
30	8.5	5.0
40	11.3	6.7
50	14.2	8.3

″	9	8
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

Proportional parts

125° (305°) (234°) **54°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

36° (216) (323°) **143°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.76 922	17	9.86 126	27	10.13 874	9.90 796	9	60
1	9.76 939	18	9.86 153	26	10.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	10.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	10.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	10.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	10.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	27	10.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	10.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	10.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	10.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	10.13 608	9.90 704	10	50
11	9.77 112	18	9.86 418	27	10.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	10.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	10.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	10.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	10.13 476	9.90 657	9	45
16	9.77 199	17	9.86 551	26	10.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	10.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	10.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	10.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	10.13 344	9.90 611	9	40
21	9.77 285	17	9.86 683	26	10.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	10.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	10.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	10.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	10.13 211	9.90 565	10	35
26	9.77 370	17	9.86 815	27	10.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	10.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	10.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	10.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	10.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	27	10.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	10.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	10.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	10.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	10.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	27	10.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	10.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	10.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	10.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	10.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	27	10.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	10.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	10.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	10.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	10.12 683	9.90 377	9	15
46	9.77 711	17	9.87 343	26	10.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	10 12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	10.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	10.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	10.12 552	9.90 330	10	10
51	9.77 795	17	9.87 475	26	10.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	10.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	10.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	10.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	10.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	27	10.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	10.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	10.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	10.12 315	9.90 244	9	1
60	9.77 946		9.87 711		10.12 289	9.90 235		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

126° (306°) (233°) **53°**

Proportional parts

″	27	26
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

″	18	17	16
1	0.3	0.3	0.3
2	0.6	0.6	0.5
3	0.9	0.8	0.8
4	1.2	1.1	1.1
5	1.5	1.4	1.3
6	1.8	1.7	1.6
7	2.1	2.0	1.9
8	2.4	2.3	2.1
9	2.7	2.6	2.4
10	3.0	2.8	2.7
20	6.0	5.7	5.3
30	9.0	8.5	8.0
40	12.0	11.3	10.7
50	15.0	14.2	13.3

″	10	9
1	0.2	0.2
2	0.3	0.3
3	0.5	0.4
4	0.7	0.6
5	0.8	0.8
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

Proportional parts

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

37° (217°) (322°) **142°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.77 946	17	9.87 711	27	10.12 289	9.90 235	10	60
1	9.77 963	17	9.87 738	26	10.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	10.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	10.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	10.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	10.12 157	9.90 187	9	55
6	9.78 047	16	9.87 869	26	10.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	10.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	10.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	10.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	10.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	27	10.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	10.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	10.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	10.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	10.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	27	10.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	10.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	10.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	10.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	10.11 764	9.90 043	9	40
21	9.78 296	17	9.88 262	27	10.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	10.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	10.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	10.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	10.11 633	9.89 995	10	35
26	9.78 379	16	9.88 393	27	10.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	10.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	10.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	10.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	10.11 502	9.89 947	10	30
31	9.78 461	17	9.88 524	26	10.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	10.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	10.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	10.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	10.11 371	9.89 898	10	25
36	9.78 543	17	9.88 655	26	10.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	10.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	10.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	10.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	10.11 241	9.89 849	9	20
41	9.78 625	17	9.88 786	26	10.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	10.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	10.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	10.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	10.11 110	9.89 801	10	15
46	9.78 707	16	9.88 916	26	10.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	10.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	10.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	10.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	10.10 980	9.89 752	10	10
51	9.78 788	17	9.89 046	27	10.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	10.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	10.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	10.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	10.10 849	9.89 702	9	5
56	9.78 869	17	9.89 177	26	10.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	10.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	10.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	10.10 745	9.89 663	10	1
60	9.78 934		9.89 281		10.10 719	9.89 653		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	27	26
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

″	17	16
1	0.3	0.3
2	0.6	0.5
3	0.8	0.8
4	1.1	1.1
5	1.4	1.3
6	1.7	1.6
7	2.0	1.9
8	2.3	2.1
9	2.6	2.4
10	2.8	2.7
20	5.7	5.3
30	8.5	8.0
40	11.3	10.7
50	14.2	13.3

″	10	9
1	0.2	0.2
2	0.3	0.3
3	0.5	0.4
4	0.7	0.6
5	0.8	0.8
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

Proportional parts

127° (307°) (232°) **52°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

38° (218°) (321°) **141°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.78 934	16	9.89 281	26	10.10 719	9.89 653	10	60
1	9.78 950	17	9.89 307	26	10.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	10.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	10.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	10.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	10.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	10.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	10.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	10.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	10.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	10.10 459	9.89 554	10	50
11	9.79 111	17	9.89 567	26	10.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	10.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	10.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	10.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	10.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	10.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	10.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	10.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	10.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	10.10 199	9.89 455	10	40
21	9.79 272	16	9.89 827	26	10.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	10.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	10.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	10.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	10.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	10.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	10.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	10.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	10.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	10.09 939	9.89 354	10	30
31	9.79 431	16	9.90 086	26	10.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	10.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	10.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	10.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	10.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	10.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	10.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	10.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	10.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	10.09 680	9.89 254	10	20
41	9.79 589	16	9.90 346	25	10.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	10.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	10.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	10.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	10.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	10.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	10.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	10.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	10.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	10.09 422	9.89 152	10	10
51	9.79 746	16	9.90 604	26	10.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	10.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	10.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	10.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	10.09 292	9.89 101	10	5
56	9.79 825	15	9.90 734	25	10.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	10.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	10.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	10.09 189	9.89 060	10	1
60	9.79 887		9.90 837		10.09 163	9.89 050		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	17	16	15
1	0.3	0.3	0.2
2	0.6	0.5	0.5
3	0.8	0.8	0.8
4	1.1	1.1	1.0
5	1.4	1.3	1.2
6	1.7	1.6	1.5
7	2.0	1.9	1.8
8	2.3	2.1	2.0
9	2.6	2.4	2.2
10	2.8	2.7	2.5
20	5.7	5.3	5.0
30	8.5	8.0	7.5
40	11.3	10.7	10.0
50	14.2	13.3	12.5

″	11	10	9
1	0.2	0.2	0.2
2	0.4	0.3	0.3
3	0.6	0.5	0.4
4	0.7	0.7	0.6
5	0.9	0.8	0.8
6	1.1	1.0	0.9
7	1.3	1.2	1.0
8	1.5	1.3	1.2
9	1.6	1.5	1.4
10	1.8	1.7	1.5
20	3.7	3.3	3.0
30	5.5	5.0	4.5
40	7.3	6.7	6.0
50	9.2	8.3	7.5

Proportional parts

128° (308°) (231°) **51°**

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

39° (219°) (320°) **140°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.79 887	16	9.90 837	26	10.09 163	9.89 050	10	60
1	9.79 903	15	9.90 863	26	10.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	10.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	10.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	10.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	10.09 034	9.88 999	10	55
6	9.79 981	15	9.90 992	26	10.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	10.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	10.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	10.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	10.08 905	9.88 948	11	50
11	9.80 058	16	9.91 121	26	10.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	10.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	10.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	10.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	10.08 776	9.88 896	10	45
16	9.80 136	15	9.91 250	26	10.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	10.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	10.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	10.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	10.08 647	9.88 844	10	40
21	9.80 213	15	9.91 379	25	10.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	10.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	10.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	10.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	10.08 518	9.88 793	11	35
26	9.80 290	15	9.91 507	26	10.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	10.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	10.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	10.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	10.08 390	9.88 741	11	30
31	9.80 366	16	9.91 636	26	10.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	10.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	10.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	10.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	10.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	26	10.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	10.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	10.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	10.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	10.08 132	9.88 636	10	20
41	9.80 519	15	9.91 893	26	10.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	10.08 081	9.88 615	10	18
43	9.80 550	15	9.91 945	26	10.08 055	9.88 605	11	17
44	9.80 565	15	9.91 971	25	10.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	10.08 004	9.88 584	11	15
46	9.80 595	15	9.92 022	26	10.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	10.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	10.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	10.07 901	9.88 542	11	11
50	9.80 656	15	9.92 125	25	10.07 875	9.88 531	10	10
51	9.80 671	15	9.92 150	26	10.07 850	9.88 521	11	9
52	9.80 686	15	9.92 176	26	10.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	10.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	10.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	10.07 747	9.88 478	10	5
56	9.80 746	16	9.92 279	25	10.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	10.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	10.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	10.07 644	9.88 436	11	1
60	9.80 807		9.92 381		10.07 619	9.88 425		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	16	15
1	0.3	0.2
2	0.5	0.5
3	0.8	0.8
4	1.1	1.0
5	1.3	1.2
6	1.6	1.5
7	1.9	1.8
8	2.1	2.0
9	2.4	2.2
10	2.7	2.5
20	5.3	5.0
30	8.0	7.5
40	10.7	10.0
50	13.3	12.5

″	11	10
1	0.2	0.2
2	0.4	0.3
3	0.6	0.5
4	0.7	0.7
5	0.9	0.8
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.6	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

129° (309°) (230°) **50°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

40° (220°) (319°) **139°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.80 807	15	9.92 381	26	10.07 619	9.88 425	10	60
1	9.80 822	15	9.92 407	26	10.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	10.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	10.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	10.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	10.07 490	9.88 372	10	55
6	9.80 897	15	9.92 535	26	10.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	10.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	10.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	10.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	10.07 362	9.88 319	11	50
11	9.80 972	15	9.92 663	26	10.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	10.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	10.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	10.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	10.07 234	9.88 266	11	45
16	9.81 047	14	9.92 792	25	10.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	10.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	10.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	10.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	10.07 106	9.88 212	11	40
21	9.81 121	15	9.92 920	25	10.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	10.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	10.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	10.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	10.06 978	9.88 158	10	35
26	9.81 195	15	9.93 048	25	10.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	10.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	10.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	10.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	10.06 850	9.88 105	11	30
31	9.81 269	15	9.93 175	26	10.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	10.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	10.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	10.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	10.06 722	9.88 051	11	25
36	9.81 343	15	9.93 303	26	10.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	10.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	10.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	20
41	9.81 417	14	9.93 431	26	10.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	10.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	10.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	25	10.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	10.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	10.06 339	9.87 887	10	10
51	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	10.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	10.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	10.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	10.06 211	9.87 833	11	5
56	9.81 636	15	9.93 814	26	10.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	10.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	10.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	10.06 109	9.87 789	11	1
60	9.81 694		9.93 916		10.06 084	9.87 778		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts		
″	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	15	14
1	0.2	0.2
2	0.5	0.5
3	0.8	0.7
4	1.0	0.9
5	1.2	1.2
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.2	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

″	11	10
1	0.2	0.2
2	0.4	0.3
3	0.6	0.5
4	0.7	0.7
5	0.9	0.8
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.6	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

Proportional parts

130° (310°) (229°) **49°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

41° (221°) (318°) **138°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11	60
1	9.81 709	14	9.93 942	25	10.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	10.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	10.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	10.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	10.05 956	9.87 723	11	55
6	9.81 781	15	9.94 069	26	10.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	10.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	10.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	10.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	10.05 829	9.87 668	11	50
11	9.81 854	14	9.94 197	25	10.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	10.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	10.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	10.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	10.05 701	9.87 613	12	45
16	9.81 926	14	9.94 324	26	10.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	10.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	10.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	10.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	10.05 574	9.87 557	11	40
21	9.81 998	14	9.94 452	25	10.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	10.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	10.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	10.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	10.05 446	9.87 501	11	35
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	10.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	10.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	10.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	10.05 319	9.87 446	12	30
31	9.82 141	14	9.94 706	26	10.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	10.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	10.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	10.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	10.05 192	9.87 390	12	25
36	9.82 212	14	9.94 834	25	10.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	10.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	10.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	10.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	10.05 065	9.87 334	12	20
41	9.82 283	14	9.94 961	25	10.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	10.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	10.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	10.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	10.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	25	10.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	10.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	10.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	10.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	10.04 810	9.87 221	12	10
51	9.82 424	15	9.95 215	25	10.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	10.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	10.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	10.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	10.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	26	10.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	10.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	10.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	10.04 582	9.87 119	12	1
60	9.82 551		9.95 444		10.04 556	9.87 107		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	15	14
1	0.2	0.2
2	0.5	0.5
3	0.8	0.7
4	1.0	0.9
5	1.2	1.2
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.2	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

″	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

131° (311°) (228°) **48°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

42° (222°) (317°) **137°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.82 551	14	9.95 444	25	10.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	26	10.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	10.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	10.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	10.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	26	10.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	10.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	10.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	10.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	10.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	10.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	10.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	10.04 175	9.86 936	12	45
16	9.82 775	13	9.95 850	25	10.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	10.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	10.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	10.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	10.04 048	9.86 879	12	40
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	10.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	10.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	10.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	10.03 922	9.86 821	12	35
26	9.82 913	14	9.96 104	25	10.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	10.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	10.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	10.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	10.03 795	9.86 763	11	30
31	9.82 982	14	9.96 231	25	10.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	10.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	10.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	10.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	10.03 668	9.86 705	11	25
36	9.83 051	14	9.96 357	26	10.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	10.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	10.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	10.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	10.03 541	9.86 647	12	20
41	9.83 120	13	9.96 484	26	10.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	10.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	10.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	10.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	10.03 414	9.86 589	12	15
46	9.83 188	14	9.96 611	25	10.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	10.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	10.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	10.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	10.03 288	9.86 530	12	10
51	9.83 256	14	9.96 738	25	10.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	10.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	10.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	10.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	10.03 161	9.86 472	12	5
56	9.83 324	14	9.96 864	26	10.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	10.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	10.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	10.03 060	9.86 425	12	1
60	9.83 378		9.96 966		10.03 034	9.86 413		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

//	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

//	14	13
1	0.2	0.2
2	0.5	0.4
3	0.7	0.6
4	0.9	0.9
5	1.2	1.1
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

//	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

Proportional parts

132° (312°) (227°) **47°**

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

43° (223°) (316°) **136°**

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.83 378	14	9.96 966	25	10.03 034	9.86 413	12	60
1	9.83 392	13	9.96 991	25	10.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	10.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	10.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	10.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	10.02 908	9.86 354	12	55
6	9.83 459	14	9.97 118	25	10.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	10.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	10.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	10.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	10.02 781	9.86 295	12	50
11	9.83 527	13	9.97 244	25	10.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	10.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	10.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	10.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	10.02 655	9.86 235	12	45
16	9.83 594	14	9.97 371	25	10.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	10.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	10.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	10.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	10.02 528	9.86 176	12	40
21	9.83 661	13	9.97 497	26	10.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	10.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	10.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	10.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	10.02 402	9.86 116	12	35
26	9.83 728	13	9.97 624	25	10.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	10.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	10.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	10.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	10.02 275	9.86 056	12	30
31	9.83 795	13	9.97 750	26	10.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	10.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	10.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	10.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	10.02 149	9.85 996	12	25
36	9.83 861	13	9.97 877	25	10.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	10.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	10.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	10.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	10.02 022	9.85 936	12	20
41	9.83 927	13	9.98 003	26	10.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	10.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	10.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	10.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	10.01 896	9.85 876	12	15
46	9.83 993	13	9.98 130	25	10.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	10.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	10.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	10.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	10.01 769	9.85 815	12	10
51	9.84 059	13	9.98 256	25	10.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	10.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	10.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	10.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	10.01 643	9.85 754	12	5
56	9.84 125	13	9.98 383	25	10.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	10.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	10.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	10.01 542	9.85 706	13	1
60	9.84 177		9.98 484		10.01 516	9.85 693		0
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Proportional parts

″	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

″	14	13
1	0.2	0.2
2	0.5	0.4
3	0.7	0.6
4	0.9	0.9
5	1.2	1.1
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

″	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

Proportional parts

133° (313°) (226°) **46°**

6.16 DEFINITION

The **exponential function** $\mathbf{e}^x$ is the inverse of the logarithm function and is given by

$$\mathbf{e}^x = \mathbf{ln}^{-1}(x).$$

6.17 DEFINITION

The **exponential function** $\mathbf{a}^x$ is defined by

$$\mathbf{a}^x = \mathbf{log}_a^{-1}(x) \quad a > 0,\ a \neq 1.$$

6.18 PROPERTIES OF EXPONENTIAL FUNCTIONS

(1) $$\mathbf{a}^x = \mathbf{e}^{x \ln a}.$$

(2) $$\mathbf{a}^{x_1}\mathbf{a}^{x_2} = \mathbf{a}^{x_1+x_2}.$$

(3) $$\frac{\mathbf{a}^{x_1}}{\mathbf{a}^{x_2}} = \mathbf{a}^{x_1-x_2}.$$

(4) $$(\mathbf{a}^{x_1})^{x_2} = \mathbf{a}^{x_1 x_2}.$$

(5) $$\mathbf{e}^{\ln x} = \mathbf{ln}\ \mathbf{e}^x = x.$$

6.19 DEFINITIONS OF HYPERBOLIC FUNCTIONS AND THEIR INVERSES

(1) $$\mathbf{sinh}\ x = \tfrac{1}{2}(\mathbf{e}^x - \mathbf{e}^{-x}).$$

(2) $$\mathbf{cosh}\ x = \tfrac{1}{2}(\mathbf{e}^x + \mathbf{e}^{-x}).$$

(3) $$\mathbf{tanh}\ x = \frac{\mathbf{sinh}\ x}{\mathbf{cosh}\ x}.$$

(4) $$\mathbf{coth}\ x = \frac{1}{\mathbf{tanh}\ x}.$$

(5) $$\mathbf{sech}\ x = \frac{1}{\mathbf{cosh}\ x}.$$

(6) $$\mathbf{csch}\ x = \frac{1}{\mathbf{sinh}\ x}.$$

(7) $$\mathbf{sinh}^{-1} x = \mathbf{ln}\,(x + \sqrt{x^2 + 1}).$$ *Domain:* $(-\infty, \infty)$.

(8) $\cosh^{-1} x = \ln (x + \sqrt{x^2 - 1})$. *Domain:* $[1, \infty)$.

(9) $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1 + x}{1 - x}\right)$. *Domain:* $(-1, 1)$.

(10) $\coth^{-1} x = \frac{1}{2} \ln \left(\frac{x + 1}{x - 1}\right)$. *Domain:* $(-\infty, -1), (1, \infty)$.

(11) $\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$. *Domain:* $(0, 1]$.

(12) $\operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{|x|}\right)$. *Domain:* $(-\infty, 0), (0, \infty)$.

6.20 PROPERTIES OF HYPERBOLIC FUNCTIONS

(1) $\cosh^2 x - \sinh^2 x = 1$.

(2) $\operatorname{sech}^2 x + \tanh^2 x = 1$.

(3) $\operatorname{csch}^2 x - \coth^2 x = -1$.

(4) $\sinh (-x) = -\sinh x$.

(5) $\cosh (-x) = \cosh x$.

(6) $\tanh (-x) = -\tanh x$.

(7) $\sinh (x_1 \pm x_2) = \sinh x_1 \cosh x_2 \pm \cosh x_1 \sinh x_2$.

(8) $\cosh (x_1 \pm x_2) = \cosh x_1 \cosh x_2 \pm \sinh x_1 \sinh x_2$.

(9) $\sinh 2x = 2 \sinh x \cosh x$.

(10) $\cosh 2x = \cosh^2 x + \sinh^2 x$.

(11) $2 \sinh^2 \frac{x}{2} = \cosh x - 1$.

(12) $2 \cosh^2 \frac{x}{2} = \cosh x + 1$.

Table 6.21

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
0.00	1.0000	1.00000	0.0000	1.0000	.00000
0.01	1.0101	0.99005	0.0100	1.0001	.01000
0.02	1.0202	.98020	0.0200	1.0002	.02000
0.03	1.0305	.97045	0.0300	1.0005	.02999
0.04	1.0408	.96079	0.0400	1.0008	.03998
0.05	1.0513	.95123	0.0500	1.0013	.04996
0.06	1.0618	.94176	0.0600	1.0018	.05993
0.07	1.0725	.93239	0.0701	1.0025	.06989
0.08	1.0833	.92312	0.0801	1.0032	.07983
0.09	1.0942	.91393	0.0901	1.0041	.08976
0.10	1.1052	.90484	0.1002	1.0050	.09967
0.11	1.1163	.89583	0.1102	1.0061	.10956
0.12	1.1275	.88692	0.1203	1.0072	.11943
0.13	1.1388	.87809	0.1304	1.0085	.12927
0.14	1.1503	.86936	0.1405	1.0098	.13909
0.15	1.1618	.86071	0.1506	1.0113	.14889
0.16	1.1735	.85214	0.1607	1.0128	.15865
0.17	1.1853	.84366	0.1708	1.0145	.16838
0.18	1.1972	.83527	0.1810	1.0162	.17808
0.19	1.2092	.82696	0.1911	1.0181	.18775
0.20	1.2214	.81873	0.2013	1.0201	.19738
0.21	1.2337	.81058	0.2115	1.0221	.20697
0.22	1.2461	.80252	0.2218	1.0243	.21652
0.23	1.2586	.79453	0.2320	1.0266	.22603
0.24	1.2712	.78663	0.2423	1.0289	.23550
0.25	1.2840	.77880	0.2526	1.0314	.24492
0.26	1.2969	.77105	0.2629	1.0340	.25430
0.27	1.3100	.76338	0.2733	1.0367	.26362
0.28	1.3231	.75578	0.2837	1.0395	.27291
0.29	1.3364	.74826	0.2941	1.0423	.28213
0.30	1.3499	.74082	0.3045	1.0453	.29131
0.31	1.3634	.73345	0.3150	1.0484	.30044
0.32	1.3771	.72615	0.3255	1.0516	.30951
0.33	1.3910	.71892	0.3360	1.0549	.31852
0.34	1.4049	.71177	0.3466	1.0584	.32748
0.35	1.4191	.70469	0.3572	1.0619	.33638
0.36	1.4333	.69768	0.3678	1.0655	.34521
0.37	1.4477	.69073	0.3785	1.0692	.35399
0.38	1.4623	.68386	0.3892	1.0731	.36271
0.39	1.4770	.67706	0.4000	1.0770	.37136
0.40	1.4918	.67032	0.4108	1.0811	.37995
0.41	1.5068	.66365	0.4216	1.0852	.38847
0.42	1.5220	.65705	0.4325	1.0895	.39693
0.43	1.5373	.65051	0.4434	1.0939	.40532
0.44	1.5527	.64404	0.4543	1.0984	.41364
0.45	1.5683	.63763	0.4653	1.1030	.42190
0.46	1.5841	.63128	0.4764	1.1077	.43008
0.47	1.6000	.62500	0.4875	1.1125	.43820
0.48	1.6161	.61878	0.4986	1.1174	.44624
0.49	1.6323	.61263	0.5098	1.1225	.45422
0.50	1.6487	.60653	0.5211	1.1276	.46212

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
0.50	1.6487	.60653	0.5211	1.1276	.46212
0.51	1.6653	.60050	0.5324	1.1329	.46995
0.52	1.6820	.59452	0.5438	1.1383	.47770
0.53	1.6989	.58860	0.5552	1.1438	.48538
0.54	1.7160	.58275	0.5666	1.1494	.49299
0.55	1.7333	.57695	0.5782	1.1551	.50052
0.56	1.7507	.57121	0.5897	1.1609	.50798
0.57	1.7683	.56553	0.6014	1.1669	.51536
0.58	1.7860	.55990	0.6131	1.1730	.52267
0.59	1.8040	.55433	0.6248	1.1792	.52990
0.60	1.8221	.54881	0.6367	1.1855	.53705
0.61	1.8404	.54335	0.6485	1.1919	.54413
0.62	1.8589	.53794	0.6605	1.1984	.55113
0.63	1.8776	.53259	0.6725	1.2051	.55805
0.64	1.8965	.52729	0.6846	1.2119	.56490
0.65	1.9155	.52205	0.6967	1.2188	.57167
0.66	1.9348	.51685	0.7090	1.2258	.57836
0.67	1.9542	.51171	0.7213	1.2330	.58498
0.68	1.9739	.50662	0.7336	1.2402	.59152
0.69	1.9937	.50158	0.7461	1.2476	.59798
0.70	2.0138	.49659	0.7586	1.2552	.60437
0.71	2.0340	.49164	0.7712	1.2628	.61068
0.72	2.0544	.48675	0.7838	1.2706	.61691
0.73	2.0751	.48191	0.7966	1.2785	.62307
0.74	2.0959	.47711	0.8094	1.2865	.62915
0.75	2.1170	.47237	0.8223	1.2947	.63515
0.76	2.1383	.46767	0.8353	1.3030	.64108
0.77	2.1598	.46301	0.8484	1.3114	.64693
0.78	2.1815	.45841	0.8615	1.3199	.65271
0.79	2.2034	.45384	0.8748	1.3286	.65841
0.80	2.2255	.44933	0.8881	1.3374	.66404
0.81	2.2479	.44486	0.9015	1.3464	.66959
0.82	2.2705	.44043	0.9150	1.3555	.67507
0.83	2.2933	.43605	0.9286	1.3647	.68048
0.84	2.3164	.43171	0.9423	1.3470	.68581
0.85	2.3396	.42741	0.9561	1.3835	.69107
0.86	2.3632	.42316	0.9700	1.3932	.69626
0.87	2.3869	.41895	0.9840	1.4029	.70137
0.88	2.4109	.41478	0.9981	1.4128	.70642
0.89	2.4351	.41066	1.0122	1.4229	.71139
0.90	2.4596	.40657	1.0265	1.4331	.71630
0.91	2.4843	.40252	1.0409	1.4434	.72113
0.92	2.5093	.39852	1.0554	1.4539	.72590
0.93	2.5345	.39455	1.0700	1.4645	.73059
0.94	2.5600	.39063	1.0847	1.4753	.73522
0.95	2.5857	.38674	1.0995	1.4862	.73978
0.96	2.6117	.38289	1.1144	1.4973	.74428
0.97	2.6379	.37908	1.1294	1.5085	.74870
0.98	2.6645	.37531	1.1446	1.5199	.75307
0.99	2.6912	.37158	1.1598	1.5314	.75735
1.00	2.7183	.36788	1.1752	1.5431	.76159

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	sinh x	cosh x	tanh x
1.00	2.7183	.36788	1.1752	1.5431	.76159
1.01	2.7456	.36422	1.1907	1.5549	.76576
1.02	2.7732	.36060	1.2063	1.5669	.76987
1.03	2.8011	.35701	1.2220	1.5790	.77391
1.04	2.8292	.35345	1.2379	1.5913	.77789
1.05	2.8577	.34994	1.2539	1.6038	.78181
1.06	2.8864	.34646	1.2700	1.6164	.78566
1.07	2.9154	.34301	1.2862	1.6292	.78946
1.08	2.9447	.33960	1.3025	1.6421	.79320
1.09	2.9743	.33622	1.3190	1.6552	.79688
1.10	3.0042	.33287	1.3356	1.6685	.80050
1.11	3.0344	.32956	1.3524	1.6820	.80406
1.12	3.0649	.32628	1.3693	1.6956	.80757
1.13	3.0957	.32303	1.3863	1.7093	.81102
1.14	3.1268	.31982	1.4035	1.7233	.81441
1.15	3.1582	.31664	1.4208	1.7374	.81775
1.16	3.1899	.31349	1.4382	1.7517	.82104
1.17	3.2220	.31037	1.4558	1.7662	.82427
1.18	3.2544	.30728	1.4735	1.7808	.82745
1.19	3.2871	.30422	1.4914	1.7957	.83058
1.20	3.3201	.30119	1.5095	1.8107	.83365
1.21	3.3535	.29820	1.5276	1.8258	.83668
1.22	3.3872	.29523	1.5460	1.8412	.83965
1.23	3.4212	.29229	1.5645	1.8568	.84258
1.24	3.4556	.28938	1.5831	1.8725	.84546
1.25	3.4903	.28650	1.6019	1.8884	.84828
1.26	3.5254	.28365	1.6209	1.9045	.85106
1.27	3.5609	.28083	1.6400	1.9208	.85380
1.28	3.5966	.27804	1.6593	1.9373	.85648
1.29	3.6328	.27527	1.6788	1.9540	.85913
1.30	3.6693	.27253	1.6984	1.9709	.86172
1.31	3.7062	.26982	1.7182	1.9880	.86428
1.32	3.7434	.26714	1.7381	2.0053	.86678
1.33	3.7810	.26448	1.7583	2.0228	.86925
1.34	3.8190	.26185	1.7786	2.0404	.87167
1.35	3.8574	.25924	1.7991	2.0583	.87405
1.36	3.8962	.25666	1.8198	2.0764	.87639
1.37	3.9354	.25411	1.8406	2.0947	.87869
1.38	3.9749	.25158	1.8617	2.1132	.88095
1.39	4.0149	.24908	1.8829	2.1320	.88317
1.40	4.0552	.24660	1.9043	2.1509	.88535
1.41	4.0960	.24414	1.9259	2.1700	.88749
1.42	4.1371	.24171	1.9477	2.1894	.88960
1.43	4.1787	.23931	1.9697	2.2090	.89167
1.44	4.2207	.23693	1.9919	2.2288	.89370
1.45	4.2631	.23457	2.0143	2.2488	.89569
1.46	4.3060	.23224	2.0369	2.2691	.89765
1.47	4.3492	.22993	2.0597	2.2896	.89958
1.48	4.3929	.22764	2.0827	2.3103	.90147
1.49	4.4371	.22537	2.1059	2.3312	.90332
1.50	4.4817	.22313	2.1293	2.3524	.90515

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	sinh x	cosh x	tanh x
1.50	4.4817	.22313	2.1293	2.3524	.90515
1.51	4.5267	.22091	2.1529	2.3738	.90694
1.52	4.5722	.21871	2.1768	2.3955	.90870
1.53	4.6182	.21654	2.2008	2.4174	.91042
1.54	4.6646	.21438	2.2251	2.4395	.91212
1.55	4.7115	.21225	2.2496	2.4619	.91379
1.56	4.7588	.21014	2.2743	2.4845	.91542
1.57	4.8066	.20805	2.2993	2.5073	.91703
1.58	4.8550	.20598	2.3245	2.5305	.91860
1.59	4.9037	.20393	2.3499	2.5538	.92015
1.60	4.9530	.20190	2.3756	2.5775	.92167
1.61	5.0028	.19989	2.4015	2.6013	.92316
1.62	5.0531	.19790	2.4276	2.6255	.92462
1.63	5.1039	.19593	2.4540	2.6499	.92606
1.64	5.1552	.19398	2.4806	2.6746	.92747
1.65	5.2070	.19205	2.5075	2.6995	.92886
1.66	5.2593	.19014	2.5346	2.7247	.93022
1.67	5.3122	.18825	2.5620	2.7502	.93155
1.68	5.3656	.18637	2.5896	2.7760	.93286
1.69	5.4195	.18452	2.6175	2.8020	.93415
1.70	5.4739	.18268	2.6456	2.8283	.93541
1.71	5.5290	.18087	2.6740	2.8549	.93665
1.72	5.5845	.17907	2.7027	2.8818	.93786
1.73	5.6407	.17728	2.7317	2.9090	.93906
1.74	5.6973	.17552	2.7609	2.9364	.94023
1.75	5.7546	.17377	2.7904	2.9642	.94138
1.76	5.8124	.17204	2.8202	2.9922	.94250
1.77	5.8709	.17033	2.8503	3.0206	.94361
1.78	5.9299	.16864	2.8806	3.0492	.94470
1.79	5.9895	.16696	2.9112	3.0782	.94576
1.80	6.0496	.16530	2.9422	3.1075	.94681
1.81	6.1104	.16365	2.9734	3.1371	.94783
1.82	6.1719	.16203	3.0049	3.1669	.94884
1.83	6.2339	.16041	3.0367	3.1972	.94983
1.84	6.2965	.15882	3.0689	3.2277	.95080
1.85	6.3598	.15724	3.1013	3.2585	.95175
1.86	6.4237	.15567	3.1340	3.2897	.95268
1.87	6.4883	.15412	3.1671	3.3212	.95359
1.88	6.5535	.15259	3.2005	3.3530	.95449
1.89	6.6194	.15107	3.2341	3.3852	.95537
1.90	6.6859	.14957	3.2682	3.4177	.95624
1.91	6.7531	.14808	3.3025	3.4506	.95709
1.92	6.8210	.14661	3.3372	3.4838	.95792
1.93	6.8895	.14515	3.3722	3.5173	.95873
1.94	6.9588	.14370	3.4075	3.5512	.95953
1.95	7.0287	.14227	3.4432	3.5855	.96032
1.96	7.0993	.14086	3.4792	3.6201	.96109
1.97	7.1707	.13946	3.5156	3.6551	.96185
1.98	7.2427	.13807	3.5523	3.6904	.96259
1.99	7.3155	.13670	3.5894	3.7261	.96331
2.00	7.3891	.13534	3.6269	3.7622	.96403

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
2.00	7.3891	.13534	3.6269	3.7622	.96403
2.01	7.4633	.13399	3.6647	3.7987	.96473
2.02	7.5383	.13266	3.7028	3.8355	.96541
2.03	7.6141	.13134	3.7414	3.8727	.96609
2.04	7.6906	.13003	3.7803	3.9103	.96675
2.05	7.7679	.12873	3.8196	3.9483	.96740
2.06	7.8460	.12745	3.8593	3.9867	.96803
2.07	7.9248	.12619	3.8993	4.0255	.96865
2.08	8.0045	.12493	3.9398	4.0647	.96926
2.09	8.0849	.12369	3.9806	4.1043	.96986
2.10	8.1662	.12246	4.0219	4.1443	.97045
2.11	8.2482	.12124	4.0635	4.1847	.97103
2.12	8.3311	.12003	4.1056	4.2256	.97159
2.13	8.4149	.11884	4.1480	4.2669	.97215
2.14	8.4994	.11765	4.1909	4.3085	.97269
2.15	8.5849	.11648	4.2342	4.3507	.97323
2.16	8.6711	.11533	4.2779	4.3932	.97375
2.17	8.7583	.11418	4.3221	4.4362	.97426
2.18	8.8463	.11304	4.3666	4.4797	.97477
2.19	8.9352	.11192	4.4116	4.5236	.97526
2.20	9.0250	.11080	4.4571	4.5679	.97574
2.21	9.1157	.10970	4.5030	4.6127	.97622
2.22	9.2073	.10861	4.5494	4.6580	.97668
2.23	9.2999	.10753	4.5962	4.7037	.97714
2.24	9.3933	.10646	4.6434	4.7499	.97759
2.25	9.4877	.10540	4.6912	4.7966	.97803
2.26	9.5831	.10435	4.7394	4.8437	.97846
2.27	9.6794	.10331	4.7880	4.8914	.97888
2.28	9.7767	.10228	4.8372	4.9395	.97929
2.29	9.8749	.10127	4.8868	4.9881	.97970
2.30	9.9742	.10026	4.9370	5.0372	.98010
2.31	10.074	.09926	4.9876	5.0868	.98049
2.32	10.176	.09827	5.0387	5.1370	.98087
2.33	10.278	.09730	5.0903	5.1876	.98124
2.34	10.381	.09633	5.1425	5.2388	.98161
2.35	10.486	.09537	5.1951	5.2905	.98197
2.36	10.591	.09442	5.2483	5.3427	.98233
2.37	10.697	.09348	5.3020	5.3954	.98267
2.38	10.805	.09255	5.3562	5.4487	.98301
2.39	10.913	.09163	5.4109	5.5026	.98335
2.40	11.023	.09072	5.4662	5.5569	.98367
2.41	11.134	.08982	5.5221	5.6119	.98400
2.42	11.246	.08892	5.5785	5.6674	.98431
2.43	11.359	.08804	5.6354	5.7235	.98462
2.44	11.473	.08716	5.6929	5.7801	.98492
2.45	11.588	.08629	5.7510	5.8373	.98522
2.46	11.705	.08543	5.8097	5.8951	.98551
2.47	11.822	.08458	5.8689	5.9535	.98579
2.48	11.941	.08374	5.9288	6.0125	.98607
2.49	12.061	.08291	5.9892	6.0721	.98635
2.50	12.182	.08208	6.0502	6.1323	.98661

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
2.50	12.182	.08208	6.0502	6.1323	.98661
2.51	12.305	.08127	6.1118	6.1931	.98688
2.52	12.429	.08046	6.1741	6.2545	.98714
2.53	12.554	.07966	6.2369	6.3166	.98739
2.54	12.680	.07887	6.3004	6.3793	.98764
2.55	12.807	.07808	6.3645	6.4426	.98788
2.56	12.936	.07730	6.4293	6.5066	.98812
2.57	13.066	.07654	6.4946	6.5712	.98835
2.58	13.197	.07577	6.5607	6.6365	.98858
2.59	13.330	.07502	6.6274	6.7024	.98881
2.60	13.464	.07427	6.6947	6.7690	.98903
2.61	13.599	.07353	6.7628	6.8363	.98924
2.62	13.736	.07280	6.8315	6.9043	.98946
2.63	13.874	.07208	6.9008	6.9729	.98966
2.64	14.013	.07136	6.9709	7.0423	.98987
2.65	14.154	.07065	7.0417	7.1123	.99007
2.66	14.296	.06995	7.1132	7.1831	.99026
2.67	14.440	.06925	7.1854	7.2546	.99045
2.68	14.585	.06856	7.2583	7.3268	.99064
2.69	14.732	.06788	7.3319	7.3998	.99083
2.70	14.880	.06721	7.4063	7.4735	.99101
2.71	15.029	.06654	7.4814	7.5479	.99118
2.72	15.180	.06587	7.5572	7.6231	.99136
2.73	15.333	.06522	7.6338	7.6991	.99153
2.74	15.487	.06457	7.7112	7.7758	.99170
2.75	15.643	.06393	7.7894	7.8533	.99186
2.76	15.800	.06329	7.8683	7.9316	.99202
2.77	15.959	.06266	7.9480	8.0106	.99218
2.78	16.119	.06204	8.0285	8.0905	.99233
2.79	16.281	.06142	8.1098	8.1712	.99248
2.80	16.445	.06081	8.1919	8.2527	.99263
2.81	16.610	.06020	8.2749	8.3351	.99278
2.82	16.777	.05961	8.3586	8.4182	.99292
2.83	16.945	.05901	8.4432	8.5022	.99306
2.84	17.116	.05843	8.5287	8.5871	.99320
2.85	17.288	.05784	8.6150	8.6728	.99333
2.86	17.462	.05727	8.7021	8.7594	.99346
2.87	17.637	.05670	8.7902	8.8469	.99359
2.88	17.814	.05613	8.8791	8.9352	.99372
2.89	17.993	.05558	8.9689	9.0244	.99384
2.90	18.174	.05502	9.0596	9.1146	.99396
2.91	18.357	.05448	9.1512	9.2056	.99408
2.92	18.541	.05393	9.2437	9.2976	.99420
2.93	18.728	.05340	9.3371	9.3905	.99431
2.94	18.916	.05287	9.4315	9.4844	.99443
2.95	19.106	.05234	9.5268	9.5791	.99454
2.96	19.298	.05182	9.6231	9.6749	.99464
2.97	19.492	.05130	9.7203	9.7716	.99475
2.98	19.688	.05079	9.8185	9.8693	.99485
2.99	19.886	.05029	9.9177	9.9680	.99496
3.00	20.086	.04979	10.018	10.068	.99505

Table 6.21 (Cont'd)

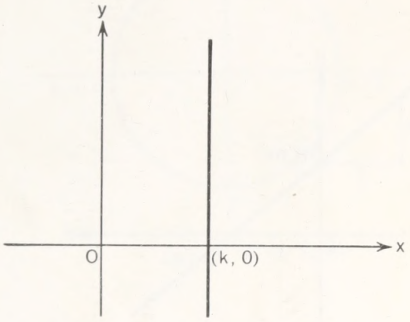
VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
3.00	20.086	.04979	10.018	10.068	.99505
3.05	21.115	.04736	10.534	10.581	.99552
3.10	22.198	.04505	11.076	11.122	.99595
3.15	23.336	.04285	11.647	11.690	.99633
3.20	24.533	.04076	12.246	12.287	.99668
3.25	25.790	.03877	12.876	12.915	.99700
3.30	27.113	.03688	13.538	13.575	.99728
3.35	28.503	.03508	14.234	14.269	.99754
3.40	29.964	.03337	14.965	14.999	.99777
3.45	31.500	.03175	15.734	15.766	.99799
3.50	33.115	.03020	16.543	16.573	.99818
3.55	34.813	.02872	17.392	17.421	.99835
3.60	36.598	.02732	18.286	18.313	.99851
3.65	38.475	.02599	19.224	19.250	.99865
3.70	40.447	.02472	20.211	20.236	.99878
3.75	42.521	.02352	21.249	21.272	.99889
3.80	44.701	.02237	22.339	22.362	.99900
3.85	46.993	.02128	23.486	23.507	.99909
3.90	49.402	.02024	24.691	24.711	.99918
3.95	51.935	.01925	25.958	25.977	.99926
4.00	54.598	.01832	27.290	27.308	.99933
4.10	60.340	.01657	30.162	30.178	.99945
4.20	66.686	.01500	33.336	33.351	.99955
4.30	73.700	.01357	36.843	36.857	.99963
4.40	81.451	.01227	40.719	40.732	.99970
4.50	90.017	.01111	45.003	45.014	.99975
4.60	99.484	.01005	49.737	49.747	.99980
4.70	109.95	.00910	54.969	54.978	.99983
4.80	121.51	.00823	60.751	60.755	.99986
4.90	134.29	.00745	67.141	67.149	.99989
5.00	148.41	.00674	74.203	74.210	.99991
5.10	164.02	.00610	82.008	82.014	.99993
5.20	181.27	.00552	90.633	90.639	.99994
5.30	200.34	.00499	100.17	100.17	.99995
5.40	221.41	.00452	110.70	110.71	.99996
5.50	244.69	.00409	122.34	122.35	.99997
5.60	270.43	.00370	135.21	135.22	.99997
5.70	298.87	.00335	149.43	149.44	.99998
5.80	330.30	.00303	165.15	165.15	.99998
5.90	365.04	.00274	182.52	182.52	.99998
6.00	403.43	.00248	201.71	201.72	.99999
6.25	518.01	.00193	259.01	259.01	.99999
6.50	665.14	.00150	332.57	332.57	1.0000
6.75	854.06	.00117	427.03	427.03	1.0000
7.00	1096.6	.00091	548.32	548.32	1.0000
7.50	1808.0	.00055	904.02	904.02	1.0000
8.00	2981.0	.00034	1490.5	1490.5	1.0000
8.50	4914.8	.00020	2457.4	2457.4	1.0000
9.00	8103.1	.00012	4051.5	4051.5	1.0000
9.50	13360.	.00007	6679.9	6679.9	1.0000
10.00	22026.	.00005	11013.	11013.	1.0000

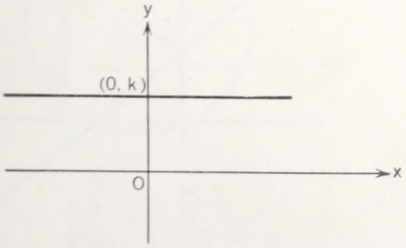
7

TWO- AND THREE-DIMENSIONAL GRAPHS

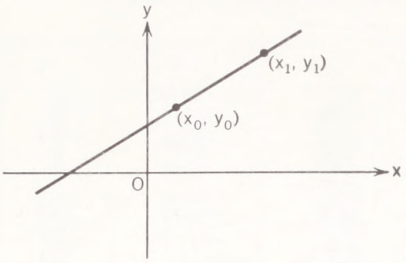
7.1 THE STRAIGHT LINE



(a) $x = k$

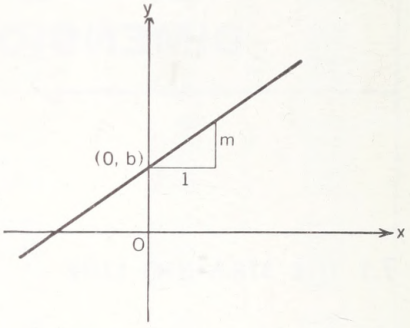


(b) $y = k$

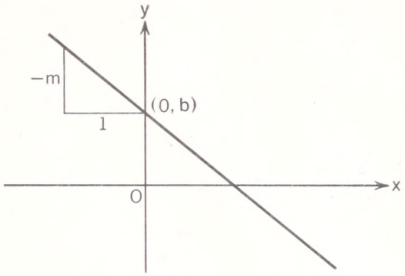


(c) $y - y_0 = \dfrac{y_1 - y_0}{x_1 - x_0}(x - x_0)$,

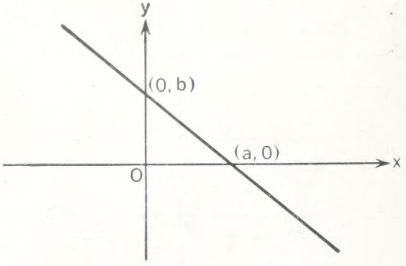
$x_1 \neq x_0$



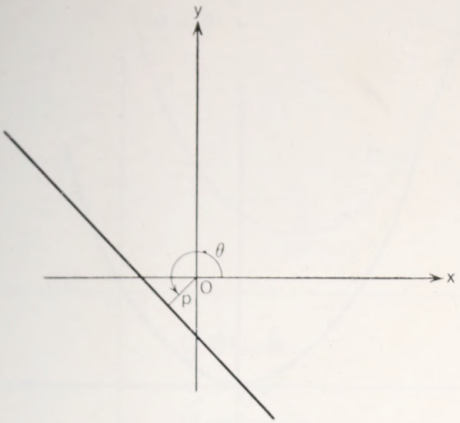
(d) $y = mx + b, \quad m > 0$



(e) $y = mx + b, \quad m < 0$

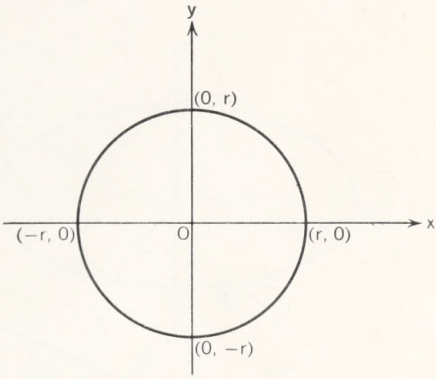


(f) $\dfrac{x}{a} + \dfrac{y}{b} = 1$

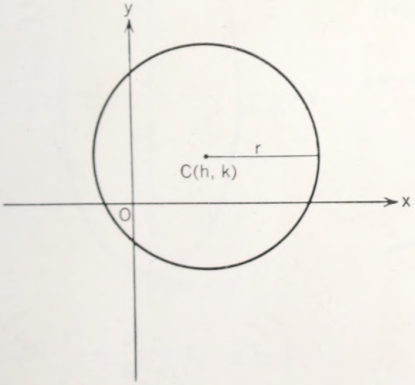


(g) $x \cos \theta + y \sin \theta - p = 0$

7.2 THE CIRCLE

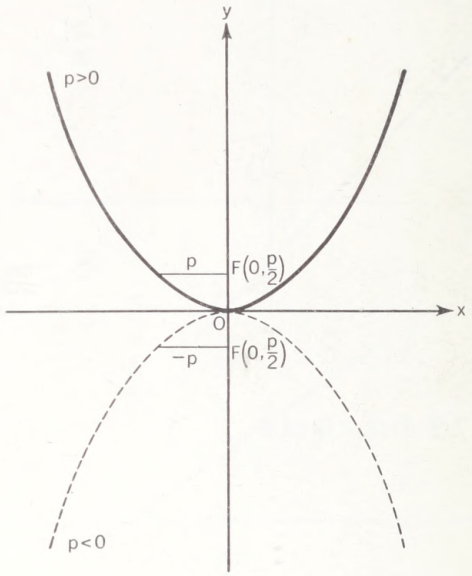


(a) $x^2 + y^2 = r^2$

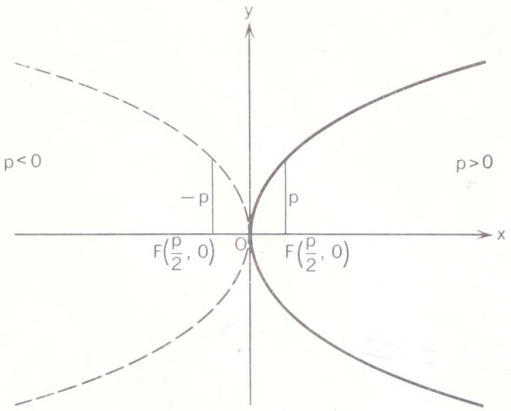


(b) $(x - h)^2 + (y - k)^2 = r^2$

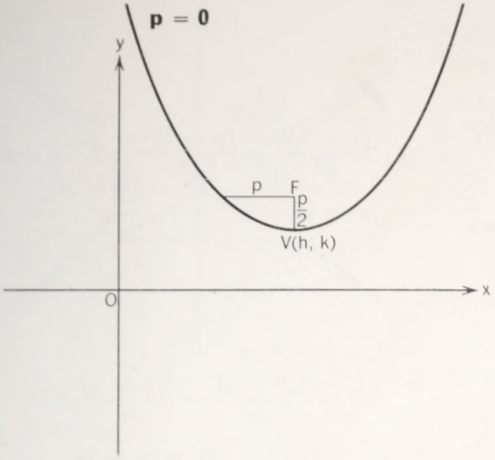
7.3 THE PARABOLA



(a) $y = \dfrac{1}{2p}x^2$

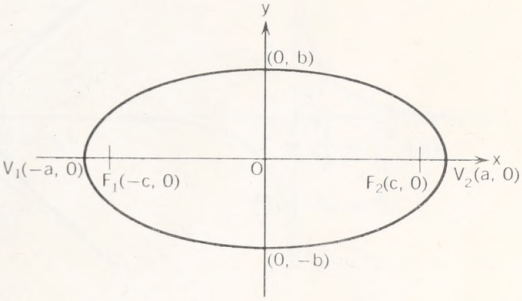


(b) $y^2 = 2px$



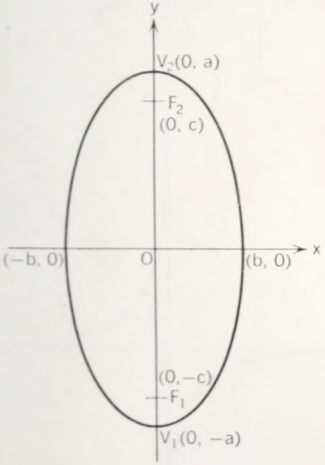
(c) $(y - k) = \dfrac{1}{2p}(x - h)^2$

7.4 THE ELLIPSE



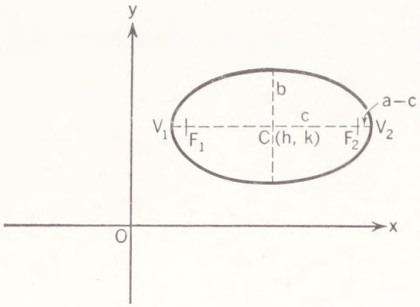
(a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \quad a > b$

$c^2 = a^2 - b^2$



(b) $\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = 1, \quad a > b$

$c^2 = a^2 - b^2$



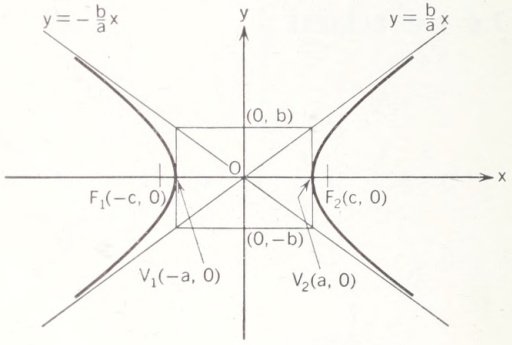
(c) $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1, \quad a > b$

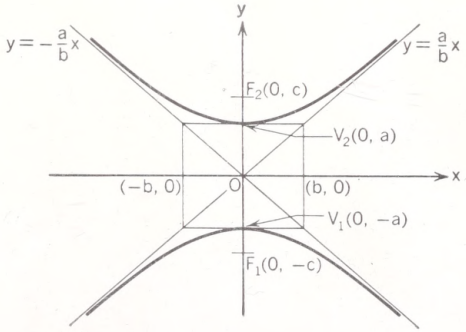
$c^2 = a^2 - b^2$

7.5 THE HYPERBOLA

(a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

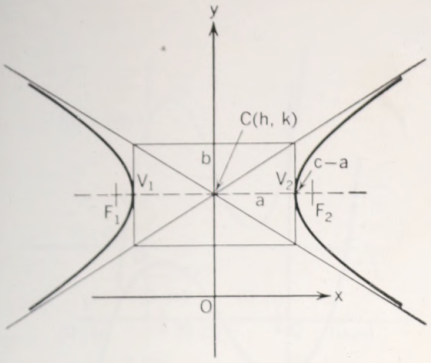
$c^2 = a^2 + b^2$





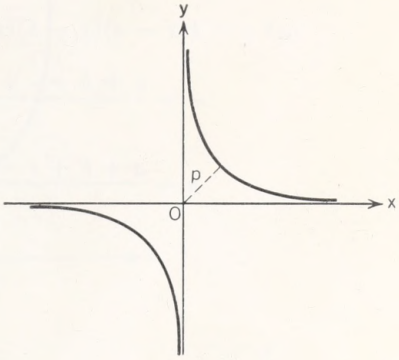
(b) $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

$c^2 = a^2 + b^2$

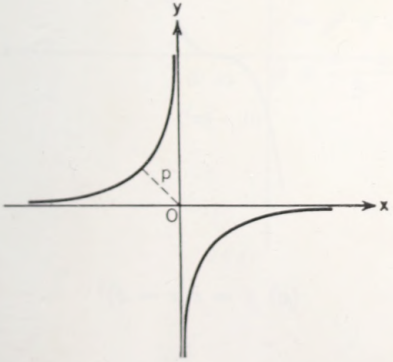


(**c**) $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

$c^2 = a^2 + b^2$

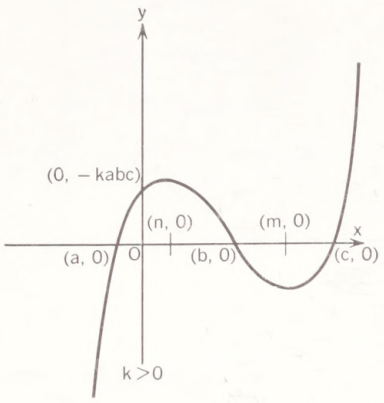


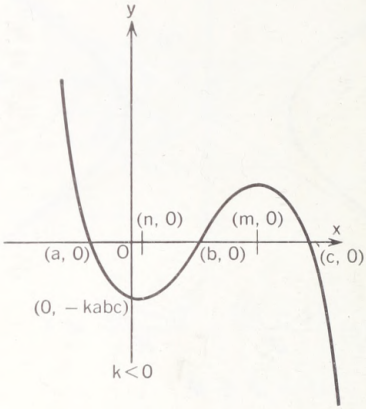
(**d**) $xy = \dfrac{p^2}{2}$



(**e**) $xy = -\dfrac{p^2}{2}$

7.6 THE CUBIC

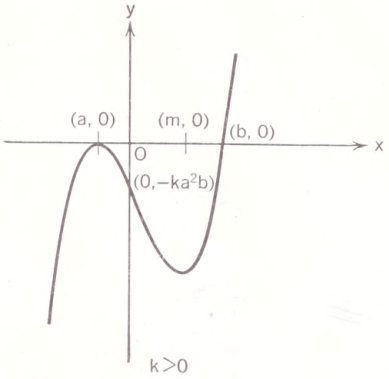




(**a**) $y = k(x - a)(x - b)(x - c)$, a, b, c all different

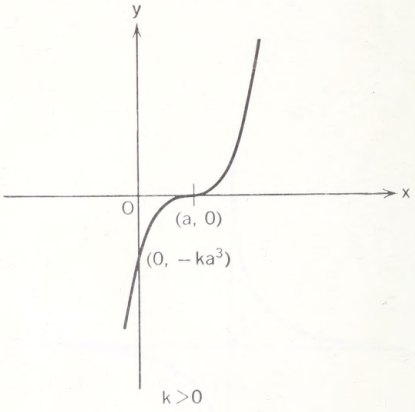
$$m = \frac{a + b + c + \sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{3}$$

$$n = \frac{a + b + c - \sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{3}$$

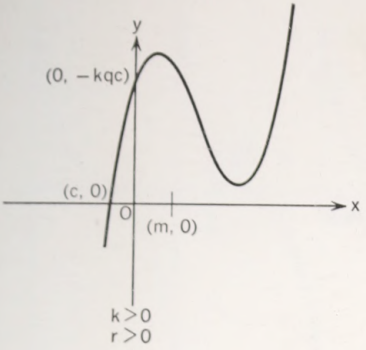


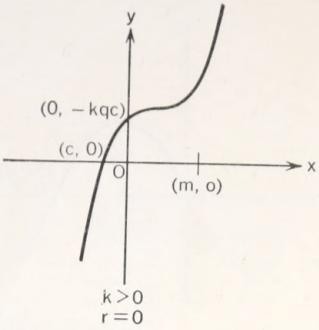
(**b**) $y = k(x - a)^2(x - b)$

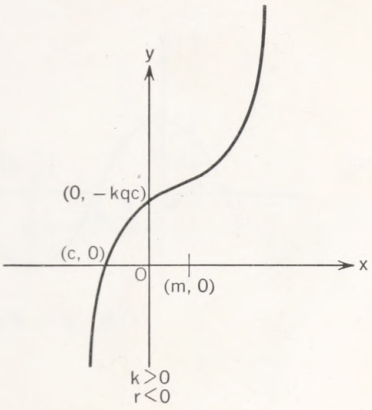
$$m = \frac{a + b}{2}$$



(**c**) $y = k(x - a)^3$





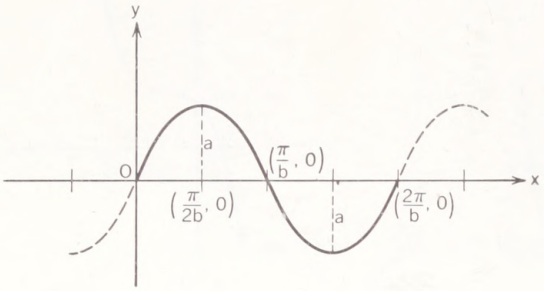


(**d**) $y = k(x^2 + px + q)(x - c), \quad p^2 - 4q < 0$

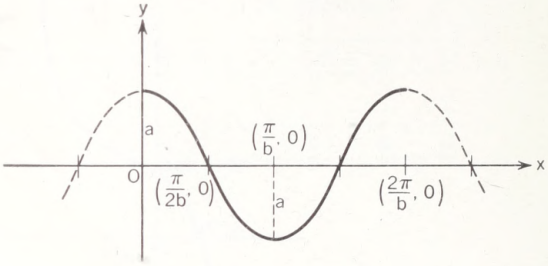
$$r = p^2 + c^2 + pc - 3q$$

$$m = \frac{c - p}{3}$$

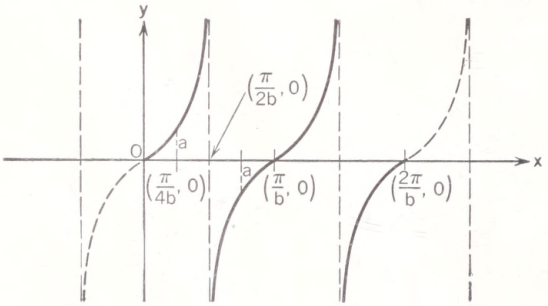
7.7 THE TRIGONOMETRIC FUNCTIONS



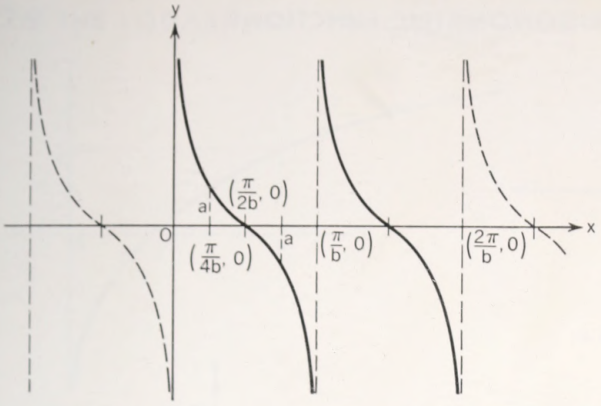
(a) $y = a \sin bx$



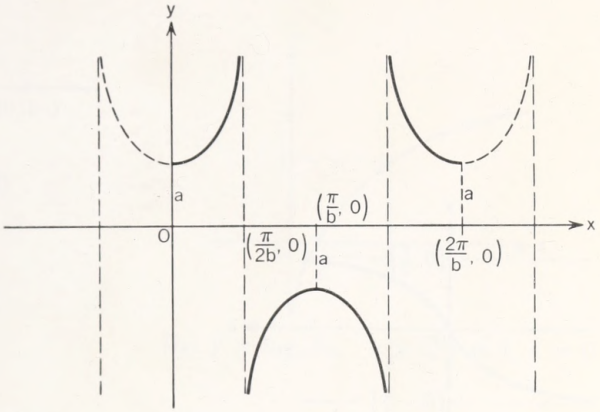
(b) $y = a \cos bx$



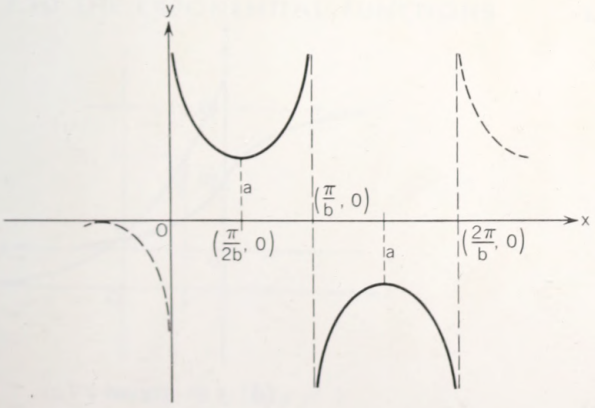
(c) $y = a \tan bx$



(**d**) $y = a$ **cot** bx

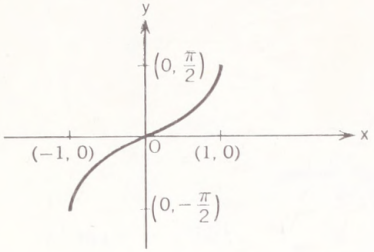


(**e**) $y = a$ **sec** bx

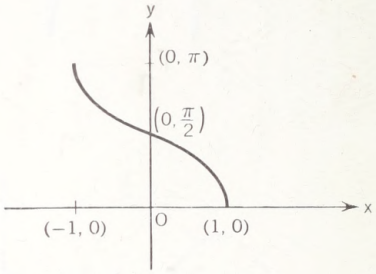


(**f**) $y = a$ **csc** bx

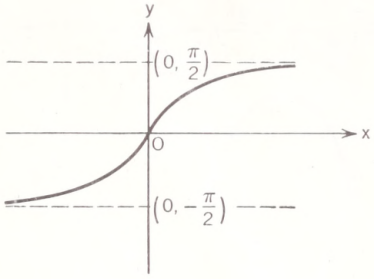
7.8 THE INVERSE TRIGONOMETRIC FUNCTIONS



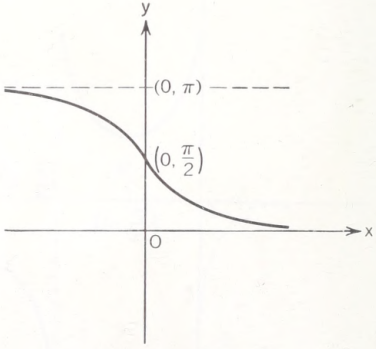
(a) $y = \arcsin x$



(b) $y = \arccos x$

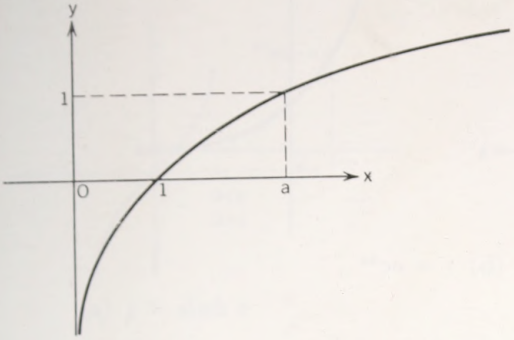


(c) $y = \arctan x$

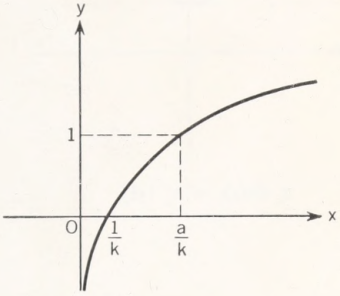


(d) $y = \operatorname{arccot} x$

7.9 THE LOGARITHMIC FUNCTIONS

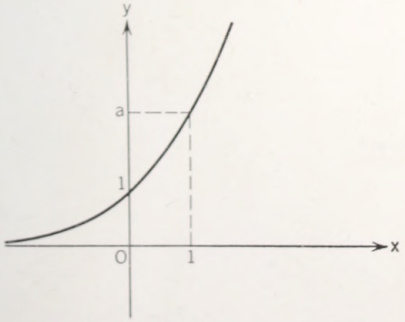


(a) $y = \log_a x, \quad a > 0, a \neq 1$

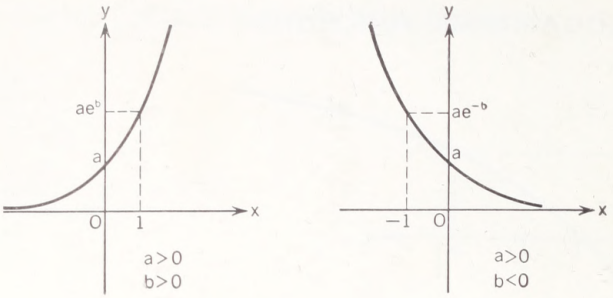


(b) $y = \log_a kx, \quad a > 0, a \neq 1, k = 0$

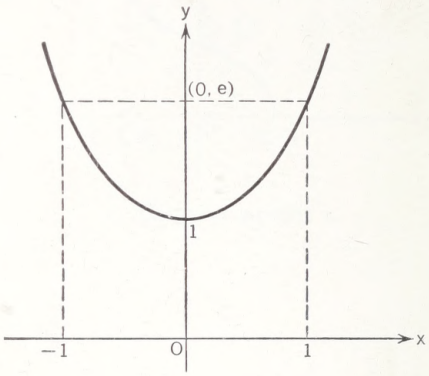
7.10 THE EXPONENTIAL FUNCTIONS



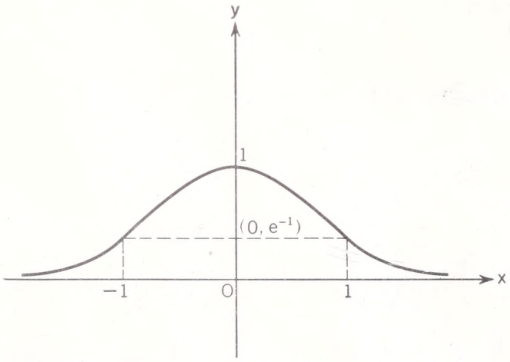
(a) $y = a^x, \quad a > 0, a \neq 1$



(b) $y = a\mathbf{e}^{bx}$

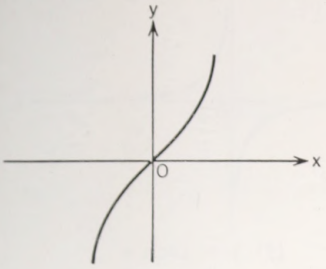


(c) $y = \mathbf{e}^{x^2}$

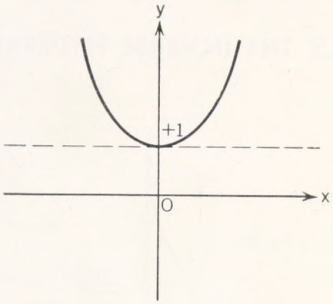


(d) $y = \mathbf{e}^{-x^2}$

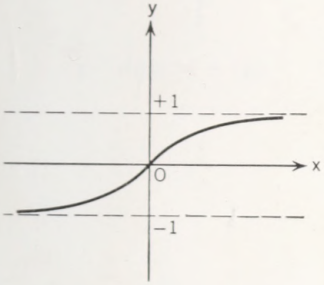
7.11 THE HYPERBOLIC FUNCTIONS



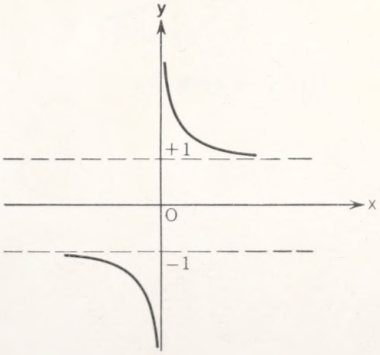
(a) $y = \sinh x$



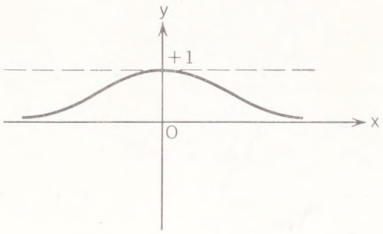
(b) $y = \cosh x$



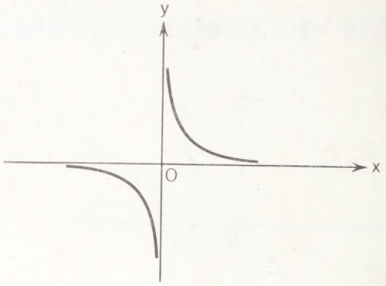
(c) $y = \tanh x$



(d) $y = \coth x$

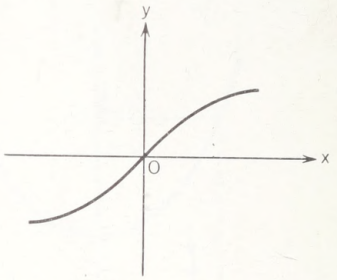


(e) $y = \mathbf{sech}\ x$

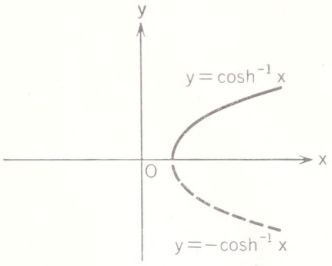


(f) $y = \mathbf{csch}\ x$

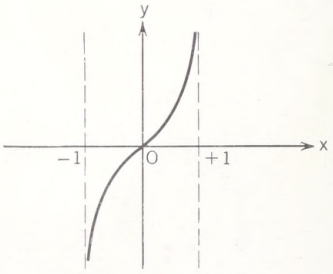
7.12 THE INVERSE HYPERBOLIC FUNCTIONS



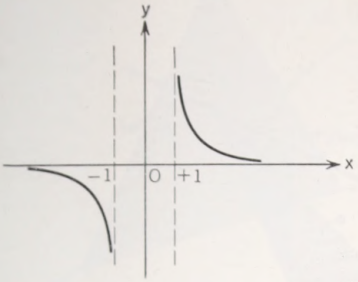
(a) $y = \mathbf{sinh}^{-1}\ x$



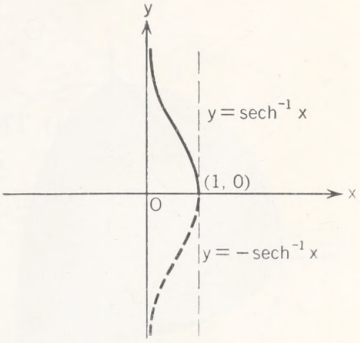
(b) $y = \mathbf{cosh}^{-1}\ x$



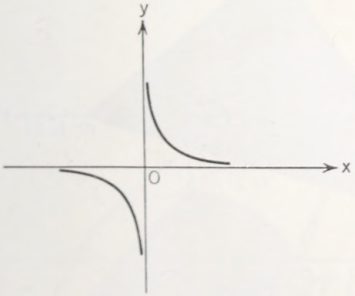
(c) $y = \mathbf{tanh}^{-1}\ x$



(d) $y = \coth^{-1} x$

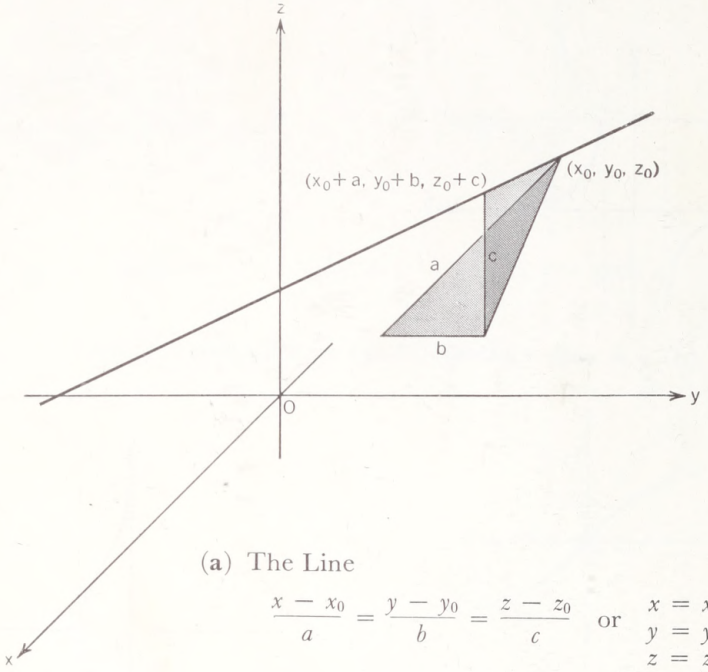


(e) $y = \operatorname{sech}^{-1} x$



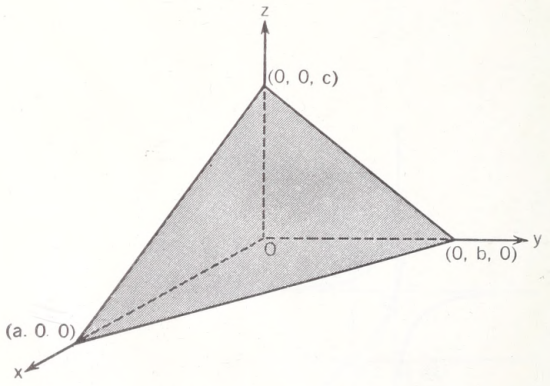
(f) $y = \operatorname{csch}^{-1} x$

7.13 THREE-DIMENSIONAL GRAPHS



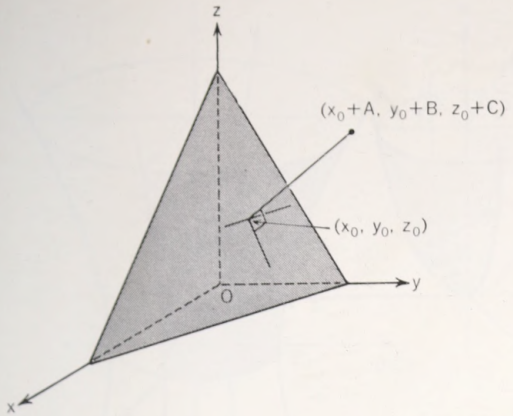
(a) The Line

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad \text{or} \quad \begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$



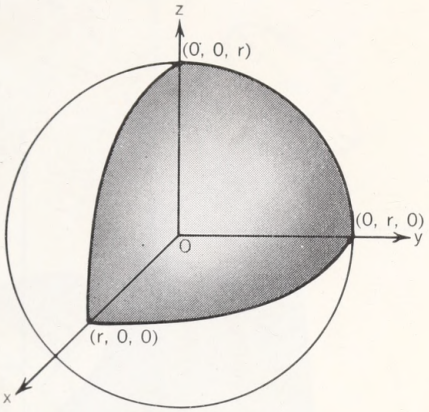
(b) The Plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



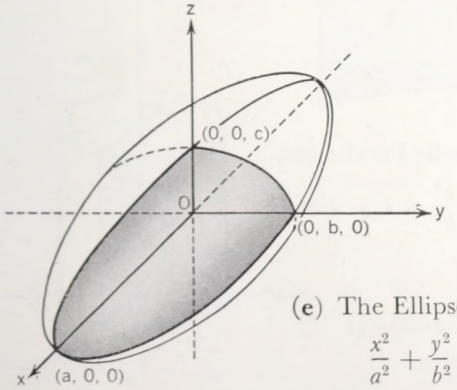
(c) The Plane

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$



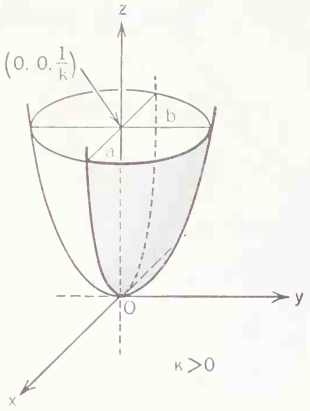
(d) The Sphere

$$x^2 + y^2 + z^2 = r^2$$



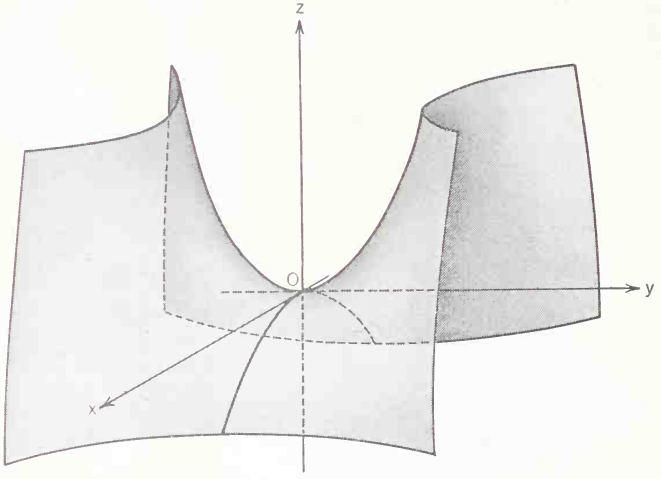
(e) The Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



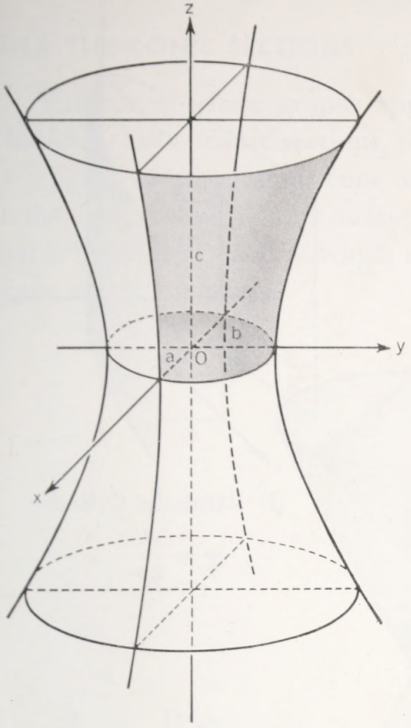
(f) Elliptic Paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = kz$$



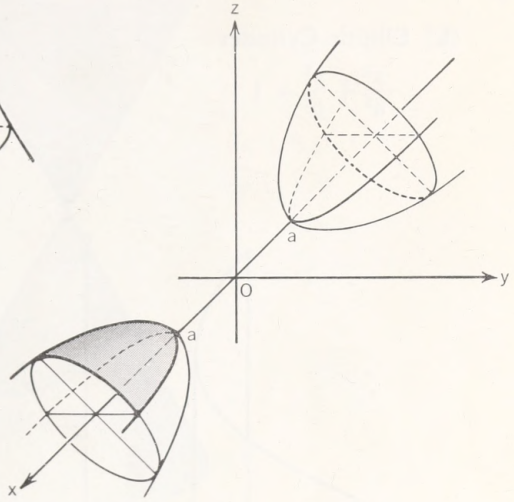
(g) Hyperbolic Paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = kz, \quad k < 0$$



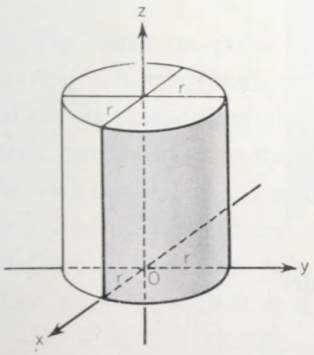
(h) Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



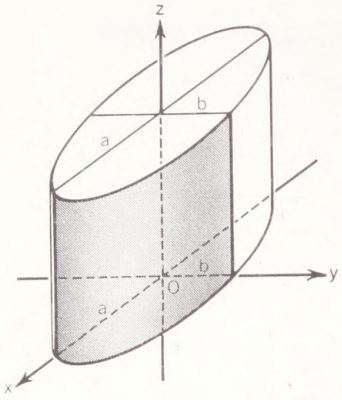
(i) Hyperboloid of Two Sheets

$$\frac{x^2}{a} - \frac{y}{b^2} - \frac{z^2}{c^2} = 1$$



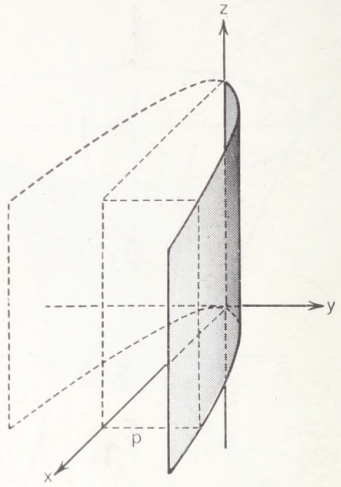
(j) Circular Cylinder

$$x^2 + y^2 = r^2$$



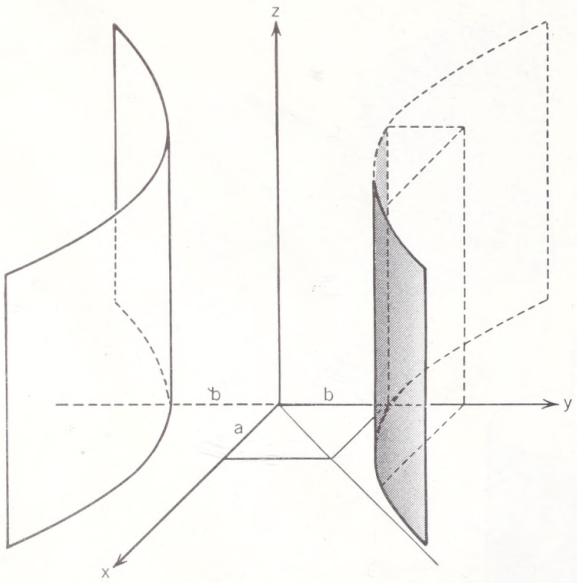
(**k**) Elliptic Cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



(**l**) Parabolic Cylinder

$$\mathbf{y} = \frac{x^2}{2p}$$

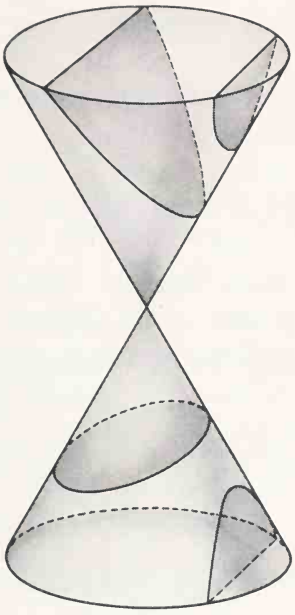


(**m**) Hyperbolic Cylinder

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

7.14 THE CONIC SECTIONS

The curves whose graphs appear in Secs. 7.2 to 7.5 are known classically as the **conic sections,** since each of them is the intersection of a plane with a circular cone of two nappes. Such a cone appears in the figure below. On its surface are sketched an ellipse, a parabola, and a hyperbola, each of which is obtained as the curve in which a plane cuts the cone.



The common point of the two nappes of the cone is called its **vertex;** a line through the vertex lying on the surface of the cone is called an **element** of the cone. The line through the vertex which forms equal angles at the vertex with all elements of the cone is called the **axis** of the cone. Let the angle between this line and an element be called α. Let P be a plane containing the axis of the cone, and let Q be a plane that intersects P in a line perpendicular to the axis at an angle β, with $0 \leqq \beta \leqq 90°$.

(1) If $\alpha < \beta \leqq 90°$:

(a) If $\beta \leqq 90°$ and Q does not contain the vertex, then the intersection of Q and the cone is an ellipse.

(b) If $\beta \leqq 90°$ and Q does contain the vertex, then the intersection of Q and the cone is a point, called a degenerate ellipse.

(c) If $\beta = 90°$ and Q does not contain the vertex, then the intersection of Q and the cone is a circle. Thus, the circle is a special case of an ellipse.

(d) If $\beta = 90°$ and Q does contain the vertex, then the intersection of Q and the cone is a point, called a degenerate circle.

(2) If $\beta = \alpha$:

(a) If Q does not contain the vertex, then the intersection of Q and the cone is a parabola.

(b) If Q does contain the vertex, then the intersection of Q and the cone is a straight line, called a degenerate parabola.

The degenerate parabola obtained here is a special case of the general degenerate parabola, which consists of two parallel lines. Here, the "parallel" lines are coincident. Although the list of conic sections in this section is complete, it does not include all second-degree loci. It omits only the general degenerate parabola and the null locus, both of which may be obtained as a plane section of a circular cylinder.

(3) If $0 \leqq \beta < \alpha$:

(a) If Q does not contain the vertex, then the intersection of Q and the cone is a hyperbola.

(b) If Q does contain the vertex, then the intersection of Q and the cone is a pair of intersecting straight lines, called a degenerate hyperbola.

8

TYPES OF FUNCTIONAL EQUATIONS

THE TWO principal types of functional equations we shall take up are differential equations and difference equations, defined in Secs. 4.4 and 4.5.

8.1 DERIVATIVES, ANTIDERIVATIVES, AND INDEFINITE INTEGRALS

The simplest kind of differential equation is $\mathbf{Df} = \mathbf{g}$. If $\mathbf{f}$ is given and we want to find $\mathbf{g}$, we take the derivative of $\mathbf{f}$. The basic laws of differentiation and the derivatives of basic functions are given in Chap. 9. If $\mathbf{g}$ is given, then the general solution for $\mathbf{f}$ is the indefinite integral of $\mathbf{g}$, while any particular solution is an antiderivative of $\mathbf{g}$. Basic properties of indefinite integrals are given in Chap. 10, which contains a table of integrals, Table 10.4.

8.2 DEFINITE INTEGRALS

From the fundamental theorem of the integral calculus, the evaluation of a definite integral

$$\int_a^b \mathbf{f}(x)\, dx$$

can sometimes be accomplished by first finding an antiderivative $\mathbf{F}(x)$

of $\mathbf{f}(x)$. Then

$$\int_a^b \mathbf{f}(x)\,dx = \mathbf{F}(b) - \mathbf{F}(a).$$

When it is possible to find an antiderivative, this procedure is usually the best way of evaluating the definite integral. Table 10.7 lists some definite integrals directly; many of these cannot be obtained conveniently by the method of first finding an antiderivative.

8.3 OTHER CLASSES OF DIFFERENTIAL EQUATIONS

A differential equation of the form

$$\mathbf{D}^n\mathbf{f} + \mathbf{h}_1(x)\,\mathbf{D}^{n-1}\mathbf{f} + \mathbf{h}_2(x)\,\mathbf{D}^{n-2}\mathbf{f} + \ldots + \mathbf{h}_{n-1}(x)\,\mathbf{D}\mathbf{f} + \mathbf{h}_n(x)\mathbf{f} = \mathbf{g}(x),$$

where n is a positive integer and $\mathbf{h}_1(x) \ldots \mathbf{h}_n(x)$ and $\mathbf{g}(x)$ are all given functions, is called a **linear differential equation.** A differential equation which cannot be written in this form is called **nonlinear.** If each of the functions $\mathbf{h}_i(x)$ is a constant, the equation is said to be **linear with constant coefficients;** these equations are treated in Chap. 11. The special case in which each $\mathbf{h}_i(x) = 0$ can be handled by n applications of Table 10.4; a few specific repeated indefinite integrals are given directly in Table 10.10. Some more general linear differential equations are discussed in Chap. 13.

Differential equations can also be classified according to their order. A general treatment of first- and second-order differential equations appears in Chap. 12.

8.4 DIFFERENCES, ANTIDIFFERENCES, AND INDEFINITE FINITE INTEGRALS

The simplest kind of difference equation is $\Delta\mathbf{f} = \mathbf{g}$. If $\mathbf{f}$ is given and we want to find $\mathbf{g}$, we find the difference of $\mathbf{f}$. The basic laws of differences are given in Chap. 13, where a table of differences of basic functions is given. If $\mathbf{g}$ is given, then the general solution for $\mathbf{f}$ is the indefinite finite integral of $\mathbf{g}$, while any particular solution is an antidifference of $\mathbf{g}$. Basic properties of indefinite finite integrals are given in Chap. 14, and Table 14.2 lists some of these finite integrals.

8.5 FINITE SUMS

In analogy with the application of the fundamental theorem of the integral calculus to the evaluation of definite integrals, it is sometimes possible to evaluate a finite sum with the aid of an antidifference. Thus to evaluate $\sum_{a}^{b} \mathbf{f}(x)$, if we can find a function $\mathbf{F}(x)$ such that $\Delta\mathbf{F}(x) = \mathbf{f}(x)$, then

$$\sum_{a}^{b} \mathbf{f}(x) = \mathbf{F}(b + 1) - \mathbf{F}(a).$$

Some properties and a table of finite sums appear in Chap. 14.

8.6 OTHER CLASSES OF DIFFERENCE EQUATIONS

For most other classes of difference equations, it is more convenient to express the equation in terms of the operator $\mathbf{E}$ instead of Δ. Conversion from one form to the other can be accomplished by the relation $\mathbf{E} = \Delta + 1$ (see Sec. 3.5). A difference equation of the form

$$\mathbf{E}^n\mathbf{f} + \mathbf{h}_1(x)\,\mathbf{E}^{n-1}\mathbf{f} + \mathbf{h}_2(x)\,\mathbf{E}^{n-2}\mathbf{f} + \ldots + \mathbf{h}_{n-1}(x)\,\mathbf{E}\mathbf{f} + \mathbf{h}_n(x)\mathbf{f} = \mathbf{g}(x),$$

is called a **linear difference equation.** A difference equation which cannot be written in this form is called **nonlinear.** If each of the functions $\mathbf{h}_i(x)$ is a constant, the equation is said to be **linear with constant coefficients;** these equations are treated in Chap. 15. Some more general linear difference equations are discussed in Chap. 16.

9

DERIVATIVES

9.1 TABLE: PROPERTIES OF DERIVATIVES

(1) $\mathbf{D}cf(x) = c\mathbf{D}f(x)$ for any real number c.

(2) $\mathbf{D}[f(x) \pm g(x)] = \mathbf{D}f(x) \pm \mathbf{D}g(x)$.

(3) $\mathbf{D}[f(x)g(x)] = g(x)\,\mathbf{D}f(x) + f(x)\,\mathbf{D}g(x)$.

(4) $$\mathbf{D}\frac{f(x)}{g(x)} = \frac{g(x)\,\mathbf{D}f(x) - f(x)\,\mathbf{D}g(x)}{[g(x)]^2}.$$

(5) $\mathbf{D}f(g(x)) = f'(g(x)) \cdot \mathbf{D}g(x)$, where $f' = \mathbf{D}f$.

9.2 TABLE: DERIVATIVES OF BASIC FUNCTIONS

(1) $\mathbf{D}c = 0$, c a real number.

(2) $\mathbf{D}x = 1$.

(3) $\mathbf{D}x^p = px^{p-1}$, p any real number.

(4) $\mathbf{D}\sin x = \cos x$.

(5) $\mathbf{D}\cos x = -\sin x$

(6) $\mathbf{D}\tan x = \sec^2 x$.

(7) $\mathbf{D}\cot x = -\csc^2 x$.

(8) $\mathbf{D}\sec x = \sec x \tan x$.

(9) $\mathbf{D}\csc x = -\csc x \cot x$

(10) $\mathbf{D}\arcsin x = \dfrac{1}{\sqrt{1 - x^2}}.$

(11) $\mathbf{D}\arccos x = \dfrac{-1}{\sqrt{1 - x^2}}.$

(12) $\mathbf{D}\arctan x = \dfrac{1}{1 + x^2}.$

(13) $\mathbf{D}\operatorname{arccot} x = \dfrac{-1}{1 + x^2}.$

(14) $\mathbf{D}\operatorname{arcsec} x = \dfrac{1}{x\sqrt{x^2 - 1}}.$

(15) $\mathbf{D}\operatorname{arccsc} x = \dfrac{-1}{|x|\sqrt{x^2 - 1}}.$

(16) $\mathbf{D}\ln x = \dfrac{1}{x}.$

(17) $\mathbf{D}\log_a x = \dfrac{1}{x \ln a}.$

(18) $\mathbf{D}e^x = e^x.$

(19) $\mathbf{D}a^x = a^x \ln a.$

(20) $\mathbf{D}\sinh x = \cosh x.$

(21) $\mathbf{D}\cosh x = \sinh x.$

(22) $\mathbf{D}\tanh x = \operatorname{sech}^2 x.$

(23) $\mathbf{D}\coth x = -\operatorname{csch}^2 x.$

(24) $\mathbf{D}\operatorname{sech} x = -\operatorname{sech} x \tanh x.$

(25) $\mathbf{D}\operatorname{csch} x = -\operatorname{csch} x \coth x.$

(26) $\mathbf{D}\sinh^{-1} x = \dfrac{1}{\sqrt{x^2 + 1}}.$

(27) $\mathbf{D}\cosh^{-1} x = \dfrac{1}{\sqrt{x^2 - 1}}.$

(28) $\mathbf{D}\tanh^{-1} x = \dfrac{1}{1 - x^2}.$

(29) $\mathbf{D}\coth^{-1} x = \dfrac{1}{1 - x^2}.$

(30) $$\mathbf{D}\ \mathbf{sech}^{-1}\ x = \frac{-1}{x\sqrt{1 - x^2}}.$$

(31) $$\mathbf{D}\ \mathbf{csch}^{-1}\ x = \frac{-1}{x\sqrt{1 + x^2}}.$$

10

INTEGRALS

10.1 TABLE: PROPERTIES OF INDEFINITE INTEGRALS

(1) $\int c\mathbf{f}(x)\,dx = c\int \mathbf{f}(x)\,dx$, c a real number.

(2) $\int [\mathbf{f}(x) \pm \mathbf{g}(x)]\,dx = \int \mathbf{f}(x)\,dx \pm \int \mathbf{g}(x)\,dx$.

(3) $\int \mathbf{f}(x)\,\mathbf{Dg}(x)\,dx = \mathbf{f}(x)\mathbf{g}(x) - \int \mathbf{g}(x)\,\mathbf{Df}(x)\,dx$, where $\mathbf{g}(x)$ is any antiderivative of $\mathbf{Dg}(x)$.

(4) $\int \mathbf{f}(\mathbf{g}(x))\,\mathbf{Dg}(x)\,dx = \mathbf{F}(\mathbf{g}(x)) + C$, where $\int \mathbf{f}(x)\,dx = \mathbf{F}(x) + C$.

(5) $\int \mathbf{Df}(x)\,dx = \mathbf{f}(x) + C$.

(6) $\mathbf{D}\int \mathbf{f}(x)\,dx = \mathbf{f}(x)$.

10.2 TECHNIQUES OF INTEGRATION

The two basic techniques of integration are substitution and integration by parts [formulas (4) and (3) of Sec. 10.1]. The former is by far the more useful technique. According to (4), an integrand of the form $\mathbf{f}(\mathbf{g}(x))\,\mathbf{Dg}(x)$ for certain functions $\mathbf{f}$ and $\mathbf{g}$ may be treated as simply $\mathbf{f}(x)$, provided that in the result of the integration x is replaced by $\mathbf{g}(x)$. In order not to confuse the various uses of 'x'

in this process, we proceed formally as follows. To integrate a function $\mathbf{h}(x)$:

(1) Attempt to find functions $\mathbf{f}$ and $\mathbf{g}$ such that

$$\mathbf{h}(x) = \mathbf{f}(\mathbf{g}(x))\,\mathbf{Dg}(x).$$

(2) Evaluate $\int \mathbf{f}(u)\,du$ as if 'u' were 'x,' to obtain a result of the form $\mathbf{F}(u) + C$.

(3) Replace 'u' in this result by $\mathbf{g}(x)$.

Example 1. Find $\int x^2\mathbf{e}^{2x^3}\,dx$. Let us choose $\mathbf{g}(x) = 2x^3$; then $\mathbf{Dg}(x) = 6x^2$. We may write the integral in the form

$$\tfrac{1}{6}\int \mathbf{e}^{2x^3}(6x^2)\,dx,$$

hence the exponential function is a good choice for $\mathbf{f}$. We evaluate

$$\tfrac{1}{6}\int \mathbf{e}^u\,du = \tfrac{1}{6}\mathbf{e}^u + C,$$

so that

$$\int x^2\mathbf{e}^{2x^3}\,dx = \tfrac{1}{6}\mathbf{e}^{2x^3} + C.$$

Example 2. Find $\displaystyle\int \frac{\mathbf{sin}\ 2x}{1 + \mathbf{cos}^2\ x}\,dx$. Since $\mathbf{sin}\ 2x = 2\ \mathbf{sin}\ x\ \mathbf{cos}\ x$, we shall evaluate $\displaystyle\int \frac{2\ \mathbf{sin}\ x\ \mathbf{cos}\ x}{1 + \mathbf{cos}^2\ x}\,dx$. We choose $\mathbf{g}(x) = \mathbf{cos}\ x$. Then $\mathbf{Dg}(x) = -\mathbf{sin}\ x$, and

$$\int \frac{2\ \mathbf{sin}\ x\ \mathbf{cos}\ x}{1 + \mathbf{cos}^2\ x}\,dx = \int \frac{-2\mathbf{Dg}(x)\cdot\mathbf{g}(x)}{1 + [\mathbf{g}(x)]^2}\,dx.$$

Now we find

$$\int \frac{-2u}{1 + u^2}\,du = -\mathbf{ln}\ (1 + u^2) + C.$$

Replacing u by $\mathbf{cos}\ x$,

$$\int \frac{\mathbf{sin}\ 2x}{1 + \mathbf{cos}^2\ x}\,dx = -\mathbf{ln}\ (1 + \mathbf{cos}^2\ x) + C.$$

Another valuable technique is that of integration by parts, making use of formula (3) of Sec. 10.1,

$$\int \mathbf{f}(x)\ \mathbf{Dg}(x)\,dx = \mathbf{f}(x)\mathbf{g}(x) - \int \mathbf{g}(x)\ \mathbf{Df}(x)\,dx.$$

To make good use of this method of finding the indefinite integral of a function $\mathbf{h}(x)$, we must write $\mathbf{h}(x)$ as a product of two functions $\mathbf{f}(x)$ and $\mathbf{Dg}(x)$, where it is possible to find the indefinite integral of $\mathbf{Dg}(x)$, and where $\mathbf{g}(x)\ \mathbf{Df}(x)$ is easier to integrate than the given function. In many cases, more than one choice of $\mathbf{f}$ and $\mathbf{Dg}$ can be made; usually only one of these leads to an essential simplification of the problem. There are no rules which specify the choices to be made, but only a little practice is needed to learn to reject most bad choices.

Example 3. Find $\int x\mathbf{e}^x\,dx$. We choose $\mathbf{f}(x) = x$ and $\mathbf{Dg}(x) = \mathbf{e}^x$. Then $\mathbf{Df}(x) = 1$ and $\mathbf{g}(x) = \mathbf{e}^x$. Applying formula (3) of Sec. 10.1, we obtain

$$\int x\mathbf{e}^x\,dx = x\mathbf{e}^x - \int \mathbf{e}^x\,dx = x\mathbf{e}^x - \mathbf{e}^x + C.$$

If we had chosen $\mathbf{f}(x) = \mathbf{e}^x$ and $\mathbf{Dg}(x) = x$, then $\mathbf{Df}(x) = \mathbf{e}^x$ and $\mathbf{g}(x) = x^2/2$ and we would have

$$\int x\mathbf{e}^x\,dx = \tfrac{1}{2}x^2\mathbf{e}^x - \tfrac{1}{2}\int x^2\mathbf{e}^x\,dx.$$

While this is a correct application of the rule, it is not a useful one, since the new integral is more complicated than the original.

There are cases in which two or more applications of the process of integration by parts yield a new integral which is a multiple of the original one. In some of these cases, we can solve for the integral by some elementary algebraic manipulations. In case this procedure results in a particular integral, the indefinite integral can be obtained by adding an arbitrary constant.

Example 4. We shall apply integration by parts to $\int \mathbf{e}^x \mathbf{sin}\ x\,dx$. Let $\mathbf{f}(x) = \mathbf{e}^x$ and $\mathbf{Dg}(x) = \mathbf{sin}\ x$; then $\mathbf{Df}(x) = \mathbf{e}^x$, $\mathbf{g}(x) = -\mathbf{cos}\ x$, and

$$\int \mathbf{e}^x \mathbf{sin}\ x\,dx = -\mathbf{e}^x \mathbf{cos}\ x + \int \mathbf{e}^x \mathbf{cos}\ x\,dx.$$

Applying the same process to $\int \mathbf{e}^x \mathbf{cos}\ x\,dx$, let $\mathbf{f}(x) = \mathbf{e}^x$ and $\mathbf{Dg}(x) = \mathbf{cos}\ x$; then $\mathbf{Df}(x) = \mathbf{e}^x$, $\mathbf{g}(x) = \mathbf{sin}\ x$, and

$$\int \mathbf{e}^x \mathbf{cos}\ x\,dx = \mathbf{e}^x \mathbf{sin}\ x - \int \mathbf{e}^x \mathbf{sin}\ x\,dx.$$

We now have two equations in the unknowns $\int \mathbf{e}^x \mathbf{sin}\ x\,dx$ and

$\int \mathbf{e}^x \cos x\, dx$. If we substitute the expression for $\int \mathbf{e}^x \cos x\, dx$ from the second equation into the first and solve for $\int \mathbf{e}^x \sin x\, dx$, we obtain

$$\int \mathbf{e}^x \sin x\, dx = \tfrac{1}{2}\mathbf{e}^x (\sin x - \cos x) + C,$$

where C was added to obtain the indefinite integral. Solving for the other integral, we get

$$\int \mathbf{e}^x \cos x\, dx = \tfrac{1}{2}\mathbf{e}^x (\sin x + \cos x) + C.$$

If an integrand involves x and one of the expressions $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$, then the integration may often be accomplished by the replacement of x by a trigonometric function $\mathbf{g}(x)$ and by integrating an associated expression. In this expression, 'x' is replaced by 'u' in order not to confuse the various uses of 'x.' Such replacements are justified by the converse of formula (4) of Sec. 10.1, which holds if the inverse of $\mathbf{g}$ is a function. We have the rule: To evaluate $\int \mathbf{f}(x)\, dx$, where $\mathbf{f}$ involves one of the above expressions, replace x by a function of the form $\mathbf{g}(u)$, where $\mathbf{g}$ is chosen according to the rules below, and evaluate $\int \mathbf{f}(\mathbf{g}(u))\, \mathbf{Dg}(u)\, du$. In this result, replace 'u' by $\mathbf{g}^{-1}(x)$. The replacement procedures are given in detail in the following rules:

(1) If $\mathbf{f}(x)$ involves x and $\sqrt{a^2 - x^2}$, substitute $a \sin u$ for x, $a \cos u$ for $\sqrt{a^2 - x^2}$, and let $\mathbf{Dg}(u) = a \cos u$. After substitution, the integrated expression is in terms of u. To state it in terms of x, write

$$\arcsin \frac{x}{a} \quad \text{for} \quad u,$$

$$\frac{x}{a} \quad \text{for} \quad \sin u,$$

$$\frac{\sqrt{a^2 - x^2}}{a} \quad \text{for} \quad \cos u,$$

to obtain the final result.

(2) If $\mathbf{f}(x)$ involves x and $\sqrt{a^2 + x^2}$, substitute $a \tan u$ for x, $a \sec u$ for $\sqrt{a^2 + x^2}$ and let $\mathbf{Dg}(h) = a \sec^2 u$. After substitution, the integrated expression is in terms of u. To state it in terms of x, write

$$\begin{array}{ll} \textbf{arctan}\,\dfrac{x}{a} & \text{for} \quad u, \\ \dfrac{x}{a} & \text{for} \quad \textbf{tan}\,u, \\ \dfrac{\sqrt{a^2 + x^2}}{a} & \text{for} \quad \textbf{sec}\,u. \end{array}$$

(3) If $\mathbf{f}(x)$ involves x and $\sqrt{x^2 - a^2}$, substitute $a\,\textbf{sec}\,u$ for x, $a\,\textbf{tan}\,u$ for $\sqrt{x^2 - a^2}$, and let $\mathbf{Dg}(u) = a\,\textbf{sec}\,u\,\textbf{tan}\,u$. After substitution, the integrated expression is in terms of u. To state it in terms of x, write

$$\begin{array}{ll} \textbf{arcsec}\,\dfrac{x}{a} & \text{for} \quad u, \\ \dfrac{x}{a} & \text{for} \quad \textbf{sec}\,u, \\ \dfrac{\sqrt{x^2 - a^2}}{a} & \text{for} \quad \textbf{tan}\,u. \end{array}$$

In each case, use trigonometric identities to find what expressions in x replace the trigonometric functions of u in the integrated expression.

Example 5. Evaluate

$$\int \frac{\sqrt{9 - x^2}\,dx}{x}$$

According to formula (1) of Sec. 10.1, let $\mathbf{g}(u) = 3\,\textbf{sin}\,u$. Then $\sqrt{9 - 9\mathbf{g}^2(u)} = 3\,\textbf{cos}\,u$, and $\mathbf{Dg}(u) = 3\,\textbf{cos}\,u$. Pass to the integral

$$\int \frac{3\,\textbf{cos}\,u}{3\,\textbf{sin}\,u}\,(3\,\textbf{cos}\,u)\,du = 3\int \frac{\textbf{cos}^2\,u\,du}{\textbf{sin}\,u}.$$

Integrating by means of Table 10.4, we obtain

$$\textbf{cos}\,u + \textbf{ln}\left|\textbf{tan}\,\frac{u}{2}\right| + C.$$

From the trigonometric identities,

$$\textbf{tan}\,\frac{u}{2} = \frac{\textbf{sin}\,(u/2)}{\textbf{cos}\,(u/2)} = \sqrt{\frac{1 - \textbf{cos}\,u}{1 + \textbf{cos}\,u}} = \frac{1 - \textbf{cos}\,u}{\textbf{sin}\,u}.$$

Making the replacements indicated in formula (1) of Sec. 10.1,

$$\int \frac{\sqrt{9 - x^2}\, dx}{x} = \frac{\sqrt{9 - x^2}}{3} + \ln \left| \frac{3 - \sqrt{9 - x^2}}{x} \right| + C.$$

10.3 SUGGESTIONS FOR THE USE OF THE TABLE OF INTEGRALS

A particular order for listing integrals has been used in Tables 10.4 and 10.7, so that the reader will be able to find any integrand quickly if he knows the order. The order of integrands is as follows: algebraic functions, trigonometric functions, inverse trigonometric functions, logarithms, exponentials, and hyperbolic functions. Integrands involving more than one function will be placed immediately following whichever function is listed last in this ordering.

In these tables we have adopted the notation that a, b, c, and p designate real numbers; m and n designate integers. C is an arbitrary (real) constant.

To integrate a rational function, it is almost always advisable to perform first a partial fractions decomposition of the integrand (see Sec. 2.8).

A number of integrals in Table 10.4 are recursion formulas; they reduce the problem of finding the integral of a function involving an n-th power to one involving a lower or more convenient power. After a sufficient number of applications of these recursion formulas, the integrand will be in a form which can be found elsewhere in the tables; a reference to such an entry will be found following the recursion formula.

Example 6. Find $\int \sin^7 x\, dx$. Use formula (74) of Table 10.4 with $n = 7$ to obtain

$$\int \sin^7 x\, dx = -\tfrac{1}{7}(\sin^6 x \cos x - 6 \int \sin^5 x\, dx).$$

Using the same formula with $n = 5$, we find

$$\int \sin^5 x\, dx = -\tfrac{1}{5}(\sin^4 x \cos x - 4 \int \sin^3 x\, dx).$$

Finally, since n is odd, we use formula (72) of Table 10.4 to obtain

$$\int \sin^3 x\,dx = -\tfrac{1}{3}\cos x(\sin^2 x + 2) + C.$$

Combining these results we obtain

$$\int \sin^7 x\,dx = -\frac{\sin^6 x\cos x}{7} - \frac{6\sin^4 x\cos x}{35} - \frac{8\cos x\,(\sin^2 x + 2)}{35} + C.$$

If an integrand is not found in the table, one should try to make a substitution which will form an integrand which is in the table. However, one should bear in mind that many integrands are of such a nature that no integral of them can be written in terms of a finite number of elementary functions.

10.4 TABLE: INDEFINITE INTEGRALS

Algebraic Functions

(1) $$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C, \quad \text{if } p \neq -1.$$

(2) $$\int \frac{dx}{x} = \ln|x| + C.$$

Algebraic Functions of x and $ax + b$

(3) $$\int (ax+b)^p\,dx = \frac{(ax+b)^{p+1}}{a(p+1)} + C, \quad \text{if } p \neq -1.$$

(4) $$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C.$$

(5) $$\int \frac{x\,dx}{ax+b} = \frac{1}{a^2}[ax - b\ln|ax+b|] + C.$$

(6) $$\int \frac{x\,dx}{(ax+b)^2} = \frac{1}{a^2}\left[\ln|ax+b| + \frac{b}{ax+b}\right] + C.$$

(7) $$\int \frac{x\,dx}{(ax+b)^3} = \frac{1}{a^2}\left[\frac{b}{2(ax+b)^2} - \frac{1}{ax+b}\right] + C.$$

(8) $$\int x(ax+b)^p\,dx = \frac{1}{a^2(p+2)}(ax+b)^{p+2} - \frac{b}{a^2(p+1)}(ax+b)^{p+1} + C \quad \text{if } p \neq -1,\ p \neq -2.$$

$$(9)\quad \int x^m(ax+b)^n\,dx = \frac{1}{m+n+1}\left[x^{m+1}(ax+b)^n + nb\int x^m(ax+b)^{n-1}\,dx\right], \quad \text{if } m+n+1 \neq 0.$$

See (1).

$$(10)\quad \int x^m(ax+b)^n\,dx = \frac{1}{a(m+n+1)}\left[x^m(ax+b)^{n+1} - mb\int x^{m-1}(ax+b)^n\,dx\right], \quad \text{if } m+n+1 \neq 0.$$

See (3).

$$(11)\quad \int \frac{x^2\,dx}{ax+b} = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2 \ln|ax+b|\right] + C.$$

$$(12)\quad \int \frac{x^2\,dx}{(ax+b)^2} = \frac{1}{a^3}\left[ax + b - 2b\ln|ax+b| - \frac{b^2}{ax+b}\right] + C.$$

$$(13)\quad \int \sqrt{ax+b}\,dx = \frac{2}{3a}\sqrt{(ax+b)^3} + C.$$

$$(14)\quad \int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a} + C.$$

$$(15)\quad \int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)\sqrt{(ax+b)^3}}{15a^2} + C.$$

$$(16)\quad \int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}.$$

See (17) if $b > 0$; (18) if $b > 0$.

$$(17)\quad \int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C = -\frac{2}{\sqrt{b}}\tanh^{-1}\sqrt{\frac{ax+b}{b}} + C, \quad \text{if } b > 0.$$

$$(18)\quad \int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}}\arctan\sqrt{\frac{ax-b}{b}} + C, \quad \text{if } b > 0.$$

$$(19)\quad \int x^2\sqrt{ax+b}\,dx = \frac{2}{105a^3}\sqrt{(ax+b)^3}\,(15a^2x^2 - 12abx + 8b^2) + C.$$

(20) $$\int \frac{x^n\,dx}{\sqrt{ax+b}} = \frac{2}{a(2n+1)}\left[x^n\sqrt{ax+b} - nb\int \frac{x^{n-1}\,dx}{\sqrt{ax+b}}\right].$$

See (14).

(21) $$\int \frac{dx}{x^n\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{dx}{x^{n-1}\sqrt{ax+b}}.$$

See (14).

Algebraic Functions of x and $ax^2 + b$

(22) $$\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}}\,\mathbf{arctan}\,\frac{x\sqrt{ab}}{b} + C, \quad \text{if } ab > 0.$$

(23) $$\int \frac{dx}{ax^2+b} = \frac{1}{2\sqrt{-ab}}\,\mathbf{ln}\left|\frac{b + x\sqrt{-ab}}{b - x\sqrt{-ab}}\right| + C$$

$$= \frac{1}{\sqrt{-ab}}\,\mathbf{tanh}^{-1}\,\frac{x\sqrt{-ab}}{b} + C, \quad \text{if } ab < 0.$$

(24) $$\int \frac{dx}{ax^2-b} = \frac{1}{2\sqrt{ab}}\,\mathbf{ln}\left|\frac{x\sqrt{a} - \sqrt{b}}{x\sqrt{a} + \sqrt{b}}\right| + C, \quad \text{if } a > 0,\ b > 0.$$

(25) $$\int \frac{dx}{-ax^2+b} = \frac{1}{2\sqrt{ab}}\,\mathbf{ln}\left|\frac{\sqrt{b} + x\sqrt{a}}{\sqrt{b} - x\sqrt{a}}\right| + C, \quad \text{if } a > 0,\ b > 0.$$

(26) $$\int \frac{dx}{(ax^2+b)^{m+1}} = \frac{1}{2mb}\left[\frac{x}{(ax^2+b)^m} + (2m-1)\int \frac{dx}{(ax^2+b)^m}\right].$$

See (23) or (24) if $b > 0$; (23) if $b < 0$.

(27) $$\int x(ax^2+b)^p\,dx = \frac{1}{2a}\,\frac{(ax^2+b)^{p+1}}{p+1} + C, \quad \text{if } p \neq -1.$$

(28) $$\int \frac{x\,dx}{ax^2+b} = \frac{1}{2a}\,\mathbf{ln}\,|ax^2+b| + C.$$

(29) $$\int \frac{x\,dx}{(ax^2+b)^{p+1}} = \frac{-1}{2pa(ax^2+b)^p} + C, \quad \text{if } p > 0.$$

(30) $$\int \frac{dx}{x(ax^n+b)} = \frac{1}{bn}\,\mathbf{ln}\left|\frac{x^n}{ax^n+b}\right| + C.$$

(31) $$\int x^p(ax^n + b)^m\,dx = \frac{1}{nm + p + 1}\,[x^{p+1}(ax^n + b)^m + bnm \int x^p(ax^n + b)^{m-1}\,dx] \quad \text{if } m > 0 \text{ and } nm + p + 1 \neq 0.$$

See (1).

(32) $$\int \sqrt{ax^2 + b}\,dx = \frac{x}{2}\sqrt{ax^2 + b} + \frac{b}{2\sqrt{a}} \ln |x\sqrt{a} + \sqrt{ax^2 + b}| + C, \quad \text{if } a > 0.$$

(33) $$\int \sqrt{ax^2 + b}\,dx = \frac{x}{2}\sqrt{ax^2 + b} + \frac{b}{2\sqrt{-a}} \arcsin\left(x\sqrt{\frac{-a}{b}}\right) + C, \quad \text{if } a < 0.$$

Algebraic Functions of x and $ax^2 + bx + c$

$$X = ax^2 + bx + c, \qquad q = 4ac - b^2.$$

(34) $$\int \frac{dx}{ax^2 + bx + c} = \frac{2}{\sqrt{q}} \arctan \frac{2ax + b}{\sqrt{q}} + C, \quad \text{if } q > 0.$$

(35) $$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{\sqrt{-q}} \ln \left|\frac{2ax + b - \sqrt{-q}}{2ax + b + \sqrt{-q}}\right| + C = \frac{-2}{\sqrt{-q}} \tanh^{-1} \frac{2ax + b}{\sqrt{-q}} + C, \quad \text{if } q < 0.$$

(36) $$\int \frac{dx}{(ax^2 + bx + c)^{n+1}} = \frac{2ax + b}{nqX^n} + \frac{2(2n - 1)a}{nq} \int \frac{dx}{X^n}.$$

See (33) if $q > 0$; (34) if $q < 0$.

(37) $$\int \sqrt{ax^2 + bx + c}\,dx = \frac{2ax + b}{4a}\sqrt{X} + \frac{q}{8a} \int \frac{dx}{\sqrt{X}}.$$

See (37) if $a > 0$; (38) if $a < 0$.

(38) $$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{a}} \ln \left|X + x\sqrt{a} + \frac{b}{2\sqrt{a}}\right| + C = \frac{1}{\sqrt{a}} \sinh^{-1} \frac{2ax + b}{\sqrt{q}} + C, \quad \text{if } a > 0.$$

(39) $$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{-a}} \arcsin \frac{-2ax - b}{\sqrt{-q}} + C, \quad \text{if } a < 0.$$

(40) $$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\left[\ln|X| - b\int\frac{dx}{X}\right].$$

See (33) if $q > 0$; (34) if $q < 0$.

Algebraic Functions of x and $x^2 \pm a^2$

(41) $$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C.$$

(42) $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C = -\frac{1}{a}\coth^{-1}\frac{x}{a} + C.$$

(43) $$\int \sqrt{x^2\pm a^2}\,dx = \tfrac{1}{2}[x\sqrt{x^2\pm a^2} \pm a^2\ln|x+\sqrt{x^2\pm a^2}|] + C.$$

(44) $$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln|x+\sqrt{x^2\pm a^2}| + C.$$

(45) $$\int x\sqrt{x^2\pm a^2}\,dx = \tfrac{1}{3}\sqrt{(x^2\pm a^2)^3} + C.$$

(46) $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| + C.$$

(47) $$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\operatorname{arcsec}\frac{x}{a} + C.$$

(48) $$\int \frac{x\,dx}{\sqrt{x^2\pm a^2}} = \sqrt{x^2\pm a^2} + C.$$

(49) $$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{x^2+a^2}+a}{x}\right| + C$$

$$= -\frac{1}{a}\operatorname{arcsinh}\frac{a}{x} + C.$$

(50) $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{x}{a} + C = \frac{1}{a}\arccos\frac{a}{x} + C.$$

(51) $$\int \sqrt{(x^2\pm a^2)^3}\,dx = \tfrac{1}{8}[2x\sqrt{(x^2\pm a^2)^3} \pm 3a^2x\sqrt{x^2\pm a^2}$$

$$+ 3a^4\ln|x-\sqrt{x^2\pm a^2}|] + C.$$

(52) $$\int \frac{dx}{\sqrt{(x^2\pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2\pm a^2}} + C.$$

(53) $$\int x\sqrt{(x^2\pm a^2)^3}\,dx = \tfrac{1}{5}\sqrt{(x^2\pm a^2)^5} + C.$$

$$(54)\quad \int \frac{x\,dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{x^2 \pm a^2}} + C.$$

Algebraic Functions of x and $a^2 - x^2$

$$(55)\quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a + x}{a - x} \right| + C$$

$$= \frac{1}{a} \tanh^{-1} \frac{x}{a} + C.$$

$$(56)\quad \int \sqrt{a^2 - x^2}\,dx = \tfrac{1}{2} \left(x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) + C.$$

$$(57)\quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$= -\arccos \frac{x}{a} + C.$$

$$(58)\quad \int x\sqrt{a^2 - x^2}\,dx = -\tfrac{1}{3}\sqrt{(a^2 - x^2)^3} + C.$$

$$(59)\quad \int \frac{\sqrt{a^2 - x^2}}{x}\,dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C.$$

$$(60)\quad \int \frac{x\,dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C.$$

$$(61)\quad \int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$= -\frac{1}{a} \cosh^{-1} \frac{a}{x} + C.$$

$$(62)\quad \int \sqrt{(a^2 - x^2)^3}\,dx = \tfrac{1}{8} \Big[2x\sqrt{(a^2 - x^2)^3} + 3a^2 x\sqrt{a^2 - x^2} + 3a^4 \arcsin \frac{x}{a} \Big] + C.$$

$$(63)\quad \int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2\sqrt{a^2 - x^2}} + C.$$

$$(64)\quad \int x\sqrt{(a^2 - x^2)^3}\,dx = -\tfrac{1}{5}\sqrt{(a^2 - x^2)^5} + C.$$

$$(65)\quad \int \frac{x\,dx}{\sqrt{(a^2 - x^2)^3}} = \frac{1}{\sqrt{a^2 - x^2}} + C.$$

Other Algebraic Functions

$$(66)\quad \int \sqrt{2ax - x^2}\,dx = \tfrac{1}{2}\left[(x - a)\sqrt{2ax - x^2} + a^2 \operatorname{arcsin} \frac{x - a}{a}\right] + C.$$

$$(67)\quad \int \frac{dx}{\sqrt{2ax - x^2}} = \operatorname{arccos} \frac{a - x}{a} + C.$$

$$(68)\quad \int \sqrt{\frac{1 + x}{1 - x}}\,dx = \operatorname{arcsin} x - \sqrt{1 - x^2} + C.$$

Trigonometric Functions

$$(69)\quad \int \sin x\,dx = -\cos x + C.$$

$$(70)\quad \int \sin (ax + b)\,dx = -\frac{1}{a}\cos (ax + b) + C.$$

$$(71)\quad \int \sin^2 (ax + b)\,dx = \frac{x}{2} - \frac{1}{2a}\cos (ax + b)\sin (ax + b) + C$$
$$= \frac{x}{2} - \frac{\sin 2(ax + b)}{4a} + C_1.$$

$$(72)\quad \int \sin^3 (ax + b)\,dx = -\frac{1}{3a}\cos (ax + b)[\sin^2 (ax + b) + 2] + C.$$

$$(73)\quad \int \sin^4 (ax + b)\,dx = \frac{3x}{8} - \frac{3 \sin 2(ax + b)}{16a} - \frac{\sin^3 (ax + b)\cos (ax + b)}{4a} + C.$$

(74) $$\int \sin^n (ax + b)\, dx = -\frac{1}{an}\left[\sin^{n-1} (ax + b) \cos (ax + b) - a(n - 1) \int \sin^{n-2} (ax + b)\, dx\right].$$

See (73) if n is even; (72) if n is odd.

(75) $$\int x \sin (ax + b)\, dx = \frac{1}{a^2} \sin (ax + b) - \frac{x}{a} \cos (ax + b) + C.$$

(76) $$\int x \sin^2 (ax + b)\, dx = \frac{x^2}{4} - \frac{x \sin 2(ax + b)}{4a} - \frac{\cos 2(ax + b)}{8a^2} + C.$$

(77) $$\int x^2 \sin^2 (ax + b)\, dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right) \sin 2(ax + b) - \frac{x \cos 2(ax + b)}{4a^2} + C.$$

(78) $$\int x^n \sin (ax + b)\, dx = -\frac{1}{a}\left[x^n \cos (ax + b) - n \int x^{n-1} \cos (ax + b)\, dx\right].$$

See (91) if n is even; (75) if n is odd.

(79) $$\int \frac{\sin ax\, dx}{x} = ax - \frac{(ax)^3}{3(3!)} + \frac{(ax)^5}{5(5!)} - + \ldots + C.$$

(80) $$\int \frac{\sin (ax + b)\, dx}{x^n} = \frac{-1}{n - 1} \frac{\sin (ax + b)}{x^{n-1}} + \frac{a}{n - 1} \int \frac{\cos (ax + b)\, dx}{x^{n-1}}, \quad \text{if } n > 1.$$

See (70) if n is even; (86) if n is odd.

(81) $$\int \frac{dx}{1 \pm \sin (ax + b)} = \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \pm \frac{ax + b}{2}\right) + C.$$

(82) $$\int \sqrt{1 + \sin x}\, dx = \pm 2 \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + C.$$

Use + sign if $(8n - 1)\frac{\pi}{2} < x \leqq (8n + 3)\frac{\pi}{2}$ for some integer n; otherwise use − sign.

(83) $$\int \sqrt{1 - \sin x}\, dx = \pm 2\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) + C.$$

Use + sign if $(8n - 3)\frac{\pi}{2} < x \leqq (8n + 1)\frac{\pi}{2}$ for some integer n; otherwise use − sign.

(84) $$\int \sin ax \sin bx\, dx = \frac{\sin (a - b)x}{2(a - b)} - \frac{\sin (a + b)x}{2(a + b)} + C,$$

if $a^2 \neq b^2$.

(85) $$\int \cos x\, dx = \sin x + C.$$

(86) $$\int \cos (ax + b)\, dx = \frac{\sin (ax + b)}{a} + C.$$

(87) $$\int \cos^2 (ax + b)\, dx = \frac{1}{2a} \sin (ax + b) \cos (ax + b) + \frac{x}{2} + C.$$

$$= \frac{1}{4a} \sin 2(ax + b) + \frac{x}{2} + C.$$

(88) $$\int \cos^3 (ax + b)\, dx = \frac{1}{a} \sin (ax + b) - \frac{1}{3a} \sin^3 (ax + b) + C$$

(89) $$\int \cos^4 (ax + b)\, dx = \frac{3x}{8} + \frac{3 \sin 2(ax + b)}{16a} + \frac{\cos^3 (ax + b) \sin (ax + b)}{4a} + C.$$

(90) $$\int \cos^n (ax + b)\, dx = \frac{1}{na}\left[\cos^{n-1} (ax + b) \sin (ax + b) + a(n - 1) \int \cos^{n-2} (ax + b)\, dx\right].$$

See (89) if n is even; (88) if n is odd.

(91) $$\int x \cos (ax + b)\, dx = \frac{1}{a^2} \cos (ax + b) + \frac{x}{a} \sin (ax + b) + C.$$

(92) $$\int x \cos^2 (ax + b)\, dx = \frac{x^2}{4} + \frac{x \sin 2(ax + b)}{4a} + \frac{\cos 2(ax + b)}{8a^2} + C.$$

(93) $$\int x^2 \cos^2 (ax + b)\, dx = \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right) \sin 2(ax + b) + \frac{x \cos 2(ax + b)}{4a^2} + C.$$

(94) $$\int x^n \cos (ax + b)\, dx = \frac{1}{a}\left[x^n \sin (ax + b) - n \int x^{n-1} \sin (ax + b)\, dx\right].$$

See (75) if n is even; (91) if n is odd.

(95) $$\int \frac{\cos ax\, dx}{x} = \ln |ax| - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - + \ldots + C.$$

(96) $$\int \frac{\cos (ax + b)}{x^n}\, dx = \frac{-1}{n - 1} \frac{\cos (ax + b)}{x^{n-1}} - \frac{a}{n - 1} \int \frac{\sin (ax + b)}{x^{n-1}}\, dx, \quad \text{if } n > 1.$$

See (86) if n is even; (70) if n is odd.

(97) $$\int \frac{dx}{1 + \cos (ax + b)} = \frac{1}{a} \tan \frac{ax + b}{2} + C.$$

(98) $$\int \frac{dx}{1 - \cos (ax + b)} = -\frac{1}{a} \cot \frac{ax + b}{2} + C.$$

(99) $$\int \sqrt{1 + \cos x}\, dx = \pm 2\sqrt{2} \sin \frac{x}{2} + C.$$

Use + sign if $(4n - 1)\pi < x \leqq (4n + 1)\pi$ for some integer n; otherwise use − sign.

(100) $$\int \sqrt{1 - \cos x}\, dx = \pm 2\sqrt{2} \cos \frac{x}{2} + C.$$

Use + sign if $(4n - 2)\pi < x \leqq 4n\pi$ for some integer n; otherwise use − sign.

(101) $$\int \cos ax \cos bx\, dx = \frac{\sin (a - b)x}{2(a - b)} + \frac{\sin (a + b)x}{2(a + b)} + C, \quad \text{if } a^2 \neq b^2.$$

(102) $$\int \sin ax \cos bx\, dx = -\frac{\cos (a - b)x}{2(a - b)} - \frac{\cos (a + b)x}{2(a + b)} + C, \quad \text{if } a^2 \neq b^2.$$

(103) $$\int \sin(ax+b)\cos(ax+b)\,dx = \frac{1}{2a}\sin^2(ax+b) + C.$$

(104) $$\int \sin^p(ax+b)\cos(ax+b)\,dx = \frac{\sin^{p+1}(ax+b)}{a(p+1)} + C, \quad \text{if } p \neq -1.$$

(105) $$\int \sin(ax+b)\cos^p(ax+b)\,dx = -\frac{\cos^{p+1}(ax+b)}{a(p+1)} + C, \quad \text{if } p \neq -1.$$

(106) $$\int \sin^2(ax+b)\cos^2(ax+b)\,dx = -\frac{1}{32a}\sin 4(ax+b) + \frac{x}{8} + C.$$

(107) $$\int \sin^m(ax+b)\cos^n(ax+b)\,dx = -\frac{1}{a(m+n)}\left[\sin^{m-1}(ax+b)\cos^{n+1}(ax+b) - (m-1)a\int \sin^{m-2}(ax+b)\cos^n(ax+b)\,dx\right], \quad \text{if } m > 0,\ m+n \neq 0.$$

See (90) if m is even; (105) if m is odd.

(108) $$\int \sin^m(ax+b)\cos^n(ax+b)\,dx = \frac{1}{a(m+n)}\left[\sin^{m+1}(ax+b)\cos^{n-1}(ax+b) + (n-1)a\int \sin^m(ax+b)\cos^{n-2}(ax+b)\,dx\right], \quad \text{if } n > 0,\ m+n \neq 0.$$

See (74) if n is even; (104) if n is odd.

(109) $$\int \frac{dx}{\sin(ax+b)\cos(ax+b)} = \frac{1}{a}\ln|\tan(ax+b)| + C.$$

(110) $$\int \frac{dx}{\sin^n(ax+b)\cos(ax+b)} = \frac{-1}{a(n-1)}\frac{1}{\sin^{n-1}(ax+b)} + \int \frac{dx}{\sin^{n-2}(ax+b)\cos(ax+b)}, \quad \text{if } n > 1.$$

See (128) if n is even; (109) if n is odd.

(111) $$\int \frac{dx}{\sin(ax+b)\cos^n(ax+b)} = \frac{1}{a(n-1)}\frac{1}{\cos^{n-1}(ax+b)} + \int \frac{dx}{\sin(ax+b)\cos^{n-2}(ax+b)}, \quad \text{if } n > 1.$$

See (135) if n is even; (109) if n is odd.

(112) $$\int \frac{\sin(ax+b)}{\cos^2(ax+b)}\,dx = \frac{1}{a\cos(ax+b)} + C = \frac{1}{a}\sec(ax+b) + C.$$

(113) $$\int \frac{\sin^2(ax+b)}{\cos(ax+b)}\,dx = -\frac{1}{a}\left[\sin(ax+b) - \ln\left|\tan\left(\frac{ax+b}{2}+\frac{\pi}{4}\right)\right|\right] + C.$$

(114) $$\int \frac{\cos(ax+b)}{\sin^2(ax+b)}\,dx = \frac{-1}{a\sin(ax+b)} + C = -\frac{1}{a}\csc(ax+b) + C.$$

(115) $$\int \frac{\cos^2(ax+b)}{\sin(ax+b)}\,dx = \frac{1}{a}\left[\cos(ax+b) + \ln\left|\tan\frac{ax+b}{2}\right|\right] + C.$$

(116) $$\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C.$$

(117) $$\int \tan(ax+b)\,dx = -\frac{1}{a}\ln|\cos(ax+b)| + C.$$

(118) $$\int \tan^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) - x + C.$$

(119) $$\int \tan^3(ax+b)\,dx = \frac{1}{2a}[\tan^2(ax+b) + 2\ln|\cos(ax+b)|] + C.$$

(120) $$\int \tan^n(ax+b)\,dx = \frac{1}{a(n-1)}\tan^{n-1}(ax+b) - \int \tan^{n-2}(ax+b)\,dx, \quad n \geqq 2.$$

See (118) if n is even; (119) if n is odd.

(121) $$\int \cot x\,dx = \ln|\sin x| + C = -\ln|\csc x| + C.$$

(122) $$\int \cot(ax+b)\,dx = \frac{1}{a}\ln|\sin(ax+b)| + C.$$

(123) $$\int \cot^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) - x + C.$$

(124) $$\int \cot^3(ax+b)\,dx = -\frac{1}{2a}\left[\cot^2(ax+b) + 2\ln|\sin(ax+b)|\right] + C.$$

(125) $$\int \cot^n(ax+b)\,dx = -\frac{1}{a(n-1)}\cot^{n-1}(ax+b) - \int \cot^{n-2}(ax+b)\,dx, \quad n \geqq 2.$$

See (123) if n is even; (124) if n is odd.

(126) $$\int \sec x\,dx = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C = \ln|\sec x + \tan x| + C.$$

(127) $$\int \sec^2 x\,dx = \tan x + C.$$

(128) $$\int \sec(ax+b)\,dx = \frac{1}{a}\ln\left|\tan\left(\frac{ax+b}{2}+\frac{\pi}{4}\right)\right| + C.$$

(129) $$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C.$$

(130) $$\int \sec^3(ax+b)\,dx = \frac{1}{2a}\left[\sec(ax+b)\tan(ax+b) + \ln\left|\tan\left(\frac{ax+b}{2}+\frac{\pi}{4}\right)\right|\right] + C.$$

(131) $$\int \sec^n(ax+b)\,dx = \frac{1}{a(n-1)}\frac{\sin(ax+b)}{\cos^{n-1}(ax+b)} + \frac{n-2}{n-1}\int \sec^{n-2}(ax+b)\,dx, \quad n \geqq 2.$$

See (129) if n is even; (130) if n is odd.

(132) $$\int \sec x\tan x\,dx = \sec x + C.$$

(133) $$\int \csc x\,dx = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C.$$

(134) $$\int \csc^2 x\,dx = -\cot x + C.$$

(135) $$\int \csc(ax+b)\,dx = \frac{1}{a}\ln\left|\tan\frac{ax+b}{2}\right| + C.$$

(136) $$\int \csc^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C.$$

(137) $$\int \csc^3(ax+b)\,dx = \frac{1}{2a}\left[-\csc(ax+b)\cot(ax+b) + \ln\left|\tan\frac{ax+b}{2}\right|\right] + C.$$

(138) $$\int \csc^n(ax+b)\,dx = \frac{-1}{a(n-1)}\frac{\cos(ax+b)}{\sin^{n-1}(ax+b)} + \frac{n-2}{n-1}\int \csc^{n-2}(ax+b)\,dx, \quad n \geqq 2.$$

See (136) if n is even; (137) if n is odd.

(139) $$\int \csc x \cot x\,dx = -\csc x + C.$$

Inverse Trigonometric Functions

(140) $$\int \arcsin\frac{x}{a}\,dx = x\arcsin\frac{x}{a} + \sqrt{a^2 - x^2} + C.$$

(141) $$\int (\arcsin ax)^2\,dx = x(\arcsin ax)^2 - 2x + \frac{2}{a}\sqrt{1 - a^2x^2}\arcsin ax + C.$$

(142) $$\int x \arcsin ax\,dx = \frac{1}{4a^2}\left[(2a^2x^2 - 1)\arcsin ax + ax\sqrt{1 - a^2x^2}\right] + C.$$

(143) $$\int \frac{\arcsin ax}{x^2}\,dx = a\ln\left|\frac{1 - \sqrt{1 - a^2x^2}}{ax}\right| - \frac{\arcsin ax}{x} + C.$$

(144) $$\int \arccos\frac{x}{a}\,dx = x\arccos\frac{x}{a} - \sqrt{a^2 - x^2} + C.$$

$$(145)\quad \int (\arccos ax)^2\,dx = x(\arccos ax)^2 - 2x - \frac{2}{a}\sqrt{1 - a^2x^2}\arccos ax + C.$$

$$(146)\quad \int \arctan\frac{x}{a}\,dx = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2 + x^2) + C.$$

$$(147)\quad \int \operatorname{arccot}\frac{x}{a}\,dx = x\operatorname{arccot}\frac{x}{a} + \frac{a}{2}\ln(a^2 + x^2) + C.$$

$$(148)\quad \int \operatorname{arcsec}\frac{x}{a}\,dx = x\operatorname{arcsec}\frac{x}{a} - a\ln\left|x + \sqrt{x^2 - a^2}\right| + C.$$

$$(149)\quad \int \operatorname{arccsc}\frac{x}{a}\,dx = x\operatorname{arccsc}\frac{x}{a} + a\ln\left|x + \sqrt{x^2 - a^2}\right| + C.$$

Logarithmic Functions

$$(150)\quad \int \ln|x|\,dx = x\ln|x| - x + C.$$

$$(151)\quad \int \log_a|x|\,dx = x\log_a|x| - \frac{x}{\ln a} + C, \quad \text{if } a \neq 1,\ a > 0.$$

$$(152)\quad \int \ln|ax + b|\,dx = \frac{ax + b}{a}\ln|ax + b| - x + C.$$

$$(153)\quad \int (\ln|x|)^2\,dx = x(\ln|x|)^2 - 2x\ln|x| + 2x + C.$$

$$(154)\quad \int (\ln|ax + b|)^n\,dx = \frac{ax + b}{a}(\ln|ax + b|)^n - n\int (\ln|ax + b|)^{n-1}\,dx.$$

See (152).

$$(155)\quad \int x\ln|x|\,dx = \frac{x^2}{2}\ln|x| - \frac{x^2}{4} + C.$$

$$(156)\quad \int \frac{dx}{x\ln|x|} = \ln|\ln|x|| + C.$$

$$(157)\quad \int x^p\ln|x|\,dx = x^{p+1}\left[\frac{\ln|x|}{p + 1} - \frac{1}{(p + 1)^2}\right] + C, \quad \text{if } p \neq -1.$$

$$(158)\quad \int \frac{(\ln|x|)^p}{x}\,dx = \frac{1}{p + 1}(\ln|x|)^{p+1} + C, \quad \text{if } p \neq -1.$$

(159) $$\int \sin \ln |x|\, dx = \frac{x}{2}\,[\sin \ln |x| - \cos \ln |x|] + C.$$

(160) $$\int \cos \ln |x|\, dx = \frac{x}{2}\,[\sin \ln |x| + \cos \ln |x|] + C.$$

Exponential Functions

(161) $$\int e^x\, dx = e^x + C.$$

(162) $$\int e^{ax}\, dx = \frac{e^{ax}}{a} + C.$$

(163) $$\int x e^{ax}\, dx = \frac{e^{ax}}{a^2}\,(ax - 1) + C.$$

(164) $$\int x^m e^{ax}\, dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1} e^{ax}\, dx, \quad m \geqq 2.$$

See (163).

(165) $$\int \frac{e^{ax}\, dx}{x} = \ln |x| + ax + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \ldots + C.$$

(166) $$\int e^{ax} \sin bx\, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C.$$

(167) $$\int e^{ax} \cos bx\, dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C.$$

(168) $$\int \frac{dx}{1 + e^x} = x - \ln (1 + e^x) + C.$$

(169) $$\int \frac{dx}{ae^{px} + b} = \frac{x}{b} - \frac{1}{bp} \ln |ae^{px} + b| + C, \quad \text{if } b \neq 0,\ p \neq 0.$$

(170) $$\int \frac{dx}{ae^{px} + be^{-px}} = \frac{1}{p\sqrt{ab}} \arctan \left(e^{px} \sqrt{\frac{a}{b}}\right) + C, \text{ if } ab > 0.$$

(171) $$\int e^{ax} \ln |bx|\, dx = \frac{1}{a}\, e^{ax} \ln |bx| - \frac{1}{a}\int \frac{e^{ax}}{x}\, dx.$$

See (165).

(172) $$\int a^x\, dx = \frac{a^x}{\ln a} + C, \text{ if } a > 0,\ a \neq 1.$$

(173) $$\int a^{bx}\, dx = \frac{a^{bx}}{b \ln a} + C, \quad \text{if } a > 0,\ a \neq 1.$$

$$(174)\quad \int x\mathbf{a}^{bx} = \frac{x\mathbf{a}^{bx}}{b\ \mathbf{ln}\ a} - \frac{\mathbf{a}^{bx}}{b^2\ (\mathbf{ln}\ a)^2} + C, \quad \text{if } a > 0,\ a \neq 1.$$

Hyperbolic Functions

$$(175)\quad \int \mathbf{sinh}\ ax\ dx = \frac{1}{a}\ \mathbf{cosh}\ ax + C.$$

$$(176)\quad \int \mathbf{sinh}^2\ ax\ dx = \frac{1}{4a}\ \mathbf{sinh}\ 2ax - \tfrac{1}{2}x + C.$$

$$(177)\quad \int \mathbf{cosh}\ ax\ dx = \frac{1}{a}\ \mathbf{sinh}\ ax + C.$$

$$(178)\quad \int \mathbf{cosh}^2\ ax\ dx = \frac{1}{4a}\ \mathbf{sinh}\ 2ax + \tfrac{1}{2}x + C.$$

$$(179)\quad \int \mathbf{tanh}\ ax\ dx = \frac{1}{a}\ \mathbf{ln}\ |\mathbf{cosh}\ ax| + C.$$

$$(180)\quad \int \mathbf{tanh}^2\ ax\ dx = x - \frac{1}{a}\ \mathbf{tanh}\ ax + C.$$

$$(181)\quad \int \mathbf{coth}\ ax\ dx = \frac{1}{a}\ \mathbf{ln}\ |\mathbf{sinh}\ ax| + C.$$

$$(182)\quad \int \mathbf{coth}^2\ ax\ dx = x - \frac{1}{a}\ \mathbf{coth}\ ax + C.$$

$$(183)\quad \int \mathbf{sech}\ ax\ dx = \frac{1}{a}\ \mathbf{arctan}\ (\mathbf{sinh}\ ax) + C.$$

$$(184)\quad \int \mathbf{sech}^2\ ax\ dx = \frac{1}{a}\ \mathbf{tanh}\ ax + C.$$

$$(185)\quad \int \mathbf{csch}\ ax\ dx = -\frac{1}{a}\ \mathbf{ln}\ |\mathbf{coth}\ ax + \mathbf{csch}\ ax| + C$$

$$= \frac{1}{a}\ \mathbf{ln}\left|\mathbf{tanh}\ \frac{ax}{2}\right| + C.$$

$$(186)\quad \int \mathbf{csch}^2\ ax\ dx = -\frac{1}{a}\ \mathbf{coth}\ ax + C.$$

$$(187)\quad \int \mathbf{sech}\ ax\ \mathbf{tanh}\ ax\ dx = -\frac{1}{a}\ \mathbf{sech}\ ax + C.$$

$$(188)\quad \int \mathbf{csch}\ ax\ \mathbf{coth}\ ax\ dx = -\frac{1}{a}\ \mathbf{csch}\ ax + C.$$

10.5 DEFINITE INTEGRALS

Table 10.7 contains some definite integrals, many of which cannot be obtained by the computation of the antiderivative of the integrand. Many of these are **improper integrals,** that is, integrals whose intervals of integration are infinite or for which the integrand is undefined at an end point of the interval of integration. Provided the limit exists in each case, we have

$$\int_a^{\infty} \mathbf{f}(x)\,dx = \lim_{b\to\infty} \int_a^b \mathbf{f}(x)\,dx;$$

$$\int_{-\infty}^b \mathbf{f}(x)\,dx = \lim_{a\to-\infty} \int_a^b \mathbf{f}(x)\,dx;$$

$$\int_{-\infty}^{\infty} \mathbf{f}(x)\,dx = \lim_{a\to-\infty} \int_a^c \mathbf{f}(x)\,dx + \lim_{b\to\infty} \int_c^b \mathbf{f}(x)\,dx.$$

If $\mathbf{f}(x)$ is undefined at $x = b$, then if $b > a$,

$$\int_a^b \mathbf{f}(x)\,dx = \lim_{h\to 0} \int_a^{b-h} \mathbf{f}(x)\,dx, \quad \text{where } h > 0.$$

If $\mathbf{f}(x)$ is undefined at $x = a$, then if $b > a$,

$$\int_a^b \mathbf{f}(x)\,dx = \lim_{h\to 0} \int_{a+h}^b \mathbf{f}(x)\,dx, \quad \text{where } h > 0.$$

10.6 TABLE: PROPERTIES OF DEFINITE INTEGRALS

(1) $\displaystyle\int_a^b c\mathbf{f}(x)\,dx = c\int_a^b \mathbf{f}(x)\,dx,$ c any real number.

(2) $\displaystyle\int_a^b [\mathbf{f}(x) \pm \mathbf{g}(x)]\,dx = \int_a^b \mathbf{f}(x)\,dx \pm \int_a^b \mathbf{g}(x)\,dx.$

(3) $\displaystyle\int_a^a \mathbf{f}(x)\,dx = 0.$

(4) $\displaystyle\int_a^c \mathbf{f}(x)\,dx = \int_a^b \mathbf{f}(x)\,dx + \int_b^c \mathbf{f}(x)\,dx.$

(5) $\displaystyle\int_a^b \mathbf{f}(x)\,dx = -\int_b^a \mathbf{f}(x)\,dx.$

10.7 TABLE: DEFINITE INTEGRALS

(1) $$\int_0^\infty \frac{\sin x\,dx}{\sqrt{x}} = \int_0^\infty \frac{\cos x\,dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}.$$

(2) $$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \frac{\pi}{2}, \quad \text{if } n \text{ is even}$$

$$= \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdots n}, \quad \text{if } n \text{ is odd.}$$

(3) $$\int_0^\pi \sin^2 ax\,dx = \int_0^\pi \cos^2 ax\,dx = \frac{\pi}{2}.$$

(4) $$\int_0^\infty \frac{\sin ax}{x}\,dx = \frac{\pi}{2}, \qquad \text{if } a > 0,$$

$$= 0, \qquad \text{if } a = 0,$$

$$= -\frac{\pi}{2}, \quad \text{if } a < 0.$$

(5) $$\int_0^\pi \sin ax \sin bx\,dx = \int_0^\pi \cos ax \cos bx\,dx = 0, \quad a \neq b.$$

(6) $$\int_0^\infty \frac{\cos ax}{1 + x^2}\,dx = \frac{\pi}{2} e^{-a}, \quad a > 0$$

$$= \frac{\pi}{2} e^{a}, \qquad a < 0.$$

(7) $$\int_0^\infty \cos (x^2)\,dx = \int_0^\infty \sin (x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$

(8) $$\int_0^{\pi/2} \frac{dx}{1 + a \cos x} = \frac{\arccos a}{\sqrt{1 - a^2}}, \quad a^2 < 1.$$

(9) $$\int_0^\pi \frac{dx}{1 + a \cos x} = \frac{\pi}{\sqrt{1 - a^2}}, \quad a^2 < 1.$$

(10) $$\int_0^\infty \frac{\tan x}{x}\,dx = \frac{\pi}{2}.$$

(11) $$\int_0^\infty e^{-ax} \cos bx\,dx = \frac{a}{a^2 + b^2}, \quad a > 0.$$

(12) $$\int_0^\infty \mathbf{e}^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}, \quad a > 0.$$

(13) $$\int_0^\infty \mathbf{e}^{-a^2x^2} \cos bx \, dx = \frac{\sqrt{\pi}\, \mathbf{e}^{-b^2/4a^2}}{2a}, \quad a > 0.$$

10.8 TABLE: DEFINITE INTEGRALS OF SPECIAL INTEREST IN PROBABILITY

$$\text{Let } \mathbf{f}(x) = \frac{1}{a}\, \mathbf{e}^{-x/a}.$$

(1a) $$\int_0^\infty \mathbf{f}(x)\, dx = 1.$$

(1b) $$\int_0^\infty x\mathbf{f}(x)\, dx = a.$$

(1c) $$\int_0^\infty x^2\mathbf{f}(x)\, dx = 2a^2.$$

$$\text{Let } \mathbf{f}(x) = \frac{1}{\sigma\sqrt{2\pi}}\, \mathbf{e}^{-(x-\mu)^2/2\sigma^2}.$$

(2a) $$\int_{-\infty}^\infty \mathbf{f}(x)\, dx = 1.$$

(2b) $$\int_{-\infty}^\infty x\mathbf{f}(x)\, dx = \mu.$$

(2c) $$\int_{-\infty}^\infty (x - \mu)^2\mathbf{f}(x)\, dx = \sigma^2.$$

10.9 ANTIDERIVATIVES OF ORDER n

Given a function $\mathbf{g}(x)$, the general solution form of the equation $\mathbf{D}^n\mathbf{f} = \mathbf{g}(x)$ is the sum of a function $\mathbf{h}(x)$ and an expression of the form

$$C_1x^n + C_2x^{n-1} + \ldots + C_{n-1}x + C_n.$$

The function $\mathbf{h}(x)$ is obtained by integrating successive particular integrals of $\mathbf{g}(x)$, the process continuing for n steps. The function $\mathbf{h}(x)$ is called the **antiderivative of $\mathbf{g}(x)$ of order n** and is denoted by $\mathbf{I}^n\mathbf{g}(x)$. Algebraic properties of the repeated indefinite integral are:

(1) $$\mathbf{I}^n(c\mathbf{g}(x)) = c \cdot \mathbf{I}^n\mathbf{g}(x), \quad c \text{ a constant,}$$

and

(2) $$\mathbf{I}^n[\mathbf{g}_1(x) \pm \mathbf{g}_2(x)] = \mathbf{I}^n\mathbf{g}_1(x) \pm \mathbf{I}^n\mathbf{g}_2(x).$$

10.10 TABLE: ANTIDERIVATIVES OF ORDER n

(1) $$\mathbf{I}^n(x^p) = \frac{x^{p+n}}{(p+1)(p+2)\dots(p+n)}, \quad p \neq -1, -2, \dots, -n.$$

(2) $$\mathbf{I}^n(\mathbf{sin}\ ax) = \frac{\mathbf{sin}\ ax}{a^n}, \quad \text{if } n = 4k.$$

(3) $$\mathbf{I}^n(\mathbf{sin}\ ax) = \frac{-\mathbf{cos}\ ax}{a^n}, \quad \text{if } n = 4k+1.$$

(4) $$\mathbf{I}^n(\mathbf{sin}\ ax) = \frac{-\mathbf{sin}\ ax}{a^n}, \quad \text{if } n = 4k+2.$$

(5) $$\mathbf{I}^n(\mathbf{sin}\ ax) = \frac{\mathbf{cos}\ ax}{a^n}, \quad \text{if } n = 4k+3.$$

(6) $$\mathbf{I}^n(\mathbf{cos}\ ax) = \frac{\mathbf{cos}\ ax}{a^n}, \quad \text{if } n = 4k.$$

(7) $$\mathbf{I}^n(\mathbf{cos}\ ax) = \frac{\mathbf{sin}\ ax}{a^n}, \quad \text{if } n = 4k+1.$$

(8) $$\mathbf{I}^n(\mathbf{cos}\ ax) = \frac{-\mathbf{cos}\ ax}{a^n}, \quad \text{if } n = 4k+2.$$

(9) $$\mathbf{I}^n(\mathbf{cos}\ ax) = \frac{-\mathbf{sin}\ ax}{a^n}, \quad \text{if } n = 4k+3.$$

(10) $$\mathbf{I}^n(\mathbf{e}^{ax}) = \frac{\mathbf{e}^{ax}}{a^n}.$$

11

LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

THE GENERAL form of such an equation is

$$\mathbf{D}^n\mathbf{f}(x) + a_1\mathbf{D}^{n-1}\mathbf{f}(x) + \ldots + a_{n-1}\mathbf{D}\mathbf{f}(x) + a_n\mathbf{f}(x) = \mathbf{g}(x),$$

which we rewrite

$$[\mathbf{D}^n + a_1\mathbf{D}^{n-1} + \ldots + a_{n-1}\mathbf{D} + \mathbf{a}_n]\mathbf{f}(x) = \mathbf{g}(x).$$

The quantity in brackets is a polynomial in $\mathbf{D}$ which we shall denote by $\mathbf{p}(\mathbf{D})$. The differential equation now takes the form

$$[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}(x).$$

If $\mathbf{g}(x) = 0$, the equation is said to be **homogeneous.** If $\mathbf{g}(x) \neq 0$, then the homogeneous equation $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = 0$ obtained by replacing $\mathbf{g}(x)$ by 0 is called the **reduced equation** of the given equation.

If $\mathbf{f}^*(x)$ is a solution of the given equation, then, for any other solution $\mathbf{f}_1^*(x)$, the function $\mathbf{f}_1(x) = \mathbf{f}_1^*(x) - \mathbf{f}^*(x)$ is a solution of its reduced equation, since

$$[\mathbf{p}(\mathbf{D})](\mathbf{f}_1^*(x) - \mathbf{f}^*(x)) = \mathbf{g}(x) - \mathbf{g}(x) = 0.$$

Thus the general solution form (GSF) of the given equation is found by adding any particular solution (PS) of the given equation to the

general solution form of the reduced equation. There are two major steps to the solution of the problem:

(1) Find the general solution of the reduced equation.

(2) Find a particular solution for the given equation.

11.1 THE AUXILIARY EQUATION

To obtain the general solution of a homogeneous linear differential equation with constant coefficients $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = 0$, we first solve the auxiliary equation

$$\mathbf{p}(m) = 0,$$

where $\mathbf{p}(m)$ is the polynomial in m obtained by replacing $\mathbf{D}$ by m in $\mathbf{p}(\mathbf{D})$.

11.2 HOMOGENEOUS EQUATIONS OF FIRST ORDER

The first-order equation takes the form

$$[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = [\mathbf{D} - \mathbf{a}]\mathbf{f}(x) = 0, \quad a \text{ a real number.}$$

The general solution form is

$$\mathbf{f}(x) = C\mathbf{e}^{ax}.$$

11.3 HOMOGENEOUS EQUATIONS OF SECOND ORDER

The general solution form of the second-order equation $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = 0$ is tabulated in Table 11.4 according to the nature of the roots of the auxiliary equation $\mathbf{p}(m) = 0$.

11.4 TABLE: GENERAL SOLUTION FORM OF THE LINEAR HOMOGENEOUS EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENTS

Let m_1 and m_2 be the roots of $\mathbf{p}(m) = 0$.

(1) $m_1 \neq m_2$, both real:

GSF: $\mathbf{f}(x) = C_1\mathbf{e}^{m_1 x} + C_2\mathbf{e}^{m_2 x}$.

(2) $m_1 = m_2$:

GSF: $\mathbf{f}(x) = (C_1 + C_2x)\mathbf{e}^{m_1x}$.

(3) The roots of the auxiliary equation are complex:

$m_1 = a + bi, \quad m_2 = a - bi.$

GSF: $\mathbf{f}(x) = \mathbf{e}^{ax}(C_1 \mathbf{sin}\ bx + C_2 \mathbf{cos}\ bx),$

or $\mathbf{f}(x) = \mathbf{e}^{ax}C_1 \mathbf{sin}\ (bx + C_2),$

or $\mathbf{f}(x) = \mathbf{e}^{ax}C_1 \mathbf{cos}\ (bx + C_2),$

or $\mathbf{f}(x) = \mathbf{e}^{ax}(C_1\mathbf{e}^{ibx} + C_2\mathbf{e}^{-ibx}).$

The equivalence of the first three of these forms is discussed in Example 8 of Sec. 4.9.

11.5 EXAMPLES

(1) $[\mathbf{D}^2 + 2\mathbf{D} + 5]\mathbf{f}(x) = 0$. We first solve the auxiliary equation $m^2 + 2m + 5 = 0$, obtaining $m = -1 \pm 2i$. Then $a = -1$, $b = 2$, and the general solution form is

$$\mathbf{f}(x) = \mathbf{e}^{-x}(C_1 \mathbf{sin}\ 2x + C_2 \mathbf{cos}\ 2x).$$

(2) $[\mathbf{D}^2 - 3\mathbf{D} - 4]\mathbf{f} = 0$. Solving $m^2 - 3m - 4 = 0$, we obtain $m_1 = -1$ and $m_2 = +4$. The general solution form is

$$\mathbf{f}(x) = C_1\mathbf{e}^{-x} + C_2\mathbf{e}^{4x}.$$

(3) $[\mathbf{D}^2 + 6\mathbf{D} + 9]\mathbf{f} = 0$. The roots of $m^2 + 6m + 9 = 0$ are both -3. Hence, the general solution form is

$$\mathbf{f}(x) = (C_1 + C_2x)\mathbf{e}^{-3x}.$$

11.6 HOMOGENEOUS EQUATIONS OF ORDER n

To find the general solution form for the homogeneous equation of order n, $[\mathbf{p}(\mathbf{D})]\mathbf{f} = 0$, one should first find the roots of the auxiliary equation $\mathbf{p}(m) = 0$. Let the distinct real roots of this equation be denoted by $m_1, m_2, \ldots, m_k$, and let their multiplicities be $r_1, r_2, \ldots, r_k$, respectively. Let the pairs of distinct complex roots be denoted by

$a_{k+1} \pm b_{k+1}i$, $a_{k+2} \pm b_{k+2}i$, . . . , $a_{k+l} \pm b_{k+l}i$, with multiplicities r_{k+1}, r_{k+2}, . . . , r_{k+l}, respectively. Then the general solution form of the equation is the sum of the general solution forms of the equations

$$[(\mathbf{D} - \mathbf{m}_j)^{r_j}]\mathbf{f}(x) = 0, \qquad 1 \leqq j \leqq k,$$

and

$$[(\mathbf{D} - \mathbf{a}_j - \mathbf{b}_j i)^{r_j}(\mathbf{D} - \mathbf{a}_j + \mathbf{b}_j i)^{r_j}]\mathbf{f}(x) = 0, \quad k + 1 \leqq j \leqq k + l,$$

where the arbitrary constants appearing in the solutions will be renumbered so that all have different subscripts. These general solution forms are listed in Table 11.7.

11.7 TABLE: GENERAL SOLUTION FORMS FOR HOMOGENEOUS DIFFERENTIAL EQUATIONS OF ORDER n WITH CONSTANT COEFFICIENTS (See Sec. 11.6)

(1) m_j is real, and $r_j = 1$:

GSF: $\mathbf{f}_j(x) = C\mathbf{e}^{m_j x}$.

(2) m_j is real, and $r_j > 1$:

GSF: $\mathbf{f}_j(x) = (C_1 + C_2 x + \ldots + C_{r_j} x^{r_j - 1})\mathbf{e}^{m_j x}$.

(3) $a_j \pm b_j i$ are complex roots, and $r_j = 1$:

GSF: $\mathbf{f}_j(x) = \mathbf{e}^{a_j x}(C_1 \mathbf{sin}\ b_j x + C_2 \mathbf{cos}\ b_j x)$,

or $\mathbf{f}_j(x) = C_1 \mathbf{e}^{a_j x} \mathbf{sin}\ (b_j x + C_2)$,

or $\mathbf{f}_j(x) = C_1 \mathbf{e}^{a_j x} \mathbf{cos}\ (b_j x + C_2)$,

or $\mathbf{f}_j(x) = \mathbf{e}^{a_j x}(C_1 \mathbf{e}^{i b_j x} + C_2 \mathbf{e}^{-i b_j x})$.

(4) $a_j \pm b_j i$ are complex roots, and $r_j > 1$.

GSF: $\mathbf{f}_j(x) = (C_1 + C_2 x + \ldots + C_{r_j} x^{r_j - 1})\mathbf{e}^{a_j x} \mathbf{sin}\ b_j x$
$+ (C_{r_j+1} + C_{r_j+2} x + \ldots + C_{2r_j} x^{r_j - 1})\mathbf{e}^{a_j x} \mathbf{cos}\ b_j x.$

11.8 EXAMPLE

$[\mathbf{D}(\mathbf{D} - 2)^3(\mathbf{D} + 1)]\mathbf{f} = 0$. The roots (and their respective multiplicities) of $m(m - 2)^3(m + 1) = 0$ are

$$m_1 = 0, \qquad r_1 = 1,$$
$$m_2 = -1, \qquad r_2 = 1,$$
$$m_3 = 2, \qquad r_3 = 3.$$

GSF: $\mathbf{f}(x) = C_1 + C_2\mathbf{e}^{-x} + (C_3 + C_4x + C_5x^2)\mathbf{e}^{2x}$.

11.9 PARTICULAR SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

A particular solution of the equation $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}(x)$ can often be found if one can guess the form such a solution is likely to take. Table 11.10 lists a number of aids to good guesses for particular solutions. The constants A and B (with or without subscripts) are real numbers to be determined by substituting the particular solution $\mathbf{f}^*(x)$ into the given equation. The examples of Sec. 11.12 illustrate how the table is used.

It will appear that it is necessary to check whether certain numbers are roots of the auxiliary equation $\mathbf{p}(m) = 0$. One should recall, however, that all roots of this equation were obtained in finding the general solution of the reduced equation. In particular, in each case, the number in question is a root if and only if $\mathbf{g}(x)$ is a solution of the reduced equation.

11.10 TABLE: PARTICULAR SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

(1) $\mathbf{g}(x) = k$, k a real number:

PS: $\mathbf{f}^*(x) = A$ unless 0 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = Ax^r$ if 0 is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(2) $\mathbf{g}(x) = \mathbf{X}$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = A_0 + A_1x + \ldots + A_sx^s$ unless 0 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r$ if 0 is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(3) $\mathbf{g}(x) = p \sin qx$:

PS: $\mathbf{f}^*(x) = A \sin qx + B \cos qx$ unless iq is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A \sin qx + B \cos qx)x^r$ if iq is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(4) $\mathbf{g}(x) = p \cos qx$:

PS: $\mathbf{f}^*(x) = A \sin qx + B \cos qx$ unless iq is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A \sin qx + B \cos qx)x^r$ if iq is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(5) $\mathbf{g}(x) = p\mathbf{e}^{qx}$:

PS: $\mathbf{f}^*(x) = A\mathbf{e}^{qx}$ unless q is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = Ax^r\mathbf{e}^{qx}$ if q is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(6) $\mathbf{g}(x) = k\mathbf{e}^{px} \sin qx$:

PS: $\mathbf{f}^*(x) = A\mathbf{e}^{px} \sin qx + B\mathbf{e}^{px} \cos qx$ unless $p + iq$ is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A\mathbf{e}^{px} \sin qx + B\mathbf{e}^{px} \cos qx)x^r$ if $p + iq$ is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(7) $\mathbf{g}(x) = k\mathbf{e}^{px} \cos qx$:

PS: $\mathbf{f}^*(x) = A\mathbf{e}^{px} \sin qx + B\mathbf{e}^{px} \cos qx$ unless $p + iq$ is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A\mathbf{e}^{px} \sin qx + B\mathbf{e}^{px} \cos qx)x^r$ if $p + iq$ is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(8) $\mathbf{g}(x) = \mathbf{X}\mathbf{e}^{qx}$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\mathbf{e}^{qx}$ unless q is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\mathbf{e}^{qx}$ if q is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(9) $\mathbf{g}(x) = \mathbf{X}\ \mathbf{sin}\ qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)\ \mathbf{cos}\ qx$ unless iq is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)x^r\ \mathbf{cos}\ qx$ if iq is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(10) $\mathbf{g}(x) = \mathbf{X}\ \mathbf{cos}\ qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)\ \mathbf{cos}\ qx$ unless iq is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)x^r\ \mathbf{cos}\ qx$ if iq is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(11) $\mathbf{g}(x) = \mathbf{X}\mathbf{e}^{px}\ \mathbf{sin}\ qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\mathbf{e}^{px}\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)\mathbf{e}^{px}\ \mathbf{cos}\ qx$ unless $p + iq$ is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\mathbf{e}^{px}\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)x^r\mathbf{e}^{px}\ \mathbf{cos}\ qx$ if $p + iq$ is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(12) $\mathbf{g}(x) = \mathbf{X}\mathbf{e}^{px}\ \mathbf{cos}\ qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\mathbf{e}^{px}\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)\mathbf{e}^{px}\ \mathbf{cos}\ qx$ unless $p + iq$ is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\mathbf{e}^{px}\ \mathbf{sin}\ qx + (B_0 + B_1x + \ldots + B_sx^s)x^r\mathbf{e}^{px}\ \mathbf{cos}\ qx$ if $p + iq$ is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

11.11 REMARK ON THE EXTENSION OF TABLE 11.10

Suppose that

$$\mathbf{g}(x) = \mathbf{g}_1(x) + \mathbf{g}_2(x) + \ldots + \mathbf{g}_n(x),$$

where particular solutions for $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}_i(x)$ can be found

in Table 11.10 (or elsewhere). Then a particular solution for $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}(x)$ can be obtained by adding the particular solutions for each of the equations $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}_i(x)$.

11.12 EXAMPLES

(1) $[\mathbf{D}^2 + 2\mathbf{D} + 5]\mathbf{f}(x) = 5x^2 + 8x + 6$. Since $\mathbf{g}(x)$ is a polynomial of degree 2, and since 0 is not a root of $m^2 + 2m + 5 = 0$, formula (2) of Table 11.10 tells us that a particular solution of the given equation has the form $A_0 + A_1x + A_2x^2$. Substituting this in the given equation, we obtain

$$2A_2 + 2A_1 + 4A_2x + 5A_0 + 5A_1x + 5A_2x^2 = 5x^2 + 8x + 6.$$

Equating coefficients of like powers of x,

$$\begin{aligned} 5A_2 &= 5, \\ 4A_2 + 5A_1 &= 8, \\ 2A_2 + 2A_1 + 5A_0 &= 6. \end{aligned}$$

Solving, we get $A_2 = 1$, $A_1 = \frac{4}{5}$, $A_0 = \frac{1}{2}\frac{2}{5}$. Adding our particular solution to the general solution form found in Example 1 of Sec. 11.5, we find that the general solution form for the given equation is

$$\mathbf{f}(x) = \mathbf{e}^{-x}(C_1 \sin 2x + C_2 \cos 2x) + x^2 + \tfrac{4}{5}x + \tfrac{1}{2}\tfrac{2}{5}.$$

(2) $[\mathbf{D}^2 + 6\mathbf{D} + 9]\mathbf{f}(x) = \mathbf{e}^{-3x}$. In Example 3 of Sec. 11.5, we found that -3 is a double root of $m^2 + 6m + 9 = 0$. Thus, a particular solution, according to (5) of Table 11.10, has the form $\mathbf{f}^*(x) = Ax^2\mathbf{e}^{-3x}$. Substituting this for $\mathbf{f}(x)$ in the given equation, we get

$$A(9x^2 - 12x + 2)\mathbf{e}^{-3x} + 6A(-3x^2 + 2x)\mathbf{e}^{-3x} + 9Ax^2\mathbf{e}^{-3x} = \mathbf{e}^{-3x}.$$

The coefficients of $x^2\mathbf{e}^{-3x}$ and $x\mathbf{e}^{-3x}$ are both 0. Equating coefficients of $\mathbf{e}^{-3x}$, we get $A = \frac{1}{2}$. Using this particular solution and the general solution form found in Example 3 of Sec. 11.5 for the reduced equation, the general solution form for the given equation is

$$\mathbf{f}(x) = (C_1 + C_2x)\mathbf{e}^{-3x} + \tfrac{1}{2}x^2\mathbf{e}^{-3x}.$$

(3) $[\mathbf{D}^2 - 3\mathbf{D} - 4]\mathbf{f}(x) = 25 \sin 3x$. Since $3i$ is not a root of $m^2 - 3m - 4$, we use (3) of Table 11.10 and find that a particular

solution has the form A **sin** $3x + B$ **cos** $3x$. Substituting this for $\mathbf{f}(x)$ in the given equation, we get

$$(-9A - 9B - 4A) \textbf{ sin } 3x + (-9B + 9A - 4B) \textbf{ cos } 3x = 25 \textbf{ sin } 3x.$$

Equating coefficients of **sin** $3x$ and **cos** $3x$, we get

$$9A - 13B = 0,$$
$$-13A - 9B = 25.$$

Solving these two equations together, we find $A = -1.3$, $B = -0.9$. Adding the general solution form of the reduced equation found in Example 2 of Sec. 11.5 to the particular solution found here, we get

$$\mathbf{f}(x) = C_1\mathbf{e}^{-x} + C_2\mathbf{e}^{4x} - 1.3 \textbf{ sin } 3x - 0.9 \textbf{ cos } 3x.$$

12

DIFFERENTIAL EQUATIONS OF FIRST AND SECOND ORDER

12.1 FUNCTIONS OF MORE THAN ONE ARGUMENT

A function of more than one argument is a mapping from sets of real numbers to real numbers. A value of a function of n arguments is determined by specifying n real numbers, one for each argument. Functions of more than one argument will be denoted by $\mathbf{g}$ or a capital letter. Arguments, which can be values of functions, will be denoted by $w, x, y, z, \ldots$, or by the symbols which denote functions. In particular, if $\mathbf{f}$ is a function of a single argument x, we shall frequently consider a function $\mathbf{g}(x, \mathbf{f})$ of the two arguments x and $\mathbf{f}(x)$.

12.2 PARTIAL DERIVATIVES

Let $\mathbf{F}$ be a function of n arguments $x_1, \ldots, x_n$. From this function, we can form a new function of any one of its arguments (say, x_i) by keeping all the other arguments fixed. If this function has a derivative, its derivative is called the **partial derivative** of $\mathbf{F}$ with respect to x_i, written $\partial\mathbf{F}/\partial x_i$. In general, $\partial\mathbf{F}/\partial x_i$ can be regarded as a function of the same n arguments as $\mathbf{F}$, since the values of $\partial\mathbf{F}/\partial x_i$ are determined not only by x_i but also by the other arguments.

12.3 PARTIAL INTEGRATION

If $\mathbf{G}$ is a function for which $\partial \mathbf{G}/\partial x_i = \mathbf{F}$, then $\mathbf{G}$ is called a **partial integral** of $\mathbf{F}$ with respect to x_i, written

$$\mathbf{G}(x_1, \ldots, x_n) = \int \mathbf{F}(x_1, \ldots, x_n)\, dx_i.$$

12.4 EXAMPLES

(1) If $\mathbf{F}(x, y) = x \sin y - y \cos x$, then

$$\frac{\partial \mathbf{F}}{\partial x} = \sin y + y \sin x \quad \text{and} \quad \frac{\partial \mathbf{F}}{\partial y} = x \cos y - \cos x.$$

(2) If $\mathbf{g}(x, \mathbf{f}) = x^2\mathbf{f} - 5\mathbf{f}^3 + 3x$, then

$$\frac{\partial \mathbf{g}}{\partial x} = 2x\mathbf{f} + 3 \quad \text{and} \quad \frac{\partial \mathbf{g}}{\partial \mathbf{f}} = x^2 - 15\mathbf{f}^2.$$

12.5 LINEAR DEPENDENCE AND INDEPENDENCE OF FUNCTIONS

Let $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n$ be a set of n functions. If there exists a set of n constants, $k_1, k_2, \ldots, k_n$, not all zero such that the sum

$$k_1\mathbf{f}_1 + k_2\mathbf{f}_2 + \ldots + k_n\mathbf{f}_n = \mathbf{0},$$

then the functions of the set are said to be **linearly dependent.** On the other hand, if the equation

$$k_1\mathbf{f}_1 + k_2\mathbf{f}_2 + \ldots + k_n\mathbf{f}_n = 0$$

implies that $k_1 = k_2 = \ldots k_n = 0$, then the functions of the set are said to be **linearly independent.**

Any set of functions from among the following is a linearly independent set:

A single nonzero constant function;

$x, x^2, x^3, \ldots$;

$\mathbf{e}^x, \mathbf{e}^{2x}, \mathbf{e}^{3x}, \ldots$;

$\mathbf{e}^{-x}, \mathbf{e}^{-2x}, \mathbf{e}^{-3x}, \ldots$;

$\sin x, \sin 2x, \sin 3x, \ldots$;

$\cos x$, $\cos 2x$, $\cos 3x$, . . . ;

xe^{kx}, x^2e^{kx}, x^3e^{kx}, . . . ;

$x \sin kx$, $x^2 \sin kx$, $x^3 \sin kx$, . . . ;

$x \cos kx$, $x^2 \cos kx$, $x^3 \cos kx$, . . . , k any real number, $k \neq 0$.

The functions $\cos 2x$, $\sin^2 x$, and 4 form an example of a linearly dependent set of functions, since

$$4 \cos 2x - 8 \sin^2 x + 1 \cdot 4 = 0.$$

12.6 USE OF THE INTEGRAL TABLE

In the tables which follow, the procedures which lead to general solution forms of certain types of differential equations require the evaluation of one or more integrals. These integrals can often be found in Table 10.4. The constant C of integration listed with the formula in that table should be omitted when the integral is used here, since the formulas given below supply arbitrary constants where they are needed.

12.7 LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

A differential equation of first order expresses a condition to be satisfied by a function $\mathbf{f}$ and its first derivative $\mathbf{Df}$. If such an equation is linear, then it can be written in the form

$$\mathbf{Df} = \mathbf{g}(x, \mathbf{f}), \tag{1}$$

where $\mathbf{g}$ is a function of at most two arguments x and $\mathbf{f}$. Conversely, if a differential equation of first order cannot be written in the form (1) (that is, solved explicitly for $\mathbf{Df}$) then it is a nonlinear equation.

If an equation can be written in the form (1), the technique of solving it, if one exists, depends on the character of the function $\mathbf{g}(x, \mathbf{f})$. The procedure for using Table 12.9 to solve a linear differential equation of first order is:

(1) Solve the equation for $\mathbf{Df}$; that is, write the equation in the form $\mathbf{Df} = \mathbf{g}(x, \mathbf{f})$.

(2) Locate the form of the function $\mathbf{g}(x, \mathbf{f})$ in the classification list in Sec. 12.8. This refers to an entry in Table 12.9.

(3) Follow the procedure outlined in the appropriate entry of Table 12.9 to obtain the general solution form of the equation.

12.8 CLASSIFICATION LIST FOR EQUATIONS OF THE FORM $\mathbf{Df} = \mathbf{g}(x, \mathbf{f})$

(1) $\mathbf{g}(x, \mathbf{f})$ is a function of x alone (does not involve $\mathbf{f}$). Write $\mathbf{g}(x)$ for $\mathbf{g}(x, \mathbf{f})$. See formula (1) of Table 12.9.

(2) $\mathbf{g}(x, \mathbf{f})$ is a function of $\mathbf{f}$ alone (does not involve x). Write $\mathbf{g}(\mathbf{f})$ for $\mathbf{g}(x, \mathbf{f})$. See formula (2) of Table 12.9.

(3) $\mathbf{g}(x, \mathbf{f})$ is the product of a function of x alone and a function of $\mathbf{f}$ alone (separable equation). Write $\mathbf{h}(x)\mathbf{j}(\mathbf{f})$ for $\mathbf{g}(x, \mathbf{f})$. See formula (3) of Table 12.9.

(4) $\mathbf{g}(x, \mathbf{f}) = \mathbf{u}(x) + \mathbf{f}\mathbf{v}(x)$, $\mathbf{u}(x)$, and $\mathbf{v}(x)$ are functions of x alone (equation linear in $\mathbf{f}$). See formula (4) of Table 12.9.

(5) $\mathbf{g}(x, \mathbf{f}) = \mathbf{f}^n\mathbf{u}(x) + \mathbf{f}\mathbf{v}(x)$, $\mathbf{u}(x)$, and $\mathbf{v}(x)$ are functions of x alone (Bernoulli equation). See formula (5) of Table 12.9.

(6) $\mathbf{g}(x, \mathbf{f}) = \dfrac{\mathbf{p}(x, \mathbf{f})}{\mathbf{q}(x, \mathbf{f})}$, where $\mathbf{p}(tx, t\mathbf{f}) = t^n\mathbf{p}(x, \mathbf{f})$ and $\mathbf{q}(tx, t\mathbf{f}) = t^n\mathbf{q}(x, \mathbf{f})$ for some number n (homogeneous equation). See formula (6) of Table 12.9.

[*Note:* If $\mathbf{p}(x, \mathbf{f})$ and $\mathbf{q}(x, \mathbf{f})$ are polynomials in which the sums of the powers of x and $\mathbf{f}$ in each term of both are equal, the equation is homogeneous.]

(7) $\mathbf{g}(x, \mathbf{f}) = \dfrac{a_1x + b_1\mathbf{f} + c_1}{a_2x + b_2\mathbf{f} + c_2}$ (linear fractional form). See formula (7) of Table 12.9.

(8) $\mathbf{g}(x, \mathbf{f}) = \dfrac{\mathbf{M}(x, \mathbf{f})}{\mathbf{N}(x, \mathbf{f})}$, $\dfrac{\partial \mathbf{M}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{N}}{\partial x} = 0$ (equation in exact form). See formula (8) of Table 12.9.

(9) $\mathbf{g}(x, \mathbf{f}) = \dfrac{\mathbf{P}(x, \mathbf{f})}{\mathbf{Q}(x, \mathbf{f})}$, $\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x} \neq 0$.

See formula (9) of Table 12.9 for methods of finding a function

$\mathbf{R}(x, \mathbf{f})$, called an **integrating factor,** so that the equation

$$\mathbf{Df} = \frac{\mathbf{P}(x, \mathbf{f})\mathbf{R}(x, \mathbf{f})}{\mathbf{Q}(x, \mathbf{f})\mathbf{R}(x, \mathbf{f})}$$

is exact. Then apply (8) of Table 12.9. In general, the problem of finding an integrating factor is just as difficult as solving the differential equation itself. Though the entries of formula (9) will yield an integrating factor in some cases, it is more efficient to try to find one by inspection before consulting the table.

12.9 TABLE: GENERAL SOLUTION FORMS FOR LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

(1) Equation: $\mathbf{Df} = \mathbf{g}(x)$.

GSF: $\mathbf{f}(x) = \int \mathbf{g}(x)\, dx + C$.

(2) Equation: $\mathbf{Df} = \mathbf{g}(\mathbf{f})$.

GSF: $\displaystyle\int \frac{d\mathbf{f}}{\mathbf{g}(\mathbf{f})} = x + C$.

(3) Equation: $\mathbf{Df} = \mathbf{h}(x)\mathbf{j}(\mathbf{f})$.

GSF: $\displaystyle\int \frac{d\mathbf{f}}{\mathbf{j}(\mathbf{f})} = \int \mathbf{h}(x)\, dx + C$.

(4) Equation: $\mathbf{Df} = \mathbf{u}(x) + \mathbf{fv}(x)$.

Let $\mathbf{w}(x) = -\int \mathbf{v}(x)\, dx$.

GSF: $\mathbf{f}(x) = \mathbf{e}^{-\mathbf{w}(x)}[\int \mathbf{u}(x)\mathbf{e}^{\mathbf{w}(x)}\, dx + C]$.

(5) Equation $\mathbf{Df} = \mathbf{f}^n\mathbf{u}(x) + \mathbf{fv}(x)$.

Let $\mathbf{w}(x) = (n - 1)\int \mathbf{v}(x)\, dx$.

GSF: $\mathbf{f}(x) = \{\mathbf{e}^{-\mathbf{w}(x)}[(1 - n)\int \mathbf{u}(x)\mathbf{e}^{\mathbf{w}(x)}\, dx + C]\}^{1/(1-n)}$.

(6) Equation:

$$\mathbf{Df} = \frac{\mathbf{p}(x, \mathbf{f})}{\mathbf{q}(x, \mathbf{f})} \quad \text{(homogeneous)}.$$

Let

$$\mathbf{F}(v) = \int \frac{\mathbf{q}(1, v)\, dv}{\mathbf{p}(1, v) + v\mathbf{q}(1, v)}.$$

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$\mathbf{F}\left(\frac{y}{x}\right) + \ln x = C,$$

where $y = \mathbf{f}(x)$.

(7) Equation:

$$\mathbf{Df} = \frac{a_1x + b_1\mathbf{f} + c_1}{a_2x + b_2\mathbf{f} + c_2}.$$

Case 1: $a_1/a_2 = b_1/b_2$.

Let $k = a_1/a_2$, $m = b_1 + a_1k$, and $n = b_1c_1 + a_1c_2$.

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$km(a_1x + b_1y) + (mc_2 - kn)\ln|m(a_1x + b_1y) + n| = m^2x + C,$$

where $y = \mathbf{f}(x)$.

Case 2: $a_1/a_2 \neq b_1/b_2$.

Let m and n be simultaneous solutions of the linear equations

$$a_1m + b_1n + c_1 = 0,$$
$$a_2m + b_2n + c_2 = 0.$$

Let $$\mathbf{F}(v) = \int \frac{(a_2 + b_2v)\,dv}{a_1 + (a_2 + b_1)v + b_2v^2}.$$

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$\mathbf{F}\left(\frac{y - n}{x - m}\right) + \ln|x - m| = C, \quad \text{where } y = \mathbf{f}(x).$$

(8) Equation:

$$\mathbf{Df} = \frac{\mathbf{M}(x, \mathbf{f})}{\mathbf{N}(x, \mathbf{f})} \quad \text{(exact)}.$$

Let

$$\mathbf{F}(x, \mathbf{f}) = \frac{\partial}{\partial \mathbf{f}} \int \mathbf{M}(x, \mathbf{f})\,dx,$$

$$\mathbf{G}(x, \mathbf{f}) = \frac{\partial}{\partial x} \int \mathbf{N}(x, \mathbf{f})\,d\mathbf{f}.$$

GSF:

(1) $\int \mathbf{M}(x, \mathbf{f})\,dx - \int [\mathbf{N}(x, \mathbf{f}) + \mathbf{F}(x, \mathbf{f})]\,d\mathbf{f} = C.$

(2) $\int \mathbf{N}(x, \mathbf{f})\,d\mathbf{f} - \int [\mathbf{M}(x, \mathbf{f}) + \mathbf{G}(x, \mathbf{f})]\,dx = C.$

(Use either form.)

(9) Equation of the form

$$\mathbf{Df} = \frac{\mathbf{P}(x, \mathbf{f})}{\mathbf{Q}(x, \mathbf{f})}, \quad \frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} \neq 0.$$

Integrating Factors

(9a) $$\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} = k\mathbf{Q}, \quad k \text{ a real number.}$$

IF: $\mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-kx}$.

(9b) $$\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} = -\frac{k\mathbf{Q}}{x}, \quad k \text{ a real number.}$$

IF: $\mathbf{R}(x, \mathbf{f}) = x^k$.

(9c) $$\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} = -\frac{k\mathbf{Q}}{\mathbf{f}}, \quad k \text{ a real number.}$$

IF: $\mathbf{R}(x, \mathbf{f}) = \mathbf{f}^k$.

(9d) $$\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} = -a\mathbf{Q} \cot ax.$$

IF: $\mathbf{R}(x, \mathbf{f}) = \sin ax$.

(9e) $$\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x} = a\mathbf{Q} \tan ax.$$

IF: $\mathbf{R}(x, \mathbf{f}) = \cos ax$.

(9f) $$\frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{Q}} = \mathbf{h}(x),$$

where $\mathbf{h}$ is a function of the single argument x.

IF: $\mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(x)\, dx}$.

(9g) $$\frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P}} = \mathbf{h}(\mathbf{f}),$$

where $\mathbf{h}$ is a function of the single argument $\mathbf{f}$.

IF: $\mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(\mathbf{f})\, d\mathbf{f}}$.

(9h) $$\frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P} + \mathbf{Q}} = \mathbf{h}(x + \mathbf{f}),$$

where $\mathbf{h}$ is a function of the single argument $x + \mathbf{f}$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u)\, du},$$

where $u = x + \mathbf{f}$.

$$\text{(9i)} \quad \frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{Q} - \mathbf{P}} = \mathbf{h}(x - \mathbf{f}),$$

where $\mathbf{h}$ is a function of the single argument $x - \mathbf{f}$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u)\, du},$$

where $\mathrm{u} = x - \mathbf{f}$.

$$\text{(9j)} \quad \frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P} + k\mathbf{Q}} = \mathbf{h}(kx + \mathbf{f}),$$

where $\mathbf{h}$ is a function of the single argument $kx + \mathbf{f}$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u)\, du},$$

where $u = kx + \mathbf{f}$.

$$\text{(9k)} \quad \frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{x\mathbf{P} + \mathbf{f}\mathbf{Q}} = \mathbf{h}(x\mathbf{f}),$$

where $\mathbf{h}$ is a function of the single argument $x\mathbf{f}$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u)\, du},$$

where $u = x\mathbf{f}$.

$$\text{(9l)} \quad \frac{x^2 \dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P}x - \mathbf{Q}\mathbf{f}} = \mathbf{h}\left(\frac{\mathbf{f}}{x}\right),$$

where $\mathbf{h}$ is a function of the single argument $\mathbf{f}/x$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u)\, du},$$

where $u = \mathbf{f}/x$.

(9m) $\mathbf{P}(x, \mathbf{f}) = \mathbf{f}\mathbf{h}_1(x\mathbf{f})$, and $\mathbf{Q}(x, \mathbf{f}) = x\mathbf{h}_2(x\mathbf{f})$,

where $\mathbf{h}_1$ and $\mathbf{h}_2$ are functions of the single argument $x\mathbf{f}$.

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = \frac{1}{x\mathbf{f}[\mathbf{h}_1(x\mathbf{f}) + \mathbf{h}_2(x\mathbf{f})]}.$$

(9n) $$\mathbf{P}(x, \mathbf{f}) = ax^p\mathbf{f} + b\mathbf{f}^{q+1} = (ax^p + b\mathbf{f}^q)\mathbf{f},$$
$$\mathbf{Q}(x, \mathbf{f}) = cx^{p+1} + dx\mathbf{f}^q = (cx^p + d\mathbf{f}^q)x,$$

where $ad \neq bc$.

Let

$$m = -1 + \frac{c(pb - qa)}{ad - bc}, \quad n = -1 + \frac{b(qc - pd)}{ad - bc}.$$

$$\text{IF:} \quad \mathbf{R}(x, \mathbf{f}) = x^m\mathbf{f}^n.$$

12.10 EXAMPLES

(1) $$\frac{\mathbf{Df}}{\mathbf{f}} = \frac{x - 1}{x^2(1 - \mathbf{f})}.$$

Solve for **Df**, obtaining

$$\mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \frac{(x - 1)\mathbf{f}}{x^2(1 - \mathbf{f})}$$
$$= \mathbf{h}(x)\mathbf{j}(\mathbf{f}) = \frac{x - 1}{x^2}\frac{\mathbf{f}}{1 - \mathbf{f}},$$

which falls under (3) of Sec. 12.8. Hence, use (3) of Table 12.9. The general solution form is

$$\int \frac{1 - \mathbf{f}}{\mathbf{f}}\,d\mathbf{f} = \int \frac{x - 1}{x^2}\,dx + C,$$

$$\ln|\mathbf{f}| - \mathbf{f} = \ln|x| + \frac{1}{x} + C.$$

(2) $3x\mathbf{Df} = 3x\mathbf{f}^4 \ln x + \mathbf{f}.$

Solve for **Df**, obtaining

$$\mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \frac{3x\mathbf{f}^4 \ln x + \mathbf{f}}{3x} = \mathbf{f}^4 \ln x + \frac{\mathbf{f}}{3x},$$

which falls under (5) of Sec. 12.8. Hence, use (5) of Table 12.9. Let

$$\mathbf{u}(x) = \ln x, \qquad \mathbf{v}(x) = \frac{1}{3x}, \qquad n = 4.$$

Let

$$\mathbf{w}(x) = -3\int \frac{1}{3x}\,dx = -\int \frac{dx}{x} = -\ln|x|;$$

then,

$$\mathbf{e}^{\mathbf{w}(x)} = \frac{1}{x}.$$

The general solution form is

$$\mathbf{f}(x) = \left[\frac{1}{x} - 3\left(\int x \ln x\,dx + C\right)\right]^{-1/3}$$

$$= \left[\frac{1}{x} - 3\left(\frac{x^2}{2}\ln|x| - \frac{x^2}{4} + C\right)\right]^{-1/3},$$

$$\mathbf{f}(x) = \left(\frac{3}{4}x - \frac{3}{2}x\ln|x| + \frac{C}{x}\right)^{-1/3}.$$

(3) $\mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \dfrac{x\mathbf{f} - \mathbf{f}^2}{x^2 - 2x\mathbf{f}}.$

This equation falls under (6) of Sec. 12.8 with $n = 2$. Hence, use (6) of Table 12.9 with $\mathbf{p}(x, \mathbf{f}) = x\mathbf{f} - \mathbf{f}^2$ and $\mathbf{q}(x, \mathbf{f}) = x^2 - 2x\mathbf{f}$. Evaluate

$$\mathbf{F}(v) = \int \frac{(1 - 2v)\,dv}{v - v^2 + v(1 - 2v)} = \int \frac{1 - 2v}{v(2 - 3v)}\,dv.$$

This integral is evaluated by expressing the integrand as a sum of simpler fractions, using the technique of partial fractions.

$$\frac{1 - 2v}{v(2 - 3v)} = \frac{A}{v} + \frac{B}{2 - 3v}.$$

$$1 - 2v = 2A - 3Av + Bv.$$

$$A = \tfrac{1}{2}, \qquad B = -\tfrac{1}{2}.$$

Thus,

$$\mathbf{F}(v) = \int \left(\frac{1}{2v} - \frac{1}{2(2 - 3v)}\right) dv$$

$$= \tfrac{1}{2}\ln|v| + \tfrac{1}{6}\ln|2 - 3v|.$$

The solution is given implicitly by the equation

$$\tfrac{1}{2}\ln\left|\frac{y}{x}\right| + \tfrac{1}{6}\ln\left|2 - 3\frac{y}{x}\right| + \ln x = C,$$

where $y = \mathbf{f}(x)$.

(4) $\mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \dfrac{\cos x - 2x\mathbf{f}}{x^2 - 1}.$

Let $\mathbf{M}(x, \mathbf{f}) = \cos x - 2x\mathbf{f}$ and $\mathbf{N}(x, \mathbf{f}) = x^2 - 1$. Note that

$$\frac{\partial \mathbf{M}}{\partial \mathbf{f}} + \frac{\partial \mathbf{N}}{\partial x} = -2x + 2x = 0.$$

Thus, the equation falls under (8) of Sec. 12.8. Its solution is found in (8) of Table 12.9. Since

$$\int \mathbf{M}(x, \mathbf{f})\, dx = \sin x - \mathbf{f}x^2,$$

$$\mathbf{F}(x, \mathbf{f}) = \frac{\partial}{\partial \mathbf{f}}(\sin x - \mathbf{f}x^2) = -x^2.$$

The solution can be written

$$\int \mathbf{M}(x, \mathbf{f})\, dx - \int [\mathbf{N}(x, \mathbf{f}) + \mathbf{F}(x, \mathbf{f})]\, d\mathbf{f} = \sin x - \mathbf{f}x^2 - \int (x^2 - 1 - x^2)\, d\mathbf{f}$$
$$= \sin x - \mathbf{f}x^2 + \mathbf{f} = C.$$

Solving for $\mathbf{f}$,

$$\mathbf{f}(x) = \frac{\sin x - C}{x^2 - 1} = \frac{\sin x + C_1}{x^2 - 1}.$$

(5) $2x\mathbf{f}^3\mathbf{Df} + x^3\mathbf{f}^2\mathbf{Df} = x^3 + \mathbf{f}^4.$

Solving for $\mathbf{Df}$, we obtain

$$\mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \frac{x^3 + \mathbf{f}^4}{2x\mathbf{f}^3 + x^3\mathbf{f}^2}.$$

This differential equation satisfies none of the first eight conditions. Therefore, we look at (9) of Table 12.9 and seek an integrating factor. Letting $\mathbf{P}(x, \mathbf{f}) = x^3 + \mathbf{f}^4$ and $\mathbf{Q}(x, \mathbf{f}) = 2x\mathbf{f}^3 + x^3\mathbf{f}^2$, we find that condition (9f) is satisfied; for

$$\frac{\dfrac{\partial \mathbf{P}}{\partial \mathbf{f}} + \dfrac{\partial \mathbf{Q}}{\partial x}}{\mathbf{Q}} = \frac{4\mathbf{f}^3 + 2\mathbf{f}^3 + 3x^2\mathbf{f}^2}{2x\mathbf{f}^3 + x^3\mathbf{f}^2}$$
$$= \frac{3\mathbf{f}^2(2\mathbf{f} + x^2)}{x\mathbf{f}^2(2\mathbf{f} + x^2)} = \frac{3}{x}$$

is a function of x alone. Thus, $\mathbf{e}^{-\int (3/x)\,dx} = x^{-3}$ is an integrating factor. Letting $\mathbf{M}(x, \mathbf{f}) = x^{-3}\mathbf{P}(x, \mathbf{f})$ and $\mathbf{N}(x, \mathbf{f}) = x^{-3}\mathbf{Q}(x, \mathbf{f})$, we have

$$\mathbf{Df} = \frac{\mathbf{M}(x, \mathbf{f})}{\mathbf{N}(x, \mathbf{f})} = \frac{1 + \mathbf{f}^4 x^{-3}}{2\mathbf{f}^3 x^{-2} + \mathbf{f}^2},$$

which is exact. We solve for $\mathbf{f}$ by means of (10) of Table 12.9.

$$\int \mathbf{N}(x, \mathbf{f})\,d\mathbf{f} = \int (2x^{-2}\mathbf{f}^3 + \mathbf{f}^2)\,d\mathbf{f} = \frac{x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3}.$$

Then, $$\mathbf{G}(x, \mathbf{f}) = \frac{\partial}{\partial x}\left(\frac{x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3}\right) = -\mathbf{f}^4 x^{-3}.$$

The solution may now be written in the implicit form

$$\int \mathbf{N}\,d\mathbf{f} - \int (\mathbf{M} + \mathbf{G})\,dx = C.$$

$$\frac{-x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3} - \int (1 + \mathbf{f}^4 x^{-3} - \mathbf{f}^4 x^{-3})\,dx = \frac{-\mathbf{f}^4}{2x^2} + \frac{\mathbf{f}^3}{3} - x = C,$$

or $$3\mathbf{f}^4 + 6x^3 + 6Cx^2 - 2x^2\mathbf{f}^3 = 0.$$

12.11 LINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER

Linear differential equations of second order are written in the form

(1) $$[\mathbf{u}D^2 + \mathbf{v}D + \mathbf{w}]\mathbf{f} = \mathbf{t},$$

where $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{t}$ are given functions of x alone. The quadratic operator polynomials have coefficients which are functions of x. Such operators are handled in a way analogous to that in which those with constant coefficients are used.

The general solution of an equation of type (1) depends on the functions $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{t}$.

The entries of Table 12.12 are techniques of solving equations in this form for various classes of given functions. An equation must first be written in the above form. Next, it must be classified according to the types considered in Table 12.12. Finally, the procedure which leads to a general solution form as outlined in the appropriate entry of Table 12.12 should be followed.

Table 12.12 is far from exhaustive. Only the more elementary types of equations are considered here.

12.12 TABLE: GENERAL SOLUTION FORMS FOR LINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER

(1) Equation: $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = 0$, $\mathbf{v}$ and $\mathbf{w}$ constant functions.

GSF: See Table 11.4.

(2) Equation: $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = 0$, $\mathbf{v}$ is a polynomial of degree m, and $\mathbf{w}$ is a polynomial of degree n.

Let r = the greater of m and n. Use the method of undetermined coefficients to determine constants $A_0, A_1, \ldots, A_{r-1}, A_r, B_1$, and B_2, so that

$$\mathbf{u}_1 = A_0x^r + A_1x^{r-1} + \ldots + A_{r-1} + A_r$$

and

$$\mathbf{u}_2 = B_1\mathbf{e}^{B_2x}$$

are particular solutions of the equation. This is not always possible. If it can be done, then $\mathbf{u}_1$ and $\mathbf{u}_2$ are linearly independent.

GSF: $\mathbf{f}(x) = C_1\mathbf{u}_1(x) + C_2\mathbf{u}_2(x)$.

(3) Equation: $[\mathbf{u}\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = 0$.

Attempt to express $\mathbf{u}\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}$ as $[\mathbf{p}\mathbf{D} + \mathbf{q}][\mathbf{r}\mathbf{D} + \mathbf{s}]$, with the factors in that order, and $\mathbf{p}, \mathbf{q}, \mathbf{r}$, and $\mathbf{s}$ functions of x alone. These functions must satisfy the equations

$$\begin{aligned} \mathbf{pr} &= \mathbf{u}, \\ \mathbf{qr} + \mathbf{ps} + \mathbf{p} \cdot \mathbf{Dr} &= \mathbf{v}, \\ \mathbf{qs} + \mathbf{p} \cdot \mathbf{Ds} &= \mathbf{w}. \end{aligned}$$

This cannot always be done. If it can, let

$$\mathbf{h}(x) = -\int \frac{\mathbf{q}(x)}{\mathbf{p}(x)}\,dx.$$

GSF: Use (4) of Table 12.9 to solve for $\mathbf{f}(x)$ the equation

$$\mathbf{Df} = C_1\frac{\mathbf{e}^{\mathbf{h}(x)}}{\mathbf{r}(x)} - \frac{\mathbf{s}(x)}{\mathbf{r}(x)}\mathbf{f},$$

where C_1 is an arbitrary constant. Call the new arbitrary constant C_2.

(4) Equation: $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{t}$ are functions of x alone.

Use (1) or (2) above, or good guessing to find a nonzero particular solution $\mathbf{u}(x)$ of the reduced equation $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = 0$. Let $\mathbf{H}(x) = 2\,\mathbf{D}\mathbf{u}(x) + \mathbf{v}(x)\mathbf{u}(x)$. Use (4) of Table 12.9 to solve for $\mathbf{g}(x)$ the equation

$$\mathbf{D}\mathbf{g} = \frac{\mathbf{t}(x)}{\mathbf{u}(x)} - \frac{\mathbf{H}(x)}{\mathbf{u}(x)}\,\mathbf{g},$$

calling the arbitrary constant C_1.

GSF: $\mathbf{f}(x) = \mathbf{u}(x)[\int \mathbf{g}(x)\,dx + C_2]$.

(5) Equation: $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{t}$ are functions of x alone (method of variation of parameters).

Use (1) or (2) above or good guessing to find two linearly independent solutions $\mathbf{u}_1(x)$ and $\mathbf{u}_2(x)$ of $[\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = 0$.

Let $\mathbf{W}(x) = \mathbf{u}_1(x) \cdot \mathbf{D}\mathbf{u}_2(x) - \mathbf{u}_2(x) \cdot \mathbf{D}\mathbf{u}_1(x)$.

$$\text{GSF: } \mathbf{f}(x) = \mathbf{u}_2(x)\left[\int \frac{\mathbf{t}(x)\mathbf{u}_1(x)}{\mathbf{W}(x)}\,dx + C_1\right] - \mathbf{u}_1(x)\left[\int \frac{\mathbf{t}(x)\mathbf{u}_2(x)}{\mathbf{W}(x)}\,dx + C_2\right].$$

(6) Equation: $[\mathbf{u}\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{t}$ are functions of x alone, with $\mathbf{D}^2\mathbf{u} - \mathbf{D}\mathbf{v} - \mathbf{w} = 0$ (exact equation).

Let $\mathbf{H}(x) = \mathbf{v}(x) - \mathbf{D}\mathbf{u}(x)$ and $\mathbf{J}(x) = \int \mathbf{t}(x)\,dx + C_1$.

GSF: Solve by (4) of Table 12.9 for $\mathbf{f}(x)$ the equation

$$\mathbf{D}\mathbf{f} = \mathbf{J}(x) + \mathbf{H}(x)\mathbf{f},$$

calling the new arbitrary constant C_2.

(7) Equation: $[\mathbf{D}^2 + \mathbf{v}\mathbf{D}]\mathbf{f} = \mathbf{t}$, where $\mathbf{v}$ and $\mathbf{t}$ are functions of x alone.

Use (4) of Table 12.9 to solve for $\mathbf{g}(x)$ the equation

$$\mathbf{D}\mathbf{g} = \mathbf{t}(x) - \mathbf{v}(x)\mathbf{g},$$

calling the arbitrary constant C_1.

GSF: $\mathbf{f}(x) = \int \mathbf{g}(x)\,dx + C_2$.

12.13 NONLINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

The general form of the nonlinear equation of first degree is

$$\mathbf{F}(x, \mathbf{f}, \mathbf{Df}) = 0$$

where $\mathbf{F}$ is a function of three arguments. In general use of the techniques of Table 12.14, the argument character of $\mathbf{Df}$ is emphasized by replacing $\mathbf{Df}$ wherever it occurs by the single letter $\mathbf{p}$. The technique of solving these equations depends on the form of the function $\mathbf{F}$. The different forms of $\mathbf{F}$ considered in Table 12.16 are:

(1) $\mathbf{F}(x, \mathbf{f}, \mathbf{p})$ is a polynomial of degree n in $\mathbf{p}$. Write

$$[\mathbf{p} - \mathbf{G}_1(x, \mathbf{f})][\mathbf{p} - \mathbf{G}_2(x, \mathbf{f})] \ldots [\mathbf{p} - \mathbf{G}_n(x, \mathbf{f})]$$

for $\mathbf{F}(x, \mathbf{f}, \mathbf{p})$ [see (1) of Table 12.14].

(2) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for $\mathbf{f}$. Write $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (2) of Table 12.14].

(3) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for $\mathbf{f}$ and written $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ and $\mathbf{G}(x, \mathbf{p}) = \mathbf{p}x + \mathbf{H}(\mathbf{p})$ (Clairaut's equation). Write $\mathbf{f} = \mathbf{p}x + \mathbf{H}(\mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (3) of Table 12.14].

(4) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for $\mathbf{f}$ and written $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ and $\mathbf{G}(x, \mathbf{p}) = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$ (D'Alembert's equation). Write $\mathbf{f} = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (4) of Table 12.14].

(5) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for x. Write $x = \mathbf{G}(\mathbf{f}, \mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (5) of Table 12.14].

(6) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ contains no x and may be solved for $\mathbf{p}$. Write $\mathbf{p} = \mathbf{G}(\mathbf{f})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (6) of Table 12.14].

(7) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ contains no $\mathbf{f}$ and can be solved for $\mathbf{p}$. Write $\mathbf{p} = \mathbf{G}(x)$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (7) of Table 12.14].

For some equations of this type, the general solution form fails to represent all the particular solutions. A particular solution not included in the general solution form is called a **singular solution** (abbreviated SS). Singular solutions are tabulated for those equations in Table 12.14 for which the general solution form given does not include all particular solutions.

12.14 TABLE: GENERAL SOLUTION FORMS OF NONLINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

(1) Equation:

$$[\mathbf{p} - \mathbf{G}_1(x, \mathbf{f})][\mathbf{p} - \mathbf{G}_2(x, \mathbf{f})] \ldots [\mathbf{p} - \mathbf{G}_n(x, \mathbf{f})] = 0.$$

Use Table 12.9 to solve for $\mathbf{f}$ each of the equations

$$\mathbf{Df} = \mathbf{G}_i(x, \mathbf{f}), \qquad 1 \leqq i \leqq n,$$

and write each solution in the form $\mathbf{H}_i(x, \mathbf{f}, C) = 0$, using the same arbitrary constant in each solution. This defines $\mathbf{f}$ implicitly.

GSF: $\mathbf{H}_1(x, y, C)\mathbf{H}_2(x, y, C) \ldots \mathbf{H}_n(x, y, C) = 0,$

where $y = \mathbf{f}(x)$.

(2) Equation: $\mathbf{f} = G(x, \mathbf{p})$.

Let $\mathbf{M}(x, \mathbf{p}) = (\partial/\partial x)\mathbf{G}(x, \mathbf{p})$ and $\mathbf{N}(x, \mathbf{p}) = (\partial/\partial \mathbf{p})\mathbf{G}(x, \mathbf{p})$. Use Table 12.9 to solve for $\mathbf{p}$ the equation

$$\mathbf{Dp} = \frac{\mathbf{p} - \mathbf{M}(x, \mathbf{p})}{\mathbf{N}(x, \mathbf{p})},$$

writing the solution in the form $\mathbf{H}(x, \mathbf{p}, C) = 0$.

GSF: Eliminate $\mathbf{p}$ between the equations

$$\mathbf{f} = \mathbf{G}(x, \mathbf{p}),$$
$$\mathbf{H}(x, \mathbf{p}, C) = 0.$$

(3) Equation: $\mathbf{f} = \mathbf{p}x + \mathbf{H}(\mathbf{p})$.

GSF: $\mathbf{f}(x) = Cx + \mathbf{H}(C)$.

SS: Eliminate C between the equations

$$\mathbf{f}(x) = Cx + \mathbf{H}(C),$$
$$x + \mathbf{DH}(C) = 0.$$

(4) Equation: $\mathbf{f} = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$.

Let $\mathbf{M}(\mathbf{p}) = \dfrac{\mathbf{DH}(\mathbf{p})}{\mathbf{p} - \mathbf{H}(\mathbf{p})}$ and $\mathbf{N}(\mathbf{p}) = \dfrac{\mathbf{DJ}(\mathbf{p})}{\mathbf{p} - \mathbf{H}(\mathbf{p})}$. Solve for $\mathbf{g}$ by (4) of Table 12.9 the equation

$$\mathbf{Dg} = \mathbf{N}(x) - \mathbf{M}(x)\mathbf{g},$$

writing the solution in the form $\mathbf{K}(x, \mathbf{g}, C) = 0$

GSF: Eliminate $\mathbf{p}$ between the equations

$$\mathbf{f}(x) = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p}),$$
$$\mathbf{K}(\mathbf{p}, x, C) = 0.$$

(5) Equation: $x = \mathbf{G}(\mathbf{f}, \mathbf{p})$.

Let $\mathbf{M}(\mathbf{f}, \mathbf{p}) = x(\partial/\partial\mathbf{f})\mathbf{G}(\mathbf{f}, \mathbf{p})$ and $\mathbf{N}(\mathbf{f}, \mathbf{p}) = (\partial/\partial\mathbf{p})\mathbf{G}(\mathbf{f}, \mathbf{p})$. Solve for $\mathbf{p}$ by the methods of Table 12.9 the equation

$$\mathbf{Dp} = \frac{1 - \mathbf{pM}(x, \mathbf{p})}{\mathbf{p}^2\mathbf{N}(x, \mathbf{p})},$$

writing the solution in the form $\mathbf{H}(x, \mathbf{p}, C) = 0$.

GSF: Eliminate $\mathbf{p}$ between the equations

$$x = \mathbf{G}(\mathbf{f}, \mathbf{p}),$$
$$\mathbf{H}(\mathbf{f}, \mathbf{p}, C) = 0.$$

(6) Equation: $\mathbf{p} = \mathbf{G}(\mathbf{f})$.

GSF: $\displaystyle\int \frac{d\mathbf{f}}{\mathbf{G}(\mathbf{f})} = x + C.$

(7) Equation: $\mathbf{p} = \mathbf{G}(x)$.

GSF: $\mathbf{f}(x) = \displaystyle\int \mathbf{G}(x)\, dx + C.$

13

DIFFERENCES OF FUNCTIONS

DIFFERENCES OF many functions whose domains are nonnegative integers may be found by consulting Sec. 13.1 and Table 13.2. The first of these furnishes formulas for differencing algebraic combinations of simpler functions; the second furnishes the differences of some basic functions from which algebraic combinations can be formed. For a discussion of the factorial function $x^{(n)}$ and the general procedure for differencing polynomials, see Sec. 5.6.

13.1 TABLE: PROPERTIES OF DIFFERENCES

(1) $$\Delta(c\mathbf{f}(x)) = c\Delta\mathbf{f}(x), \quad c \text{ a real number.}$$

(2) $$\Delta(\mathbf{f}(x) \pm \mathbf{g}(x)) = \Delta\mathbf{f}(x) \pm \Delta\mathbf{g}(x).$$

(3) $$\Delta(\mathbf{f}(x) \cdot \mathbf{g}(x)) = \mathbf{E}\mathbf{f}(x) \cdot \Delta\mathbf{g}(x) + \mathbf{g}(x) \cdot \Delta\mathbf{f}(x).$$

(4) $$\Delta\left(\frac{\mathbf{f}(x)}{\mathbf{g}(x)}\right) = \frac{\mathbf{g}(x) \cdot \Delta\mathbf{f}(x) - \mathbf{f}(x) \cdot \Delta\mathbf{g}(x)}{\mathbf{g}(x) \cdot \mathbf{E}\mathbf{g}(x)}.$$

13.2 TABLE: FIRST DIFFERENCES

(1) $\Delta x^{(n)} = nx^{(n-1)}$.

(2) $\Delta(ax + b)^{(n)} = an(ax + b)^{(n-1)}$.

(3) $$\Delta \frac{1}{(x + n - 1)^{(n)}} = \frac{-n}{(x + n)^{(n+1)}}.$$

(4) $\Delta\binom{x}{n} = \binom{x}{n-1}$.

(5) $\Delta(x!) = x(x!)$.

(6) $\Delta \sin (ax + b) = 2 \sin \frac{a}{2} \cos \left(ax + b + \frac{a}{2}\right)$.

(7) $\Delta \cos (ax + b) = -2 \sin \frac{a}{2} \sin \left(ax + b + \frac{a}{2}\right)$.

(8) $\Delta \tan (ax + b) = \dfrac{\sin a}{\cos (ax + b) \cos (ax + a + b)}$.

(9) $\Delta \cot (ax + b) = \dfrac{-\sin a}{\sin (ax + b) \sin (ax + a + b)}$.

(10) $\Delta \sec (ax + b) = \dfrac{2 \sin (a/2) \sin (ax + b + a/2)}{\cos (ax + b) \cos (ax + a + b)}$.

(11) $\Delta \csc (ax + b) = \dfrac{-2 \sin (a/2) \cos (ax + b + a/2)}{\sin (ax + b) \sin (ax + a + b)}$.

(12) $\Delta c^x = (c - 1)c^x$.

(13) $\Delta 2^x = 2^x$.

(14) $\Delta c^{ax+b} = (c^a - 1)c^{ax+b}$.

(15) $\Delta \log_c x = \log_c \left(1 + \frac{1}{x}\right)$.

(16) $\Delta \log_c (ax + b) = \log_c \left(1 + \frac{a}{ax + b}\right)$.

14

INDEFINITE FINITE INTEGRALS, FINITE SUMS, INFINITE SERIES

INDEFINITE FINITE integrals of many functions whose domains are nonnegative integers may be found by consulting Tables 14.1 and 14.2. The first of these lists formulas for finding indefinite finite integrals of certain algebraic combinations of simpler functions; the second furnishes the indefinite finite integrals of some basic functions from which algebraic combinations can be formed.

A **finite sum** of a function $\mathbf{g}(x)$, defined for nonnegative integers, is an expression of the form

$$\sum_{a}^{b} \mathbf{g}(x) = \mathbf{g}(a) + \mathbf{g}(a+1) + \ldots + \mathbf{g}(n), \quad n \geqq a.$$

It is sometimes possible to evaluate a finite sum with the aid of an antidifference of the function being summed (see Sec. 8.5). Table 14.3 gives algebraic properties of finite sums, and specific finite sums appear in Table 14.4.

14.1 TABLE: ALGEBRAIC PROPERTIES OF INDEFINITE FINITE INTEGRALS

(1) $\Delta^{-1}(c\mathbf{f}(x)) = c\Delta^{-1}\mathbf{f}(x)$, c a real number.

(2) $\Delta^{-1}(\mathbf{f}(x) \pm \mathbf{g}(x)) = \Delta^{-1}\mathbf{f}(x) \pm \Delta^{-1}\mathbf{g}(x)$.

(3) $\Delta^{-1}(\mathbf{f}(x) \cdot \Delta\mathbf{g}(x)) = \mathbf{f}(x) \cdot \mathbf{g}(x) - \Delta^{-1}(\mathbf{E}\mathbf{g}(x) \cdot \Delta\mathbf{f}(x))$.

14.2 TABLE: INDEFINITE FINITE INTEGRALS

(1) $\Delta^{-1}x^{(n)} = \dfrac{x^{(n+1)}}{n+1} + C, \quad n \neq -1.$

(2) $\Delta^{-1}(ax+b)^{(n)} = \dfrac{(ax+b)^{(n+1)}}{a(n+1)} + C, \quad n \neq -1.$

(3) $\Delta^{-1}\dfrac{1}{(x+n-1)^{(n)}} = \dfrac{1}{(1-n)(x+n-2)^{(n-1)}} + C, \quad n \neq 1.$

(4) $\Delta^{-1}\binom{x}{n} = \binom{x}{n+1} + C.$

(5) $\Delta^{-1}x(x!) = x! + C.$

(6) $\Delta^{-1}\,\mathbf{sin}\,(ax+b) = \dfrac{-\mathbf{cos}\,(ax+b-a/2)}{2\,\mathbf{sin}\,(a/2)} + C, \quad a \neq 2n\pi.$

(7) $\Delta^{-1}\,\mathbf{cos}\,(ax+b) = \dfrac{\mathbf{sin}\,(ax+b-a/2)}{2\,\mathbf{sin}\,a/2} + C, \quad a \neq 2n\pi.$

(8) $\Delta^{-1}(\mathbf{sec}\,(ax+b)\,\mathbf{sec}\,(ax+a+b)) = \mathbf{csc}\,a\,\mathbf{tan}\,(ax+b) + C,$
$a \neq n\pi.$

(9) $\Delta^{-1}(\mathbf{csc}\,(ax+b)\,\mathbf{csc}\,(ax+a+b)) = -\mathbf{csc}\,a\,\mathbf{cot}\,(ax+b) + C,$
$a \neq n\pi.$

(10) $\Delta^{-1}\dfrac{\mathbf{sin}\,(ax+b+a/2)}{\mathbf{cos}\,(ax+b)\,\mathbf{cos}\,(ax+a+b)} = \dfrac{\mathbf{sec}\,(ax+b)}{2\,\mathbf{sin}\,(a/2)} + C,$
$a \neq 2n\pi.$

(11) $\Delta^{-1}\dfrac{\mathbf{cos}\,(ax+b+a/2)}{\mathbf{sin}\,(ax+b)\,\mathbf{sin}\,(ax+a+b)} = \dfrac{-\mathbf{csc}\,(ax+b)}{2\,\mathbf{sin}\,(a/2)} + C,$
$a \neq 2n\pi.$

(12) $\Delta^{-1}\mathbf{c}^x = \dfrac{\mathbf{c}^x}{c-1} + C, \quad c \neq 1.$

(13) $\Delta^{-1}\mathbf{2}^x = \mathbf{2}^x + C.$

(14) $\Delta^{-1}\mathbf{c}^{ax+b} = \dfrac{\mathbf{c}^{ax+b}}{c^a - 1} + C, \quad c \neq 1,\ a \neq 0.$

14.3 TABLE: PROPERTIES OF FINITE SUMS

(1) $\sum_{a}^{b} c\mathbf{g}(x) = c\sum_{a}^{b} \mathbf{g}(x)$, c a real number.

(2) $\sum_{a}^{b} [\mathbf{g}_1(x) \pm \mathbf{g}_2(x)] = \sum_{a}^{b} \mathbf{g}_1(x) \pm \sum_{a}^{b} \mathbf{g}_2(x)$.

(3) $\sum_{a}^{a} \mathbf{g}(x) = \mathbf{g}(a)$.

(4) $\sum_{a}^{c} \mathbf{g}(x) = \sum_{a}^{b} \mathbf{g}(x) + \sum_{b+1}^{c} \mathbf{g}(x)$, $a \leqq b < c$.

14.4 TABLE: FINITE SUMS

(1) $\sum_{1}^{n} \mathbf{c} = nc$, c a real number.

(2) $\sum_{1}^{n} x = \frac{n(n+1)}{2}$.

(3) $\sum_{1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$.

(4) $\sum_{1}^{n} x^3 = \frac{n^2(n+1)^2}{4}$.

(5) $\sum_{1}^{n} \frac{1+2+\ldots+x}{x} = \frac{n(n+3)}{4}$.

(6) $\sum_{1}^{n} \frac{1+2^2+\ldots+x^2}{x} = \frac{n(4n^2+15n+17)}{36}$.

(7) $\sum_{1}^{n} \frac{1}{1+2+\ldots+x} = \frac{2n}{n+1}$.

(8) $\sum_{1}^{n} \frac{1}{x(x+1)} = \frac{n}{n+1}$.

(9) $\sum_{1}^{n} x(x!) = (n+1)! - 1$.

(10) $\sum_{1}^{n} (x^2+1)x! = n(n+1)!$.

(11) $\sum_{1}^{n} \binom{n}{x} = 2^n - 1$.

(12) $\sum_{a+1}^{n} \binom{x}{a} = \binom{n+1}{a+1} - 1.$

(13) $\sum_{1}^{n} \mathbf{c}^x = \frac{c}{c-1}(c^n - 1).$

(14) $\sum_{1}^{n} x\mathbf{c}^x = \frac{nc^{n+1}}{c-1} - \frac{c(c^n-1)}{(c-1)^2}.$

(15) $\sum_{1}^{n} x^2\mathbf{c}^x = \frac{n^2c^{n+1}}{c-1} - \frac{2nc^{n+1}}{(c-1)^2} + \frac{c(c+1)(c^n-1)}{(c-1)^3}.$

(16) $\sum_{1}^{n} \mathbf{c}^{x+p} = \frac{c^{p+1}}{c-1}(c^n - 1).$

(17) $\sum_{1}^{n} x\mathbf{c}^{x+p} = \frac{nc^{n+p+1}}{c-1} - \frac{c^{p+1}(c^n-1)}{(c-1)^2}.$

(18) $\sum_{1}^{n} x^2\mathbf{c}^{x+p} = \frac{n^2c^{n+p+1}}{c-1} - \frac{2nc^{n+p+1}}{(c-1)^2} + \frac{c^{p+1}(c+1)(c^n-1)}{(c-1)^3}.$

(19) $\sum_{1}^{n} \mathbf{2}^x = 2(2^n - 1).$

(20) $\sum_{1}^{n} \mathbf{c}^{ax} = \frac{c^a}{c^a-1}(c^{an} - 1).$

(21) $\sum_{1}^{n} \mathbf{sin}\,(ax+b) = \frac{\mathbf{sin}\left(\frac{n+1}{2}a + b\right)\mathbf{sin}\,\frac{n}{2}a}{\mathbf{sin}\,(a/2)}, \quad a \neq 2n\pi.$

(22) $\sum_{1}^{n} \mathbf{cos}\,(ax+b) = \frac{\mathbf{cos}\left(\frac{n+1}{2}a + b\right)\mathbf{sin}\,\frac{n}{2}a}{\mathbf{sin}\,(a/2)}, \quad a \neq 2n\pi.$

(23) $\sum_{0}^{n} \binom{n}{x} \mathbf{a}^x\mathbf{b}^{n-x} = (a+b)^n$ (the binomial theorem).

14.5 INFINITE SERIES

An infinite series defined for a function $\mathbf{g}(x)$ whose domain consists of nonnegative integers is

$$\sum_{a}^{\infty} \mathbf{g}(x) = \lim_{n\to\infty} \sum_{a}^{n} \mathbf{g}(x).$$

We shall consider two problems for such series. The first is to determine whether this limit exists; if it does, the series is said to be **convergent.** (A series which is not convergent is called **divergent.**) The other problem is to evaluate the infinite sum if the series is convergent.

Two types of convergence arise, absolute and conditional. A series $\sum_a^\infty \mathbf{g}(x)$ is **absolutely convergent** if $\sum_a^\infty |\mathbf{g}(x)|$ is convergent. A convergent series which is not absolutely convergent is **conditionally convergent.**

Table 14.6 lists six tests for convergence or divergence. In some cases, the test may fail to indicate whether the series converges or diverges; one should then try another test.

14.6 TABLE: TESTS FOR CONVERGENCE OR DIVERGENCE OF $\sum_a^\infty \mathbf{g}(x)$

(1) *Term Test*

(a) If $\lim_{x\to\infty} \mathbf{g}(x) \neq 0$, then $\sum_a^\infty \mathbf{g}(x)$ is divergent.

(b) If $\lim_{x\to\infty} \mathbf{g}(x) = 0$, the test fails.

(2) *Comparison Test.* Let $\sum_a^\infty \mathbf{h}(x)$ be a convergent series of positive terms and $\sum_a^\infty \mathbf{k}(x)$ be a divergent series of positive terms.

(a) If there exists a positive integer N such that

$$|\mathbf{g}(x)| \leqq \mathbf{h}(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty \mathbf{g}(x)$ converges absolutely.

(b) If there exists a positive integer N such that

$$\mathbf{g}(x) \geqq \mathbf{k}(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty \mathbf{g}(x)$ diverges.

(c) If there exists a positive integer N such that

$$|\mathbf{g}(x)| \geqq \mathbf{k}(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty \mathbf{g}(x)$ does not converge absolutely, though it may converge conditionally.

(3) *Ratio Test.* Let $r = \lim_{x\to\infty} \left|\frac{\mathbf{g}(x+1)}{\mathbf{g}(x)}\right|$.

(a) If $r < 1$, then $\sum_a^\infty \mathbf{g}(x)$ converges absolutely.

(b) If $r > 1$, then $\sum_a^\infty \mathbf{g}(x)$ diverges.

(c) If $r = 1$, the test fails.

(4) *Root Test.* Let $r = \lim_{x\to\infty} [\mathbf{g}(x)]^{1/x}$.

(a) If $r < 1$, then $\sum_a^\infty \mathbf{g}(x)$ converges absolutely.

(b) If $r > 1$, then $\sum_a^\infty \mathbf{g}(x)$ diverges.

(c) If $r = 1$, the test fails.

(5) *Integral Test.* Let $\mathbf{h}(x)$ be a real-valued function such that $\mathbf{h}(x) = |\mathbf{g}(x)|$ for each x in the domain of $\mathbf{g}$. If $\mathbf{h}(x)$ has the property that, for some positive integer N, $\mathbf{h}(x + c) < \mathbf{h}(x)$ whenever $x > N$ and $c > 0$, determine whether $\lim_{b\to\infty} \int_a^b \mathbf{h}(x)\,dx$ exists, where $a > N$.

(a) If the limit exists, then $\sum_a^\infty \mathbf{g}(x)$ is absolutely convergent.

(b) If $\mathbf{g}(x)$ is a series of positive terms and the limit fails to exist, then $\sum_a^\infty \mathbf{g}(x)$ is divergent.

(c) If $\mathbf{g}(x)$ is not a series of positive terms and the limit fails to exist, then $\sum_a^\infty \mathbf{g}(x)$ is not absolutely convergent, though it may be conditionally convergent.

(6) *Alternating Series Test*

(a) If there exists a positive integer N such that

(i) the signs of $\mathbf{g}(x)$ and $\mathbf{g}(x+1)$ are opposite for every $x > N$

(ii) $|\mathbf{g}(x+1)| < |\mathbf{g}(x)|$ for every $x > N$, and if

(iii) $\lim_{x\to\infty} \mathbf{g}(x) = 0$,

then $\sum_{a}^{\infty} \mathbf{g}(x)$ converges.

(b) If at least one of the conditions (ii) and (iii) fails when (i) holds, then the series $\sum_{a}^{\infty} \mathbf{g}(x)$ diverges.

14.7 TABLE: INFINITE SERIES

Convergent Series

(1) $\sum_{1}^{\infty} \frac{(-\mathbf{1})^{x-1}}{x}$. Sum: $\mathbf{ln}\ 2$.

(2) $\sum_{1}^{\infty} \frac{(-\mathbf{1})^{x-1}}{2x-1}$. Sum: $\frac{\pi}{4}$.

(3) $\sum_{1}^{\infty} \frac{1}{x(x+1)}$. Sum: 1.

(4) $\sum_{1}^{\infty} \frac{1}{x(x+1)(x+2)}$. Sum: $\frac{1}{4}$.

(5) $\sum_{0}^{\infty} \frac{1}{x!}$. Sum: e.

(6) $\sum_{1}^{\infty} \frac{1}{x^r}$, $r > 1$.

(7) $\sum_{1}^{\infty} \frac{1}{1+x^2}$.

(8) $\sum_{2}^{\infty} \frac{1}{(\mathbf{ln}\ x)^{\mathbf{ln}\ x}}$.

(9) $\sum_{0}^{\infty} \mathbf{a}^x$, $|a| < 1$. Sum: $\frac{1}{1-a}$.

(10) $\sum_{0}^{\infty} x\mathbf{a}^x$, $|a| < 1$. Sum: $\frac{a}{(1-a)^2}$.

(11) $\sum_{0}^{\infty} x^2\mathbf{a}^x$, $|a| < 1$. Sum: $\frac{a^2+a}{(1-a)^3}$.

(12) $\sum_{0}^{\infty} \mathbf{a}^{x+p}, \quad |a| < 1.$ Sum: $\dfrac{a^p}{1-a}$.

(13) $\sum_{0}^{\infty} x\mathbf{a}^{x+p}, \quad |a| < 1.$ Sum: $\dfrac{a^{p+1}}{(1-a)^2}$.

(14) $\sum_{0}^{\infty} x^2\mathbf{a}^{x+p}, \quad |a| < 1.$ Sum: $\dfrac{a^{p+2} + a^{p+1}}{(1-a)^3}$.

(15) $\sum_{2}^{\infty} \dfrac{1}{x(\mathbf{ln}\ x)^r}, \quad r > 1.$

(16) $\sum_{2}^{\infty} \dfrac{1}{x\ \mathbf{ln}\ x\ (\mathbf{ln}\ \mathbf{ln}\ x)^r}, \quad r > 1.$

Divergent Series

(17) $\sum_{1}^{\infty} \dfrac{1}{x}$.

(18) $\sum_{1}^{\infty} \dfrac{1}{x^r}, \quad r \leqq 1.$

(19) $\sum_{0}^{\infty} \mathbf{a}^x, \quad |a| \geqq 1.$

(20) $\sum_{1}^{\infty} \dfrac{1}{ax + b}$.

(21) $\sum_{2}^{\infty} \dfrac{1}{x\ (\mathbf{ln}\ x)^r}, \quad r \leqq 1.$

(22) $\sum_{2}^{\infty} \dfrac{1}{x(\mathbf{ln}\ x)(\mathbf{ln}\ \mathbf{ln}\ x)^r}, \quad r \leqq 1.$

14.8 PROBABILITY SUMS

$$\text{Let } \mathbf{g}(x) = \frac{\mathbf{e}^{-m}\mathbf{m}^x}{x!}.$$

(1a) $\sum_{0}^{\infty} \mathbf{g}(x) = 1.$

(1b) $\sum_{0}^{\infty} x\mathbf{g}(x) = m.$

(1c) $\sum_{0}^{\infty} x^2\mathbf{g}(x) = m + m^2.$

Let $\mathbf{g}(x) = (1 - p)\mathbf{p}^x, \quad 0 \leqq p < 1.$

(2a) $\sum_0^\infty \mathbf{g}(x) = 1.$

(2b) $\sum_0^\infty x\mathbf{g}(x) = \frac{p}{1 - p}.$

(2c) $\sum_0^\infty x^2\mathbf{g}(x) = \frac{p^2 + p}{(1 - p)^2}.$

Let $\mathbf{g}(x) = \binom{n}{x}\mathbf{p}^x(\mathbf{1} - \mathbf{p})^{n-x}, \quad 0 < p < 1.$

(3a) $\sum_0^\infty \mathbf{g}(x) = 1.$

(3b) $\sum_0^\infty x\mathbf{g}(x) = np.$

(3c) $\sum_0^\infty x^2\mathbf{g}(x) = n^2p^2 + np(1 - p).$

14.9 ANTIDIFFERENCES OF ORDER n

Antidifferences of order n, denoted by $\mathbf{J}^n\mathbf{g}(x)$, are listed in Table 14.10. The integrals are particular solutions of equations of the form $\Delta^n\mathbf{g} = 0$. The general solution form of such an equation is

$$\mathbf{J}^n\mathbf{g}(x) + C_0x^n + C_1x^{n-1} + \ldots + C_{n-1}x + C_n.$$

14.10 TABLE: ANTIDIFFERENCES OF ORDER n

(1) $\mathbf{J}^n\mathbf{c} = \frac{cx^{(n)}}{n!}, \quad c$ a real number.

(2) $\mathbf{J}^n x^{(k)} = \frac{x^{(k+n)}}{(k + n)^{(n)}}.$

(3) $\mathbf{J}^n(ax + b)^{(k)} = \frac{(ax + b)^{(k+n)}}{a^n(k + n)^{(n)}}.$

(4) $\mathbf{J}^n \frac{1}{(x + k - 1)^{(k)}} = \frac{(-1)^n}{(n - k)^{(n)}(x + k - n - 1)^{(k-n)}}, \quad n \leqq k.$

(5) $\mathbf{J}^n \binom{x}{k} = \binom{x}{n + k}.$

(6) $\mathbf{J}^n \sin(ax+b) = \dfrac{\sin(ax+b-na/2)}{2^n \sin^n(a/2)}$, if $n = 4m$, $a \neq 2n\pi$.

(7) $\mathbf{J}^n \sin(ax+b) = \dfrac{-\cos(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+1$, $a \neq 2n\pi$.

(8) $\mathbf{J}^n \sin(ax+b) = \dfrac{-\sin(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+2$, $a \neq 2n\pi$.

(9) $\mathbf{J}^n \sin(ax+b) = \dfrac{\cos(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+3$, $a \neq 2n\pi$.

(10) $\mathbf{J}^n \cos(ax+b) = \dfrac{\cos(ax+b-na/2)}{2^n \sin^n(a/2)}$, if $n = 4m$, $a \neq 2n\pi$.

(11) $\mathbf{J}^n \cos(ax+b) = \dfrac{\sin(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+1$, $a \neq 2n\pi$.

(12) $\mathbf{J}^n \cos(ax+b) = \dfrac{-\cos(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+2$, $a \neq 2n\pi$.

(13) $\mathbf{J}^n \cos(ax+b) = \dfrac{-\sin(ax+b-na/2)}{2^n \sin^n(a/2)}$,

if $n = 4m+3$, $a \neq 2n\pi$.

(14) $\mathbf{J}^n c^x = \dfrac{c^x}{(c-1)^n}$.

(15) $\mathbf{J}^n 2^x = 2^x$.

(16) $\mathbf{J}^n c^{ax} = \dfrac{c^{ax}}{(c^a-1)^n}$.

14.11 TAYLOR'S SERIES

Certain functions whose domains are real numbers can be written as infinite power series, called **Taylor's series** or **Taylor's expansions,** which may be used to approximate values of these functions for certain arguments. In general, the Taylor's expansion of a function does not converge over the whole domain of the function. The set of arguments for which the Taylor's expansion does converge occupies an interval of the real axis. This interval is called the **interval of convergence** of the expansion. The Taylor's series for a number of important functions are listed in Table 14.12, along with their intervals of convergence.

14.12 TABLE: TAYLOR'S SERIES

(1) $$\frac{1}{1 \pm x} = 1 \mp x + x^2 \mp x^3 + \mp \ldots, \quad |x| < 1.$$

(2) $$\frac{1}{(1 \pm x)^2} = 1 \mp 2x + 3x^2 \mp 4x^3 + \mp \ldots, \quad |x| < 1.$$

(3) $$\sqrt{1 + x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}x^4 + - \ldots, \quad |x| < 1.$$

(4) $$\frac{1}{\sqrt{1 + x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 - + \ldots, \quad |x| < 1.$$

(5) $$(a + x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + \ldots, \quad x^2 < a^2.$$

(6) $$\mathbf{sin}\ x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \ldots, \quad \text{for all values of } x.$$

(7) $$\mathbf{cos}\ x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - \ldots, \quad \text{for all values of } x.$$

(8) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots, \quad |x| < \frac{\pi}{2}.$

(9) $\arcsin x = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^7}{7} + \ldots, \quad |x| \leq 1.$

(10) $\arccos x = \frac{\pi}{2} - \arcsin x.$

(11) $\arctan x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + - \ldots, \quad |x| \leq 1.$

(12) $\ln x = (x - 1) - \tfrac{1}{2}(x - 1)^2 + \tfrac{1}{3}(x - 1)^3 - + \ldots, \quad 0 < x \leq 2.$

(13) $\ln (1 \pm x) = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} - \frac{x^4}{4} \pm - \ldots, \quad -1 < x \leq 1.$

(14) $\ln \frac{1 + x}{1 - x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\right), \quad |x| < 1.$

(15) $\ln (x + \sqrt{1 + x^2}) = x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + - \ldots, \quad |x| \leq 1.$

(16) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,$ for all values of x.

(17) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + - \ldots,$ for all values of x.

(18) $a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \ldots,$ for all values of x.

(19) $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots,$ for all values of x.

(20) $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots,$ for all values of x.

(21) $\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + - \ldots,$ for all values of x.

(22) $\sinh^{-1} x = x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + - \ldots, \quad |x| \leq 1.$

(23) $\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots, \quad |x| < 1.$

15

LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

THE GENERAL form of such an equation is

$$\mathbf{E}^n\mathbf{f}(x) + a_1\mathbf{E}^{n-1}\mathbf{f}(x) + \ldots + a_{n-1}\mathbf{E}\mathbf{f}(x) + a_n\mathbf{f}(x) = \mathbf{g}(x),$$

which we rewrite

$$[\mathbf{E}^n + a_1\mathbf{E}^{n-1} + \ldots + a_{n-1}\mathbf{E} + a_n]\mathbf{f}(x) = \mathbf{g}(x).$$

The expression in brackets is a polynomial in $\mathbf{E}$ which we shall denote by $\mathbf{p}(\mathbf{E})$. The difference equation now takes the form

$$[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}(x).$$

Any equation written in terms of the difference operator Δ may be rewritten in terms of $\mathbf{E}$, using the relations of Sec. 3.5.

If $\mathbf{g}(x) = 0$, the equation is said to be **homogeneous.** If $\mathbf{g}(x) \neq 0$, then the homogeneous equation $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = 0$ obtained by replacing $\mathbf{g}(x)$ by 0 is called the **reduced equation** of the given equation.

If $\mathbf{f}^*(x)$ is a solution of the given equation, then, for any other solution $\mathbf{f}_1^*(x)$, the function $\mathbf{f}_1(x) = \mathbf{f}_1^*(x) - \mathbf{f}^*(x)$ is a solution of its reduced equation, since

$$[\mathbf{p}(\mathbf{E})](\mathbf{f}_1^*(x) - \mathbf{f}^*(x)) = \mathbf{g}(x) - \mathbf{g}(x) = 0.$$

Thus, the general solution form of the given equation is found by add-

ing any particular solution of the given equation to the general solution form of the reduced equation. There are two major steps to the solution of the problem:

(1) Finding the general solution of the reduced equation.

(2) Finding a particular solution of the given equation.

15.1 THE AUXILIARY EQUATION

To obtain the general solution of a homogeneous linear difference equation with constant coefficients $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = 0$, we first solve the auxiliary equation

$$\mathbf{p}(m) = 0$$

where $\mathbf{p}(m)$ is the polynomial in m obtained by replacing $\mathbf{E}$ by m in $\mathbf{p}(\mathbf{E})$.

15.2 HOMOGENEOUS EQUATIONS OF FIRST ORDER

The first-order equation takes the form

$$[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = [\mathbf{E} - \mathbf{a}]\mathbf{f}(x) = 0, \quad a \text{ a real number.}$$

The general solution form is

$$\mathbf{f}(x) = C\mathbf{a}^x.$$

15.3 HOMOGENEOUS EQUATIONS OF SECOND ORDER

The general solution form of the second-order equation

$$[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = 0$$

is tabulated in Table 15.4 according to the nature of the roots of the auxiliary equation $\mathbf{p}(m) = 0$.

15.4 TABLE: GENERAL SOLUTION FORM OF THE LINEAR HOMOGENEOUS EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENTS

Let m_1 and m_2 be the roots of $\mathbf{p}(m) = 0$.

(1) $m_1 \neq m_2$, both real:

GSF: $\mathbf{f}(x) = C_1\mathbf{m}_1^x + C_2\mathbf{m}_2^x$.

(2) $m_1 = m_2$:

GSF: $\mathbf{f}(x) = (C_1 + C_2x)\mathbf{m}_1{}^x$.

(3) The roots of the auxiliary equation are complex: $m_1 = a + bi$, $m_2 = a - bi$. Let $\rho = \sqrt{a^2 + b^2}$ and $\theta = \textbf{arctan}\ (b/a)$.

GSF: $\mathbf{f}(x) = \boldsymbol{\rho}^x(C_1\ \textbf{sin}\ x\theta + C_2\ \textbf{cos}\ x\theta)$

or $\mathbf{f}(x) = C_1(\mathbf{a} + \mathbf{b}i)^x + C_2(\mathbf{a} - \mathbf{b}i)^x$.

15.5 EXAMPLES

(1) $[\mathbf{E}^2 + 2\mathbf{E} + 5]\mathbf{f} = 0$. We first solve the auxiliary equation $m^2 + 2m + 5 = 0$, obtaining $m = -1 \pm 2i$. Then $a = -1$, $b = 2$, $\rho = \sqrt{5}$, and $\theta = \textbf{arctan}\ (-2)$. The general solution form is

$$\mathbf{f}(x) = (\sqrt{5})^x(C_1\ \textbf{sin}\ x(\textbf{arctan} - 2) + C_2\ \textbf{cos}\ x(\textbf{arctan} - 2)).$$

(2) $[\mathbf{E}^2 + 3\mathbf{E} - 4]\mathbf{f} = 0$. Solving $m^2 + 3m - 4 = 0$, we obtain $m_1 = 1$ and $m_2 = -4$. The general solution form is

$$\mathbf{f}(x) = C_1(\mathbf{1})^x + C_2(\mathbf{-4})^x = C_1 + C_2(\mathbf{-4})^x.$$

(3) $[\mathbf{E}^2 + 6\mathbf{E} + 9]\mathbf{f} = 0$. The roots of $m^2 + 6m + 9 = 0$ are both -3. The general solution form is

$$\mathbf{f}(x) = (C_1 + C_2x)(\mathbf{-3})^x.$$

15.6 HOMOGENEOUS EQUATIONS OF ORDER n

To find the general solution form for the homogeneous equation of order n, $[\mathbf{p}(\mathbf{E})]\mathbf{f} = 0$, one should first find the roots of the auxiliary equation $\mathbf{p}(m) = 0$. Let the distinct real roots of this equation be denoted by $m_1, m_2, \ldots, m_k$, and let their multiplicities be $r_1, r_2, \ldots, r_k$, respectively. Let the pairs of distinct complex roots be denoted by $a_{k+1} \pm b_{k+1}i$, $a_{k+2} \pm b_{k+2}i$, $\ldots$, $a_{k+l}b_{k+l}i$, with multiplicities $r_{k+1}, r_{k+2}, \ldots, r_{k+l}$, respectively. The general solution form of the equation is the sum of the general solution forms of the equations

$$[(\mathbf{E} - \mathbf{m}_j)^{r_j}]\mathbf{f}(x) = 0, \quad 1 \leqq j \leqq k$$

$$[(\mathbf{E} - \mathbf{a}_j - \mathbf{b}_ji)^{r_j}(\mathbf{E} - \mathbf{a}_j + \mathbf{b}_ji)^{r_j}]\mathbf{f}(x) = 0, \quad k + 1 \leqq j \leqq k + l$$

where the arbitrary constants appearing in the solutions will be numbered so that all have different subscripts. These general solution forms are listed in Table 15.7.

15.7 TABLE: GENERAL SOLUTION FORMS FOR HOMOGENEOUS DIFFERENCE EQUATIONS OF ORDER n WITH CONSTANT COEFFICIENTS (See Sec. 15.6)

(1) m_j is real, and $r_j = 1$:

GSF: $\mathbf{f}_j(x) = C\mathbf{m}_j^x$.

(2) m_j is real, and $r_j > 1$:

GSF: $\mathbf{f}_j(x) = (C_1 + C_2x + \ldots + C_jx^{r_j-1})\mathbf{m}_j^x$.

(3) $a_j \pm b_ji$ are complex roots, and $r_j = 1$. Let $\rho_j = \sqrt{a_j^2 + b_j^2}$ and $\theta_j = \mathbf{arctan}\ (b_j/a_j)$.

GSF: $\mathbf{f}_j(x) = \rho_j^x(C_1\ \mathbf{sin}\ x\theta_j + C_2\ \mathbf{cos}\ x\theta_j)$.

or $\mathbf{f}_j(x) = C_1(\mathbf{a}_j + \mathbf{b}_ji)^x + C(\mathbf{a}_j - \mathbf{b}_ji)^x$.

(4) $a_j \pm b_ji$ are complex roots, and $r_j > 1$. Let $\rho_j = \sqrt{a_j^2 + b_j^2}$ and $\theta_j = \mathbf{arctan}\ (b_j/a_j)$.

GSF: $\mathbf{f}_j(x) = (C_1 + C_2x + \ldots + C_{r_j}x^{r_j-1})\rho_j^x\ \mathbf{sin}\ x\theta_j$
$+ (C_{r_j+1} + C_{r_j+2}x + \ldots + C_{2r_j}x^{r_j-1})\rho_j^x\ \mathbf{cos}\ x\theta_j$

or $\mathbf{f}_j(x) = (C_1 + C_2x + \ldots + C_{r_j}x^{r_j-1})(\mathbf{a}_j + \mathbf{b}_ji)^x$
$+ (C_{r_j+1} + C_{r_j+2}x + \ldots C_{2r_j}x^{r_j-1})(\mathbf{a}_j - \mathbf{b}_ji)^x$.

15.8 EXAMPLE

$[\mathbf{E}(\mathbf{E} - \mathbf{2})^3(\mathbf{E} + \mathbf{1})]\mathbf{f} = 0$. The roots (and their respective multiplicities) of $m(m - 2)^3(m + 1) = 0$ are

$$m_1 = 0, \qquad r_1 = 1,$$
$$m_2 = -1, \qquad r_2 = 1,$$
$$m_3 = 2, \qquad r_3 = 3.$$

GSF: $\mathbf{f}(x) = C(\mathbf{0})^x + C_1(-\mathbf{1})^x + (C_2 + C_3x + C_4x^2)\mathbf{2}^x$. Note that m_1 makes no contribution to the general solution form of the equation. This is because there is no solution for the equation $\mathbf{Ef} = 0$ except the function $\mathbf{f}(x) = 0$.

15.9 PARTICULAR SOLUTIONS OF LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

A particular solution of the equation $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}(x)$ can often be found if one can guess the form such a solution is likely to take. Table 15.10 lists a number of aids to good guesses for particular solutions. The constants A and B (with or without subscripts) are real numbers to be determined by substituting the particular solution $\mathbf{f}^*(x)$ into the given equation. The examples of Sec. 15.12 will illustrate how the table is used.

It will appear that it is necessary to check whether certain numbers are roots of the auxiliary equation $\mathbf{p}(m) = 0$. One should recall, however, that all roots of this equation were obtained in finding the general solution of the reduced equation. In particular, in each case, the number in question is a root if and only if $\mathbf{g}(x)$ is a solution of the reduced equation.

15.10 TABLE: PARTICULAR SOLUTIONS OF LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

(1) $\mathbf{g}(x) = k$, k a real number:

PS: $\mathbf{f}^*(x) = A$, unless 1 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = Ax^r$, if 1 is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(2) $\mathbf{g}(x) = \mathbf{X}$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = A_0 + A_1x + \ldots + A_sx^s$, unless 1 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r$, if 1 is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(3) $\mathbf{g}(x) = p \sin qx$:

PS: $\mathbf{f}^*(x) = A \sin qx + B \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A \sin qx + B \cos qx)x^r$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(4) $\mathbf{g}(x) = p \cos qx$:

PS: $\mathbf{f}^*(x) = A \sin qx + B \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A \sin qx + B \cos qx)x^r$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(5) $\mathbf{g}(x) = p\mathbf{q}^x$:

PS: $\mathbf{f}^*(x) = A\mathbf{q}^x$, unless q is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = Ax^r\mathbf{q}^x$, if q is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(6) $\mathbf{g}(x) = \mathbf{X}\mathbf{q}^x$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)\mathbf{q}^x$, unless q is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r\mathbf{q}^x$, if q is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(7) $\mathbf{g}(x) = \mathbf{X} \sin qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s) \sin qx + (B_0 + B_1x + \ldots B_sx^s) \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r \sin qx + (B_0 + B_1x + \ldots + B_sx^s)x^r \cos qx$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

(8) $\mathbf{g}(x) = \mathbf{X} \cos qx$, where $\mathbf{X}$ is a polynomial in x of degree s:

PS: $\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s) \sin qx + (B_0 + B_1x + \ldots + B_sx^s) \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \ldots + A_sx^s)x^r \sin qx + (B_0 + B_1x + \ldots + B_sx^s)x^r \cos qx$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $\mathbf{p}(m) = 0$ of multiplicity r.

15.11 REMARK ON THE EXTENSION OF TABLE 15.10

Suppose that $\mathbf{g}(x) = \mathbf{g}_1(x) + \mathbf{g}_2(x) + \ldots + \mathbf{g}_n(x)$, where particular solutions for $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}_i(x)$ can be found in Table 15.10 (or else-

where). Then a particular solution for $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}(x)$ can be obtained by adding the particular solutions of each of the equations $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}_i(x)$.

15.12 EXAMPLES

(1) $[\mathbf{E}^2 + 2\mathbf{E} + 5]\mathbf{f} = 5x^2 + 8x + 6$. Since $\mathbf{g}(x)$ is a polynomial of degree 2, and since 1 is not a root of $m^2 + 2m + 5 = 0$, formula (2) of Table 15.10 tells us that a particular solution of the equation has the form $A_0 + A_1x + A_2x^2$. Substituting this in the given equation, we obtain

$$(8A_0 + 4A_1 + 6A_2) + (8A_1 + 8A_2)x + 8A_2x^2 = 5x^2 + 8x + 6.$$

Equating like powers of x,

$$\begin{aligned} 8A_2 &= 5, \\ 8A_2 + 8A_1 &= 8, \\ 6A_2 + 4A_1 + 8A_0 &= 6. \end{aligned}$$

Solving, we get

$$A_0 = \tfrac{3}{32}, \qquad A_1 = \tfrac{3}{8}, \qquad A_2 = \tfrac{5}{8}.$$

Adding our particular solution to the general solution form found in Example 1 of Sec. 15.5, we find that the general solution form for the given equation is

$$\mathbf{f}(x) = (\sqrt{5})^x[C_1 \sin x(\arctan - 2) + C_2 \cos x(\arctan - 2)] + \frac{5}{8} + \frac{3x}{8} + \frac{3x^2}{32}.$$

(2) $[\mathbf{E}^2 + 6\mathbf{E} + 9]\mathbf{f} = (-3)^x$. In Example 3 of Sec. 15.5, we found that -3 is a double root of $m^2 + 6m + 9 = 0$. Thus, the particular solution according to (5) of Table 15.10 has the form $Ax^2(-3)^x$. Substituting this for $\mathbf{f}$ in the given equation, we obtain

$$A(-3)^2(-3)^x(x^2 + 4x + 4) + A(-3)(-3)^x(x^2 + 2x + 1) + A(-3)^x x^2 = (-3)^x.$$

The coefficients of $x^2(-3)^x$ and $x(-3)^x$ are zero. Equating coefficients of $(-3)^x$, we have $33A = 1$, or $A = \frac{1}{33}$. Using this particular solution and the general solution form found in Example 3 of Sec.

15.5, the general solution form of the given equation is

$$\mathbf{f}(x) = (C_1 + C_2x)(-3)^x + \tfrac{1}{3^3}x^2(-3)^x.$$

(3) $[\mathbf{E} - 3]\mathbf{f} = 2\,\mathbf{sin}\,2x$. Using (3) of Table 15.10, a particular solution of the equation has the form $A\,\mathbf{sin}\,2x + B\,\mathbf{cos}\,2x$. Substituting this into the equation, we obtain

$$A\,\mathbf{sin}\,(2x + 2) + B\,\mathbf{cos}\,(2x + 2) - 3A\,\mathbf{sin}\,2x - 3B\,\mathbf{cos}\,2x = 2\,\mathbf{sin}\,2x.$$

Using the argument sum properties of Sec. 6.4, we obtain

$$(A\,\mathbf{cos}\,2 - B\,\mathbf{sin}\,2 - 3A)\,\mathbf{sin}\,2x + (A\,\mathbf{sin}\,2 + B\,\mathbf{cos}\,2 - 3B)\,\mathbf{cos}\,2x = 2\,\mathbf{sin}\,2x.$$

Equating coefficients of $\mathbf{sin}\,2x$ and $\mathbf{cos}\,2x$ and solving for A and B, we get

$$A = \frac{\mathbf{cos}\,2 - 3}{5 - \mathbf{cos}\,2} = -0.63, \qquad B = \frac{-\mathbf{sin}\,2}{5 - \mathbf{cos}\,2} = -0.08$$

to two decimal places. Since the general solution form of the reduced equation is $C(3)^x$, the general solution form of the given equation is

$$\mathbf{f}(x) = C(3)^x - 0.63\,\mathbf{sin}\,2x - 0.08\,\mathbf{cos}\,2x.$$

16

OTHER DIFFERENCE EQUATIONS

(1) $\mathbf{E}f(x) - \mathbf{h}(x)f(x) = \mathbf{g}(x)$ (linear first-order equation).

Let $\mathbf{H}(x) = \mathbf{h}(x) \cdot \mathbf{h}(x-1) \ldots \mathbf{h}(0)$. If $\mathbf{H}(x)$ is never 0,

$$\text{GSF:} \quad \mathbf{f}(x) = \mathbf{H}(x-1)\left(\Delta^{-1}\frac{\mathbf{g}(x)}{\mathbf{H}(x)} + C\right).$$

If for some nonnegative integers $\mathbf{H}(x) = 0$, one can solve for $\mathbf{f}(x)$ in a domain of nonnegative integers $a \leqq x \leqq b$ for which $\mathbf{H}(x) \neq 0$. In this case, let $\mathbf{H}_a(x) = \mathbf{h}(x) \cdot \mathbf{h}(x-1) \ldots \mathbf{h}(a)$.

$$\text{GSF:} \quad \mathbf{f}(x) = \mathbf{H}_a(x-1)\left[\Delta^{-1}\frac{\mathbf{g}(x)}{\mathbf{H}_a(x)} + C\right].$$

(2) $\mathbf{f}(x) \cdot \mathbf{E}f(x) + af(x) + b = 0$, a and b real numbers.
Let m be a root of the equation $z^2 + za + b = 0$.
Let $\mathbf{h}(x)$ be the general solution form of

$$\left[\mathbf{E} + \frac{m}{m+a}\right]\mathbf{h}(x) = -\frac{1}{m+a}.$$

$$\text{GSF:} \quad \mathbf{f}(x) = \frac{1}{\mathbf{h}(x)} + m.$$

(3) $\mathbf{f}(x) \cdot \mathbf{E}f(x) + a\mathbf{E}f(x) + bf(x) + c = 0$, a, b, and c real numbers.

Case 1: $(a + b)^2 \neq 4c$. Let m and n be roots of the equation $z^2 - (a - b)z + c - ab = 0$

$$\text{GSF:}\quad \mathbf{f}(x) = \frac{C\mathbf{m}^{x+1} + \mathbf{n}^{x+1}}{C\mathbf{m}^x + \mathbf{n}^x} - a.$$

Case 2: $(a + b)^2 = 4c$.

$$\text{GSF:}\quad \mathbf{f}(x) = \frac{a - b}{2(C + x)} - \frac{a + b}{2}.$$

(4) $\mathbf{p}(x)\mathbf{f}(x) \cdot \mathbf{Ef}(x) + \mathbf{q}(x)\mathbf{Ef}(x) + \mathbf{r}(x)\mathbf{f}(x) = 0.$

Let $\mathbf{h}(x)$ be the general solution form of the equation.

$$\mathbf{Eh}(x) + \frac{\mathbf{q}(x)}{\mathbf{r}(x)}\,\mathbf{h}(x) = -\frac{\mathbf{p}(x)}{\mathbf{r}(x)}.$$

$$\text{GSF:}\quad \mathbf{f}(x) = \frac{1}{\mathbf{h}(x)}.$$

(5) $\mathbf{E}^2\mathbf{f}(x) + \mathbf{a}(x)\mathbf{Ef}(x) + \mathbf{b}(x)\mathbf{f}(x) = \mathbf{c}(x)$. Let $\mathbf{v}(x)$ be a particular solution of the reduced equation

$$\mathbf{E}^2\mathbf{f}(x) + \mathbf{a}(x)\mathbf{Ef}(x) + \mathbf{b}(x)\mathbf{f}(x) = 0.$$

This can frequently be found by trial and error. Let $\mathbf{p}(x) = 1 + \mathbf{aEv}(x)/\mathbf{E}^2\mathbf{v}(x)$ and $\mathbf{q}(x) = c/\mathbf{E}^2\mathbf{v}(x)$.

Let $\mathbf{h}(x)$ be the general solution form of the equation $\mathbf{Eh}(x) + \mathbf{p}(x)\mathbf{h}(x) = \mathbf{q}(x)$, and call the arbitrary constant C_1.

$$\text{GSF:}\quad \mathbf{f}(x) = \Delta^{-1}\mathbf{h}(x) + C_2.$$

(6) $a_0(\mathbf{Ef}(x))^n + a_1(\mathbf{Ef}(x))^{n-1}\mathbf{f}(x) + \mathbf{a}_2(\mathbf{Ef}(x))^{n-2}\mathbf{f}^2(x) + \ldots + a_n\mathbf{f}^n(x) = 0.$

Solve the equation

$$a_0m^n + a_1m^{n-1} + \ldots + a_{n-1}m + a_n = 0.$$

Let the distinct (not necessarily real) roots be denoted $m_1, m_2, \ldots, m_k$ and their multiplicities $r_1, r_2, \ldots, r_k$. Let

$$\mathbf{f}_i(x) = (C_1 + C_2x + \ldots + C_{r_i}x^{r_i-1})m_i^x.$$

GSF: $\mathbf{f}(x)$ is given implicitly by

$$[y - \mathbf{f}_1(x)][y - \mathbf{f}_2(x)] \ldots [y - \mathbf{f}_k(x)] = 0,$$

where $y = \mathbf{f}(x)$.

INDEX

A

Algebraic function, 4
  definition of, 7
  integrals of, 185
Alternating series, 239
  test for convergence of, 239
Antiderivative:
  definition of, 13
  of *n*th order, table, 205
Antidifference:
  definition of, 14
  of *n*th order, table, 242
Arbitrary constants, use of, 14
Arc of a circle, length of (inside front cover)
Arccosecant, definition of, 69
Arcosine:
  definition of, 69
  graph of, 160
Arccotangent:
  definition of, 69
  graph of, 160
Arcsecant, definition of, 69
Arcsine:
  definition of, 69
  graph of, 160
Arctangent:
  definition of, 69
  graph of, 160
Area of (see inside front cover for items below):
  circle
  ellipse
  parallelogram
  rectangle
  right circular cone
  right circular cylinder
  sector of circle
  sphere
  trapezoid
  triangle
Argument:
  functions of more than one argument, 215
  of a function, 1
Auxiliary equation:
  difference equations, 247
  differential equations, 207

B

Base of logarithms, 70
Bernoulli differential equation, 218, 219
Binomial coefficients:
  definition of, 19
  table of values of, 20
Boundary conditions, 17

C

Circle:
  arc length of (inside front cover)
  area of (inside front cover)
  circumference of (inside front cover)
  graph of, 151
  sector area of (inside front cover)
Circumference (see inside front cover for items below):
  circle
  ellipse
Clairaut differential equation, 229, 230
Classification list, differential equation:
  first-order linear, 218
  first-order nonlinear, 229

Coefficients, binomial (*see* Binomial coefficients)
Comparison test for convergence, 238
Composition of functions, 3
Conditions, boundary (*see* Boundary conditions)
Cone, right circular:
  area of lateral surface of (inside front cover)
  graph of, 171
  volume of (inside front cover)
Conic sections, 171
Constant coefficients:
  linear difference equations with, 246
  linear differential equations with, 206
Constants, arbitrary (*see* Arbitrary constants)
Constants, table of important (inside front cover)
Convergence and divergence, tests for, 238
Convergence of infinite series, 237
Convergent series, sums of, 240
Cosecant:
  definition of, 25
  graph of, 159
Cosine:
  definition of, 25
  graph of, 158
Cotangent:
  definition of, 25
  graph of, 159
Cubic functions, graphs of, 156
Cylinder:
  elliptic, graph of, 170
  hyperbolic, graph of, 170
  parabolic, graph of, 170
Cylinder, right circular:
  area of lateral surface of (inside front cover)
  graph of, 169
  volume of (inside front cover)

D

D'Alembert differential equation, 229, 230
D derivative, definition of, 9
Definite integrals, 173
  improper integrals, 202
  probability integrals, 204
  properties of, 202
  table of, 203
Definition of:
  algebraic functions, 7
  antiderivative, 13
  antidifference, 14
  base of logarithms, 70
  binomial coefficients, 19
  derivative, 9
  difference, 10
  factorial function, 18
  function, 1
  functional equation, 12
  homogeneous difference equation, 246
  homogeneous differential equation, 206
  hyperbolic function, 140
  indefinite finite integral, 14
  indefinite integral, 13
  polynomial, 4
  prime polynomial, 4
  rational function, 5
  reduced difference equations, 246
  reduced differential equation, 206
  transcendental function, 24
  translate, 9
Degree of polynomials, 4
Degrees-radians, 30
Dependence, linear, of functions, 216
Derivative **D**, definition of, 9
Derivatives, partial, 215
Derivatives, table of, 176
Difference Δ:
  definition of, 10
  relation to translate, 10
  table of, 232
Difference equation, 13
  auxiliary equation, 247
  homogeneous, 246
  linear, 175
  linear with constant coefficients, 246
    auxiliary equation, 247
    first-order homogeneous, table of solutions, 247
    homogeneous, 246
    miscellaneous, table of solutions, 254
    *n*th order homogeneous, table of solutions, 249
    operator polynomials, 10, 246
    order, 13
    particular solution, 250
    particular solution forms, table of, 250
    reduced equation, 246

second-order homogeneous, table of solutions, 247
nonlinear, 175
order, 13
reduced, 246
Differential equation, 12
auxiliary equation, 207
homogeneous, 206
linear, 174
linear first-order, 215, 217
classification list, 218
integrating factors, 221
table of general solutions, 219
linear second-order, table of solutions, 227
linear with constant coefficients, 174, 206
auxiliary equations, 207
*n*th order homogeneous, table of solutions, 209
operator polynomials, 10, 206
order, 12
particular solutions, 210
particular solution forms, table of, 210
reduced equation, 206
second-order homogeneous, table of solutions, 207
nonlinear, 174
nonlinear first-order:
classification list, 229
table of general solutions, 230
order, 12
reduced, 206
Divergence of infinite series, 237
tests for, 238
Domain of:
function, 1
inverse hyperbolic functions, 140
inverse trigonometric functions, 69

E

e:
definition of, 70
value of, 70, inside front cover
E, translate, definition of, 9
Ellipse:
area of (inside front cover)
circumference of (inside front cover)
graph of, 153
Ellipsoid, 167
Elliptic cylinder, 170
Elliptic paraboloid, 168
Equations:
auxiliary, difference equation, 247
auxiliary, differential equation, 207
difference (*see* Difference equation)
differential (*see* Differential equation)
functional (*see* Functional equations)
graphs of functions and, 149
homogeneous difference, 246
homogeneous differential, 206
reduced difference, 246
reduced differential, 206
Euler's constant (inside front cover)
Evaluating indefinite integrals, techniques for, 179
Exact differential equation:
first-order, 218, 220
second-order, 228
Explicitly defined functions, 16
Exponential function:
graphs of, 161
integrals of, 200
properties of, 140
Extending table of particular solution forms of:
linear difference equations, 251
linear differential equations, 212

F

Factorial function:
definition of, 18
table of values of, 19
Factorial powers, 20
relation to ordinary powers, 21
Factors, integrating, 219
table of, 221
of polynomials, 5
multiplicity of, 5
Finite integrals, indefinite (*see* Indefinite finite integrals)
Finite sums, 175, 234
properties of, 236
table of, 236
First-order differential equations:
linear (*see* Differential equations, linear first-order)
nonlinear (*see* Differential equations, nonlinear)
Fractions, partial, decomposition of rational functions into, 5

Functional equations, 12
  arbitrary constants, use of, 14
  boundary conditions, 17
  definition of, 12
  difference equations, 13
  differential equations, 12
  general solution form of, 12
  general solution of, 12
  particular solution of, 12
  types of, 173
Functions, 1
  algebraic definitions of, 4, 7
  binomial coefficients, 19
    table of values of, 20
  composition of, 3
  definition of, 1
  domain of, 1
  factorial, definition of, 18
    table of values of, 19
  factorial powers, 20
  graphs of, 149
  hyperbolic, definition, 140
    inverses of, 140
    properties of, 141
    table of values of, 142
  implicitly defined, 16
  inverses of, 8
  inverses of hyperbolic, 140
  inverses of trigonometric, 69
  linear dependence and independence of, 216
  logarithmic, definition of, 70
    properties of, 70
    table of values of, 72
  logarithms of trigonometric, table, 96
  of more than one argument, 215
  polynomial, definition of, 4
    degree of, 4
  range of, 1
  rational, definition of, 5
    partial fractions, decomposition of, 5
  transcendental, 24
  trigonometric, 24
    graphs of, 158
    integrals of, 191
    inverses of, 69
    properties of, 25
    table of integrals of, 191
    table of values of, 28
Function-function operations, 9
  antiderivative, 13
  antidifference, 14
  arbitrary constants, use of, 14
  derivative, 9
  difference, 10
  indefinite finite integral, 14
  indefinite integral, 13
  operator polynomials, 10
  partial derivatives, 215
  partial integrals, 216
  translate, 9

G

γ (inside front cover)
General solution, 12
General solution form, 12
General solution forms, table of:
  difference equations (*see* Difference equations)
  differential equations (*see* Differential equations)
Graphs of functions, 149
  three-dimensional linear, 166
  three-dimensional quadratic, 167
  two-dimensional cubic, 156
  two-dimensional linear, 149
  two-dimensional quadratic, 151
GSF, 206

H

Homogeneous:
  difference equation (*see* Difference equation)
  differential equation (*see* Differential equation)
Hyperbola, 154
Hyperbolic cylinder, 170
Hyperbolic functions:
  graphs of, 163
  integrals of, 201
  inverses of, 140
  properties of, 141
  table of values, 142
Hyperbolic paraboloid, 168
Hyperboloid, 169

I

Implicitly defined functions, 16
Improper integrals, 202

Indefinite finite integrals:
definition of, 14
properties of, 234
table of, 235
Indefinite integrals:
definition of, 13
properties of, 179
table, suggestions for use, 184
table of, 185
techniques for evaluating, 179
Independence, linear, 216
Infinite series:
convergence of, 237
divergence of, 237
probability sums, 241
table of, 240
Taylor's series, 244
tests for convergence and divergence, 238
Integrals, definite (*see* Definite integrals)
Integrals, indefinite (*see* Indefinite integrals)
Integrals, indefinite finite (*see* Indefinite finite integrals)
Integrals, partial, 216
Integrals for probability, table of, 204
Integral test for convergence, 239
Integrating factors, 219
table of, 221
Interval of convergence, Taylor's series, 244
Inverses of:
functions, 8
hyperbolic functions, 140
trigonometric functions, 69

L

Length of arc of a circle (inside front cover)
Line, graph of:
two dimensions, 149
three dimensions, 166
Linear dependence, 216
Linear difference equations (*see* Difference equations, linear)
Linear differential equations (*see* Differential equations, linear)
Linear fractional differential equation, 218, 220
Linear independence, 216
Logarithmic functions:
integrals of, 199
properties of, 70
tables of values of, 72
Logarithms of trigonometric functions, table, 96

M

Multiplicity of:
a factor of a polynomial, 5
a root of auxiliary equation, 208, 248

N

Nonhomogeneous linear differential equation (*see* Differential equation, linear, nonhomogeneous)
Nonlinear difference equation (*see* Difference equation, nonlinear)
Nonlinear differential equation (*see* Differential equation, nonlinear)
Nonlinear first-order differential equation (*see* Differential equation, nonlinear first-order)
*n*th order difference equation (*see* Difference equation)
*n*th order differential equation (*see* Differential equation)
Numbers, Stirling, 21
first kind, table of, 22
second kind, table of, 23
use of, 21

O

Operations, function-function (*see* Function-function operations)
Operator polynomials, 10, 206, 246
Order of:
difference equations, 13
differential equations, 12

P

Parabola, graph of, 152
Parabolic cylinder, graph of, 170
Paraboloid, elliptic, 168
hyperbolic, 168
Parallelepiped, rectangular, volume of (inside front cover)
Parallelogram, area of (inside front cover)
Parameters, variation of, 228
Partial derivatives, 215
Partial fractions decomposition, 5

Partial integration, 216
Particular solutions:
  of functional equations, 12
  of difference equations,
    linear table of, 250
      extension of, 251
  of differential equations,
    linear, table of, 210
      extension of, 213
π (inside front cover)
Plane, graph of, 166
Polynomials, 4
  degree of, 4
  factors of, 4
    multiplicity of, 4
  operator, 10, 206, 246
  prime, definition of, 4
Powers, factorial, 20
Power series, 244
Probability integrals, table of, 204
Probability sums, table of, 241
Properties of:
  definite integrals, 202
  finite sums, 236
  hyperbolic functions, 141
  indefinite finite integrals, 234
  indefinite integrals, 179
  logarithmic functions, 70
  trigonometric functions, 25
PS, 206
Pyramid, volume of (inside front cover)

R

Radian, table of degrees and, 30
Range:
  of a function, definition of, 1
  of inverse hyperbolic functions, 140
  of inverse trigonometric functions, 69
Rational functions, definition of, 5
  partial fractions decomposition of, 5
Ratio test for convergence, 239
Rectangle, area of (inside front cover)
Reduced difference equation, 246
Reduced differential equation, 206
Root test for convergence, 239

S

Secant, definition of, 25
  graph of, 159
Sections, conic, graphs of, 151
Sector of a circle, area of (inside front cover)
Separable differential equation, 218, 219
Series, infinite, 237
  alternating, 240
  convergence of, 237
  divergence of, 237
  power series, 244
  table of, 240
  Taylor's series, 244
    interval of convergence, 244
    table of, 244
  tests for convergence and divergence, 238
Sine, definition of, 25
  graph of, 158
Singular solution, 229
Solution form, general (*see* General solution form)
Solution, general:
  of difference equations (*see* Difference equations, general solution of)
  of differential equations (*see* Differential equations, general solution of)
  of functional equations (*see* Functional equations, general solution of)
Solution, particular:
  of difference equations (*see* Difference equations, particular solution of)
  of differential equations (*see* Differential equations, particular solution of)
  of functional equations (*see* Functional equations, particular solution of)
Solution, singular, 229
Sphere, area of surface of (inside front cover)
  graph of, 167
  volume of (inside front cover)
SS, 229
Stirling numbers, 21
  first kind, table of, 22
  second kind, table of, 23
  use of, 21
Stirling's formula, 18
Sums, finite, 175, 234
  probability, 241
  properties of, 236
  table of, 236

T

Table:
binomial coefficients, 20
definite integrals, 203
derivatives, 176
differences, 232
difference equations:
linear, with constant coefficients, first and second-order, homogeneous, general solution form, 247
*n*th order, homogeneous, general solution form, 249
particular solutions, 250
miscellaneous, 254
differential equations:
linear, with constant coefficients, first- and second-order, homogeneous, general solution form, 207
*n*th order, homogeneous, general solution form, 209
particular solutions, 210
linear first-order, general solution form, 219
integrating factors for, 221
with constant coefficients, homogeneous, general solution form, 209
linear second-order, general solution form, 227
nonlinear, first-order, 230
factorial function, values of, 19
finite sums, 236
hyperbolic functions, values of, 142
important constants, (inside front cover)
indefinite finite integrals, 235
indefinite integrals, 185
infinite series, 240
integrating factors for linear first-order differential equations, 221
inverses of trigonometric functions, 69
logarithmic functions, values of, 72
logarithms of trigonometric functions, 96
probability integrals, 204
probability sums, 241
Stirling numbers, first kind, 22
Stirling numbers, second kind, 23
Taylor's series, 244
trigonometric functions, values of, 96
logarithms of values of, 96
Tangent, definition of, 25
graph of, 158
Taylor's series, 244
interval of convergence, 244
table of, 244
Techniques for evaluating indefinite integrals, 179
Tests for convergence and divergence of infinite series, 238
Transcendental functions, definition of, 24
Translate **E**, definition of, 9
relation to difference, 11
Trapezoid, area of (inside front cover)
Triangle, area of (inside front cover)
Trigonometric functions, definition of, 24
graphs of, 158
integrals of, 191
inverses of, 69
properties of, 25
table of integrals of, 191
table of values of, 28
table of values of logarithms of, 96
Types of functional equations, 12, 173

U

Use of arbitrary constants, 14
Use of Stirling numbers, 21
Use of table of indefinite integrals, suggestions for, 184

V

Variation of parameters, 228
Volume of (*see* inside front cover for items below):
pyramid
rectangular parallelepiped
right circular cone
right circular cylinder
sphere